

EXPRESS LETTER

A new fifth parameter for transverse isotropy

Hitoshi Kawakatsu

Earthquake Research Institute, The University of Tokyo, Tokyo 113-0032, Japan. E-mail: hitosi@eri.u-tokyo.ac.jp

Accepted 2015 November 2. Received 2015 October 30; in original form 2015 September 15

SUMMARY

Properties of a new parameter, η_κ , that is recently introduced by Kawakatsu *et al.* for transverse isotropy are examined. It is illustrated that the parameter nicely characterizes the incidence angle dependence of bodywave phase velocities for vertical transverse isotropy models that share the same *P*- and *S*-wave anisotropy. When existing models of upper-mantle radial anisotropy are compared in terms of this new parameter, PREM shows a distinct property. Within the anisotropic layer of PREM (a depth range of 24.4–220 km), $\eta_\kappa < 1$ in the upper half and $\eta_\kappa > 1$ in the lower half. If $\eta_\kappa > 1$, anisotropy cannot be attributed to a layering of homogeneous isotropic layers, and thus requires the presence of intrinsic anisotropy.

Key words: Composition of the mantle; Body waves; Seismic anisotropy.

INTRODUCTION

Kawakatsu *et al.* (2015) recently introduced a new parameter, η_κ that characterizes the incidence angle dependence (relative to the symmetry axis) of seismic bodywaves in a transverse isotropy (TI) system. While the commonly used fifth parameter in global seismology to describe TI system, $\eta = F/(A - 2L)$ (e.g. Anderson 1961; Takeuchi & Saito 1972), has no simple physical meaning, the newly defined parameter,

$$\eta_\kappa = \frac{F + L}{(A - L)^{1/2}(C - L)^{1/2}}, \quad (1)$$

where *A*, *C*, *F* and *L* denote the Love’s elastic constants for TI (Love 1927), measures the departure from the ‘elliptic condition’ (Thomsen 1986) when η_κ not equal to unity, and characterizes nicely the incidence angle dependence of bodywaves. The elliptic condition means that phase velocity surfaces of bodywaves are either circular (*SV*) or elliptic (*P* and *SH*).

Here we assume that the symmetry axis is vertical, that is, vertical transverse isotropy (VTI) or radial anisotropy, which is the common parametrization in global seismology, but the argument can be easily generalized for arbitrary orientation of the symmetry axis. In a VTI system, horizontally and vertically propagating *P* waves have phase velocities of $\alpha_H = \sqrt{A/\rho}$ and $\alpha_V = \sqrt{C/\rho}$, respectively, where ρ gives the density. As to shear waves, the situation is slightly more complicated: while horizontally and vertically polarized *horizontally propagating S waves*, respectively, have phase velocities of $\beta_H = \sqrt{N/\rho}$ and $\beta_V = \sqrt{L/\rho}$, *vertically propagating S waves* also have a phase velocity of β_V (Fig. 1). So for these horizontally or vertically traveling bodywaves, phase velocities are

given by the four elastic constants, *A*, *C*, *L* and *N*, and the ratios of these elastic constants define the degree of anisotropy,

$$\varphi = C/A = \alpha_V^2/\alpha_H^2$$

for *P* wave, and

$$\xi = N/L = \beta_H^2/\beta_V^2$$

for *S* wave (e.g. Babuska & Cara 1991). For other intermediate direction bodywaves, the fifth elastic constant, *F*, affects the incidence angle dependence of quasi-*P* and quasi-*SV* waves via η_κ .

INCIDENCE ANGLE DEPENDENCE OF BODYWAVES

The incidence angle, θ (measured from the symmetry axis), dependence of bodywave phase velocities can be shown as

