Effects of Exit Geometry on Turbulent Mixing in Supersonic Flows

by

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Introduction

The highlights of this presentation are:

- Turbulent Mixing and its Relevance to Volcanic Eruptions.
- Studies on Supersonic Nozzles of Complex Geometry
- Near Field flow using PLIF and 3D-CFD
- Far Field flow using Hot-Wire Anemometry
- Definition of entrainment from hot wire anemometry profiles.
- Implications of these studies to volcanic eruptions.
Turbulent Mixing in Volcanic Plumes

- Turbulent Entrainment and Mixing between the ejected material (volcanic gas and hot ash (solid)) from a volcanic eruption and the ambient air, is crucial in the dynamics and development of the plume and hence the kind of disaster it can cause (Suzuki and Koyaguchi, Earth Planets Space, 2013, Journal of Earth Simulator, 2007).

- The kind of volcanic plume completely dependent on:
  - Rate of density change with height.
  - Entrainment and Mixing with the ambient air.
Large entrainment and mixing between hot pyroclast and air causes global density of the plume to fall, buoyancy gives rise to very high plinian eruption.

Low entrainment does not decrease the global density and hence the eruption column collapses under gravity causing pyroclastic flow along the ground.
Explosive Volcanic Eruption as A Supersonic Free Jet

- High overpressures within the volcano causes a supersonic jet to develop. (Greg & Kenneth, 1989; Ogden et al. J. Geophysical Research, 2008)

- Effect of compressibility at high Mach numbers decreases mixing rate

- Nozzle exit geometry influences the mixing rate to a great extent due to production of streamwise vortices (Rao & Jagadeesh, ATE, 2015).
Based on flow features observed, the supersonic jet plume can be divided into the near field and the far field.

The near field is characterized by presence of shock structures, contains a supersonic core.

The far field is largely subsonic, distinct core is lost and highly turbulent.

Near Field flow (<6D) is investigated using PLIF technique, Pitot measurements.

Far field (>40D) flow using Hot Wire Anemometry.
Nozzles with complex shaped exits

(a) Type A: Conical Nozzle
(b) Type B: Beveled Conical Nozzle
(c) Type C: Serrated Nozzle
(d) Type D: ESTS Nozzle

4 different kinds of supersonic nozzles
Near Field Studies

- Experiments conducted at NPR=6.0
- Acetone PLIF @ 10 Hz, using Hamamatsu ICCD in streamwise and cross-sectional planes.
- CFD studies using FASTAR(JAXA), 8 million cells in the mesh, Spallart Almaras turbulence model.
Data Analysis

- Histogram based image decomposition of PLIF images.
- Helps to identify mixing layer growth.
- Steady flow of about 2 secs.

Histogram of Acetone PLIF image

Image Processing of PLIF images
PLIF results

- Type C and Type D nozzle show maximum difference of flow.
- Type D nozzle shows the highest increase of mixing rate.
- Exit flow star shaped in Type C, 6-lobed shape in Type D
- By about 8D the nozzles regain circular shapes
PLIF results

<table>
<thead>
<tr>
<th>Nozzle Type</th>
<th>Type A</th>
<th>Type B</th>
<th>Type C</th>
<th>Type D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth rate $dW/dZ$</td>
<td>0.072</td>
<td>0.099</td>
<td>0.159</td>
<td>0.308</td>
</tr>
<tr>
<td>% Increase</td>
<td>-</td>
<td>138%</td>
<td>222%</td>
<td>430%</td>
</tr>
</tbody>
</table>

Table of average growth rate of jet width vs Z

<table>
<thead>
<tr>
<th>Nozzle Type</th>
<th>Type A</th>
<th>Type B</th>
<th>Type C</th>
<th>Type D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z=15$ mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area (A)</td>
<td>$2.94\times10^4$</td>
<td>$3.93\times10^4$</td>
<td>$4.73\times10^4$</td>
<td>$5.85\times10^4$</td>
</tr>
<tr>
<td>$A/A_{Type A}$</td>
<td>1.00</td>
<td>1.34</td>
<td>1.61</td>
<td>1.99</td>
</tr>
<tr>
<td>$Z=40$ mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area (A)</td>
<td>$5.40\times10^4$</td>
<td>$4.60\times10^4$</td>
<td>$6.04\times10^4$</td>
<td>$1.24\times10^5$</td>
</tr>
<tr>
<td>$A/A_{Type A}$</td>
<td>1.00</td>
<td>0.85</td>
<td>1.12</td>
<td>2.29</td>
</tr>
</tbody>
</table>

Table of cross sectional areas (in pixel units) evaluated from cross-sectional PLIF images for the four nozzles.
Pitot Pressure and CFD

- Pitot pressures and their profiles from experiment and CFD are in good agreement.
- The density contours show flow features that are similar to the observations from PLIF.
Vorticity and its Production

- Streamwise vorticity high in Type C and Type D nozzles.
- Wide-spread vortices in Type D, clustered vortices in Type C.
- Pressure gradient across the lobes from throat to exit causes formation of large vortices.
- Shape of the nozzle exit plays an important role in determining entrainment and mixing rate near the nozzle exit.
The hot-wire anemometry is used for accurate velocity measurement with high frequency response.