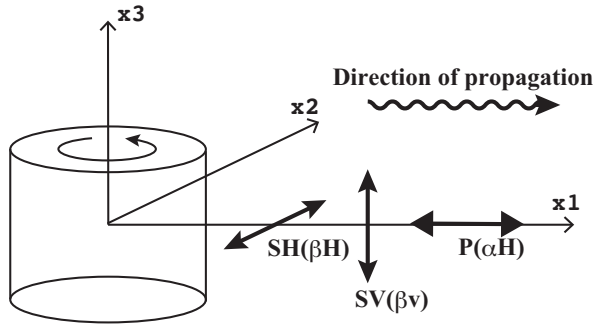
$$\begin{aligned} \rho v_{SH}^2(\theta) &= L + (N - L) \sin^2 \theta \\ \rho v_{SV}^2(\theta) &= \frac{(L + C) + (A - C) \sin^2 \theta - \sqrt{S}}{2} \\ \rho v_P^2(\theta) &= \frac{(L + C) + (A - C) \sin^2 \theta + \sqrt{S}}{2}, \end{aligned}$$

where v_{SH} , v_{SV} and v_P denote phase velocities of *SH*, quasi-*SV* and quasi-*P* waves, respectively, and

$$S = \{(C - L) + (A - C) \sin^2 \theta\}^2 + (F + L)^2 (1 - \eta_\kappa^{-2}) \sin^2 2\theta.$$

Here, the definition of *SH* and *SV* waves follows the convention of seismology (Aki & Richards 1980). When the elliptic condition,

(A) Perpendicular to symmetry axis



(B) Parallel to symmetry axis

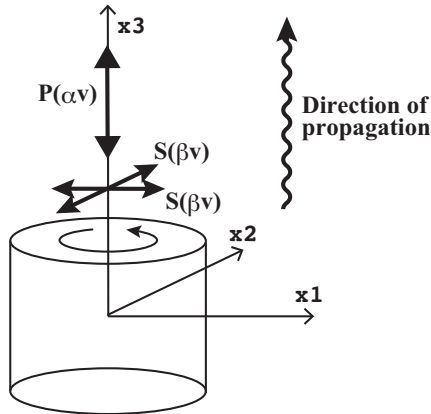


Figure 1. Schematic figure showing properties of bodywaves in a VTI system.

$\eta_\kappa = 1$, is satisfied,

$$\rho v_{SH}^2(\theta) = L + (N - L) \sin^2 \theta$$

$$\rho v_{SV}^2(\theta) = L$$

$$\rho v_P^2(\theta) = C + (A - C) \sin^2 \theta.$$

Fig. 2 exemplifies how this new parameter represents the incidence angle dependence. VTI models with common 4 per cent P -wave anisotropy ($a_p = -0.04$) and 3 per cent S -wave anisotropy ($a_s = -0.03$) are constructed from an isotropic Poisson solid whose P -wave and S -wave velocities and density are given by $\alpha_0 = 8.0 \text{ km s}^{-1}$, $\beta_0 = 4.619 \text{ km s}^{-1}$ and $\rho_0 = 3.33 \text{ g cc}^{-1}$ via following relations:

$$\alpha_{H,V} = \alpha_0(1 \mp a_p/2),$$

$$\beta_{H,V} = \beta_0(1 \mp a_s/2).$$

$\alpha_{H,V}$, $\beta_{H,V}$ are then used to calculate A, C, L and N. The fifth constant F is then calculated from specified η_κ that varies from 0.90 to 1.10 with an interval of 0.05. For five VTI models that share the same P - and S -wave anisotropy, the incidence angle dependence varies systematically as a function of η_κ . This figure should be compared with Fig. 3 of Dziewonski & Anderson (1981) where the effect of η is displayed for PREM.

η_κ OF REFERENCE EARTH MODELS

As Figs 2 and 3 nicely illustrate that the newly defined fifth parameter, η_κ , has a clear physical meaning, it is then instructive to

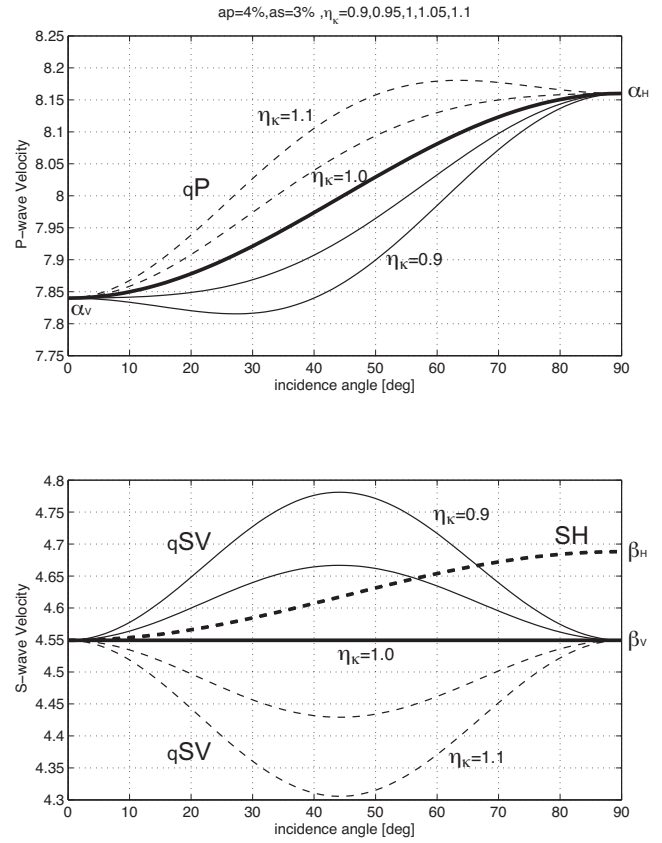


Figure 2. Incidence angle dependence of bodywave phase velocities for five VTI models that have common P - and S -wave anisotropy: (top) for quasi- P wave, (bottom) for SH and quasi- SV waves. Thick solid lines show cases when the elliptic condition is satisfied, that is, $\eta_\kappa = 1$. Thin solid (broken) lines are for cases with $\eta_\kappa < 1$ (> 1). For SH waves, five models show no difference.

investigate how it behaves in the reference Earth models of VTI or radial anisotropy. Fig. 4 shows η_κ and η as a function of depth for PREM (Dziewonski & Anderson 1981) and S362ANI model of Kustowski *et al.* (2008). While the difference in the two models is not so obvious in terms of η (Fig. 4, broken lines), when η_κ is compared (solid lines) the two models look quite different. Within the anisotropic layer of PREM (a depth range of 24.4–220 km), $\eta_\kappa < 1$ in the upper half and $\eta_\kappa > 1$ in the lower half. If $\eta_\kappa > 1$, anisotropy cannot be attributed to a layering of homogeneous isotropic layers (Berryman 1979; Backus 1962), and thus requires the presence of intrinsic anisotropy. Considering that the ocean covers nearly 70 per cent of the Earth's surface, if we associate the upper and lower halves of the anisotropic layer of PREM with (oceanic) lithosphere and asthenosphere, respectively, the former can be explained by the fine layering of homogeneous layers but the latter cannot be. Wang *et al.* (2013) also reached a similar conclusion based on a comparison of different anisotropic parameters, ξ and φ . How independent their inference is from the one made here based on the property of η_κ is an interesting question to be clarified in the future.

Recent findings of high-frequency of bodywaves in OBS data indicate that the oceanic lithosphere consists of thin horizontally elongated scatterers (Shito *et al.* 2013, 2015; Kennett & Furumura 2013; Kennett *et al.* 2014) that may be consistent with the upper half of the anisotropic layer having $\eta_\kappa < 1$. Kennett & Furumura (2013) argue that ~ 1.5 per cent of radial anisotropy can be expected from such a structure. As $\eta_\kappa < 1$ does not necessarily