- Heat transfer across a heated thin wire (≈ 5 µm) is related to the velocity.
- A fourth order polynomial fit relates voltage across the wire to velocity.
Turbulent Jet at Low velocities

- In the far field centerline velocity decays linearly
- The velocity profile is gaussian and self-similar in the far-field
- These observations match previous studies of Hussien et al. JFM, 1994.
Hot-Wire Anemometry in the Far Field

- Hot wire anemometry experiments are conducted at $X \geq 40D$
- The slope of linear profiles indicate the differences in centerline velocity decay
- Type D shows the maximum decay rate, then Type C. Type A and Type B are similar in magnitude.
- Velocity profiles in the far field are not very different.
- Type B shows the effects of slight bending by producing asymmetry of velocity profiles.
- Velocity profiles are very nearly similar in the far field. Velocity decay rate is affected by geometry and near field structure.
Entrainment ratio

- Knowing the velocity profiles the entrainment can be defined based on mass conservation

\[
E = \lim_{r \to \infty} (-2\pi r V) = \frac{d}{dx} \left[ 2\pi \int_0^\infty U_r \, dr \right] = 2\pi B(M_0)^{\frac{1}{2}} \int_0^\infty \left[ \frac{U}{U_c} \right] \eta \, d\eta.
\]

- The entrainment ratio is then defined as:

\[
\alpha = \frac{E'}{2bW},
\]

\[
b = I_1 x / (I_2)^{\frac{1}{3}},
\]

\[
W = (I_2 / I_1) B(M_0)^{\frac{1}{2}} / x,
\]

\[
E = I_1 B(M_0)^{\frac{1}{2}},
\]

\[
b^2 W = 2 \int_0^\infty U_r \, dr,
\]

\[
b^2 W^2 = 2 \int_0^\infty U^2 r \, dr,
\]

\[
E' = \frac{d}{dx} \left( I_1 \int_0^\infty U_r \, dr \right),
\]

\[
\frac{U}{U_c} = (a + b\eta^2 + c\eta^4)e^{-A\eta^2}
\]

\[
l_1 = 2 \int_0^\infty \frac{U}{U_c} \times \eta \, d\eta = \frac{a}{A} + \frac{b}{A^2} + \frac{2c}{A^3}
\]

\[
l_2 = 2 \int_0^\infty \left( \frac{U}{U_c} \right)^2 \times \eta \, d\eta = \frac{a^2}{2A} + \frac{ab}{2A^2} + \frac{b^2 + 2ac}{4A^3} + \frac{3bc}{4A^4} + \frac{3c^2}{4A^5}
\]
For the fan jet case the entrainment ratio is 0.0718

For the four different supersonic nozzles

<table>
<thead>
<tr>
<th>TYPE</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>A</th>
<th>I1</th>
<th>I2</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>-4.51887</td>
<td>5694.78</td>
<td>148.273</td>
<td>0.010033</td>
<td>0.004444</td>
<td>0.07525</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>6.14092</td>
<td>1915.59</td>
<td>104.158</td>
<td>0.013557</td>
<td>0.006239</td>
<td>0.08582</td>
</tr>
<tr>
<td>B minus</td>
<td>1</td>
<td>-8.47814</td>
<td>7329.25</td>
<td>150.047</td>
<td>0.010627</td>
<td>0.004672</td>
<td>0.07774</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>-10.776</td>
<td>7238.05</td>
<td>150.065</td>
<td>0.010469</td>
<td>0.004573</td>
<td>0.07740</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>-8.12222</td>
<td>9678.49</td>
<td>175.639</td>
<td>0.009003</td>
<td>0.003970</td>
<td>0.07144</td>
</tr>
</tbody>
</table>

Entrainment ratios are of similar orders.
Conclusion

- Turbulent Jet Entrainment is important in understanding the nature of volcanic jets.
- The effect of nozzle exit shape is studied on four different kinds of supersonic free jets.
- A very strong influence of nozzle exit shape is seen in the near field of the jet, where the complex nozzles show high rates of entrainment.
- Streamwise vorticity production is responsible for this phenomenon.
- In the far field, the rate of centerline velocity decay is affected by the nozzle exit shape.
- The velocity profiles, however, show similarity.
- The entrainment ratio evaluated from velocity profiles are about 0.08 in majority of the cases.
- Studies are being planned to include the effects of a dusty-gas in experiments and numerical simulations.
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