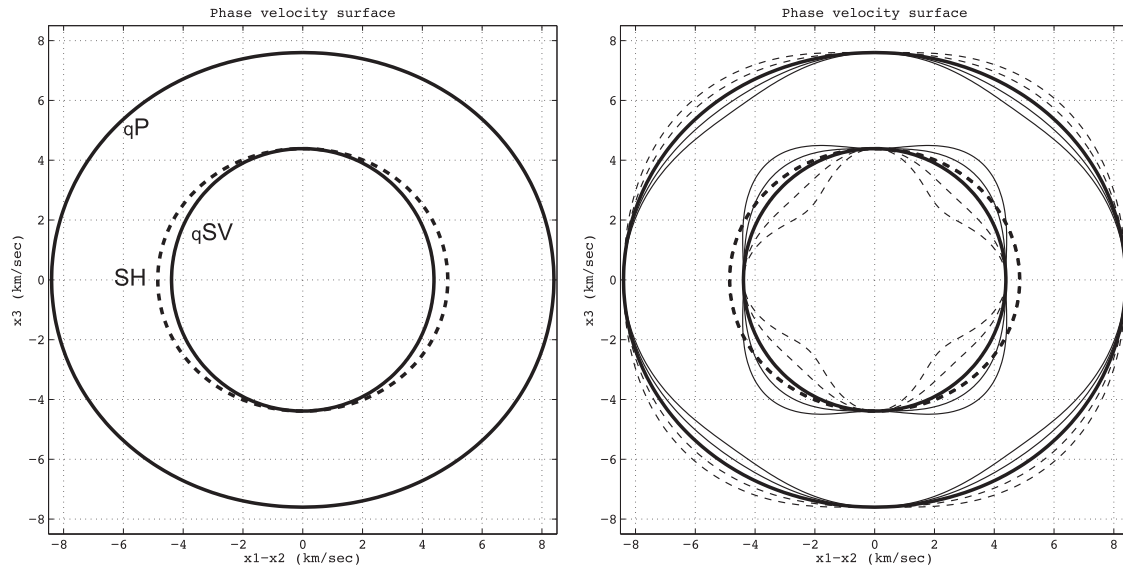


Figure 3. Similar to Fig. 2, but for phase velocity surfaces. Here anisotropy is stronger ($a_p = a_s = -0.1$) for graphical purposes. Left: for the case of the elliptical condition, right: η_κ varies from 0.60 to 1.40 with an interval of 0.2. Note that the opposite effect of η_κ on phase velocities of q-P and q-SV.

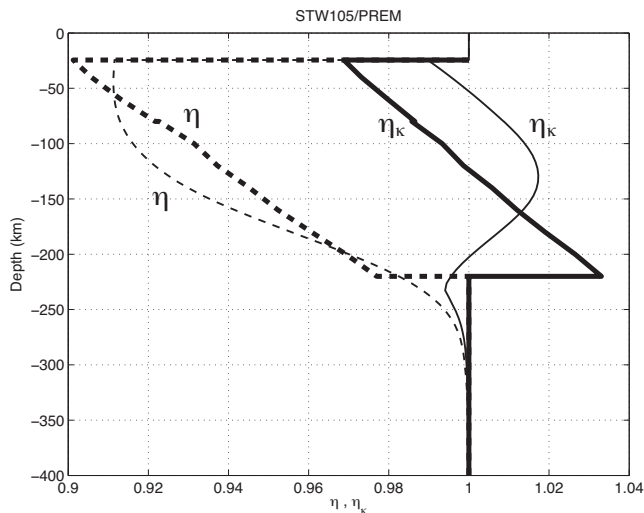


Figure 4. Comparison of η_κ (solid lines) and η (broken lines) for PREM (thick lines) and S362ANI of Kustowski *et al.* (2008) (thin lines).

require the layering for the origin of anisotropy, CPO of mantle rocks might be also responsible for radial anisotropy of the lithosphere. On the other hand, $\eta_\kappa > 1$ for the lower half of PREM and for the shallow asthenospheric depth range of S362ANI model (Kustowski *et al.* 2008), may indicate that simple application of the millefeuille model (fine layering of melt and isotropic solid layers) of Kawakatsu *et al.* (2009) for the asthenosphere is not appropriate, as suggested by Kawakatsu & Song (2012) from a slightly different context. The model of the oceanic asthenosphere of Song & Kawakatsu (2012) gives $\eta_\kappa = 1.022$ for their azimuthally averaged VTI model and Song & Kawakatsu (2013) suggest the importance of the solid fabric to explain the incidence angle dependence of their model.

How well the fifth parameter of existing VTI models is constrained from data still remains to be carefully examined. However, we now have, at least, a fifth parameter that properly characterizes the VTI system. This parameter, η_κ , may be more useful in future

surface wave and bodywave studies of mantle anisotropy, rather than the conventional η .

ACKNOWLEDGEMENTS

This research is partly supported by JSPS KAKENHI 15K13558.

REFERENCES

- Aki, K. & Richards, P., 1980. *Quantitative Seismology: Theory and Methods*, W. H. Freeman.
- Anderson, D.L., 1961. Elastic wave propagation in layered anisotropic media, *J. geophys. Res.*, **9**, 2953–2963.
- Babuska, V. & Cara, M., 1991. *Seismic Anisotropy in the Earth*, Kluwer Academic.
- Backus, G.E., 1962. Long-wave elastic anisotropy produced by horizontal layering, *J. geophys. Res.*, **67**, 4427–4440.
- Berryman, J.G., 1979. Long-wave elastic anisotropy in transversely isotropic media, *Geophysics*, **44**, 896–917.
- Dziewonski, A.M. & Anderson, D.L., 1981. Preliminary reference Earth model, *Phys. Earth planet. Inter.*, **25**, 297–356.
- Kawakatsu, H. & Song, T.-R.A., 2012. Significance of radial seismic anisotropy of the oceanic asthenosphere inferred from sub-slab shear-wave splitting observations, in *Abstract DI24A-03 presented at 2012 Fall Meeting, AGU*.
- Kawakatsu, H., Kumar, P., Takei, Y., Shinohara, M., Kanazawa, T., Araki, E. & Suyehiro, K., 2009. Seismic evidence for sharp lithosphere-asthenosphere boundaries of oceanic plates, *Science*, **324**, 499–502.
- Kawakatsu, H., Montagner, J.-P. & Song, T.-R.A., 2015. On DLA's η , in *The Interdisciplinary Earth: A Volume in Honor of Don L. Anderson*, pp. 33–38, GSA and AGU.
- Kennett, B.L.N. & Furumura, T., 2013. High-frequency Po/So guided waves in the oceanic lithosphere: I—long-distance propagation, *Geophys. J. Int.*, **195**, 1862–1877.
- Kennett, B.L.N., Furumura, T. & Zhao, Y., 2014. High-frequency Po/So guided waves in the oceanic lithosphere: II—heterogeneity and attenuation, *Geophys. J. Int.*, **199**, 614–630.
- Kustowski, B., Ekström, G. & Dziewonski, A.M., 2008. Anisotropic shear-wave velocity structure of the Earth's mantle: a global model, *J. geophys. Res.*, **113**, B06306.
- Love, A.E.H., 1927. *A Treatise on the Mathematical Theory of Elasticity*, Dover, New York.

- Shito, A., Suetsugu, D., Furumura, T., Sugioka, H. & Ito, A., 2013. Small-scale heterogeneities in the oceanic lithosphere inferred from guided waves, *Geophys. Res. Lett.*, **40**, 1708–1712.
- Shito, A., Suetsugu, D. & Furumura, T., 2015. Evolution of the oceanic lithosphere inferred from Po/So waves traveling in the Philippine Sea Plate, *J. geophys. Res.*, **120**, 5238–5248.
- Song, T.-R.A. & Kawakatsu, H., 2012. Subduction of oceanic asthenosphere: evidence from sub-slab seismic anisotropy, *Geophys. Res. Lett.*, **39**, L17301, doi:10.1029/2012GL052639.
- Song, T.-R.A. & Kawakatsu, H., 2013. Subduction of oceanic asthenosphere: a critical appraisal in Central Alaska, *Earth planet. Sci. Lett.*, **367**, 82–94.
- Takeuchi, H. & Saito, M., 1972. Seismic surface waves, in *Seismology: Surface Waves and Earth Oscillations, Methods in Computational Physics*, vol. 11, pp. 217–295, ed. Bolt, B.A., Academic Press, New York.
- Thomsen, L., 1986. Weak elastic anisotropy, *Geophysics*, **51**, 1954–1966.
- Wang, N., Montagner, J.-P., Fichtner, A. & Capdeville, Y., 2013. Intrinsic versus extrinsic seismic anisotropy: the radial anisotropy in reference Earth models, *Geophys. Res. Lett.*, **40**, 4284–4288.