PREDICTION OF STRONG ACCELERATION MOTIONS
USING EMPIRICAL GREEN'S FUNCTION

by
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Abstract

The scaling law of the source spectra was examined for the strong motion records of the sequence of 1983 Japan-Sea earthquake. The spectral shapes for events smaller than magnitude 6.1 give roughly a fit to the \( w^2 \) model. However, the spectra of larger earthquakes (\( M=7.1 \) and \( M=7.7 \)) have much larger high-frequency components than those predicted from the \( w^2 \) model. The specific barrier model is a useful model to explain rich high-frequency motions. The synthetic method of strong ground motions is studied to match the spectral scaling relation using empirical Green's function technique. In the case that both large and small earthquakes have spectral characteristics predicted from the \( w^2 \) model, the summation procedure for synthesis is made to match the moment and the stress drop of the event to be synthesized. Finally a more general method is attempt to synthesize large earthquake motions which have spectral characteristics predicted from the specific barrier model.

INTRODUCTION

We need to discuss the following two problems to predict strong acceleration motions for large earthquakes using empirical Green's function. The first problem is how the source characteristics change with the seismic moment. The scaling law of the seismic spectra is examined by calculating the spectral ratios between the pairs of earthquakes with nearly the same epicenter but different size to avoid the propagation characteristics from source to station. We consider the \( w^2 \) model as a reference model to compare the observed to the theoretical, giving moment and fault size. Next the spectra, not explained by the \( w^2 \) model, are compared to the specific barrier model, assuming additive parameters, crack size and number of cracks. The second problem is how the strong ground motions for large earthquakes can be synthesized by summing the ground motion records from small earthquakes to satisfy the scaling law of the source spectra.

We examine the above problems by using the strong ground motion records of the sequence of 1983 Japan-Sea earthquake (Akita-Oki). The data set analyzed here range in magnitude from 7.7 to 3.8 determined by JMA (Japan Meteorological Agency) scale. Finally we attempt to synthesize the strong acceleration motions for the mainshock (\( M=7.7 \)) and the second largest aftershocks (\( M=6.1 \)) using the smaller earthquake records, respectively.

SYNTHESIS OF STRONG MOTIONS

We consider a rectangular fault (length \( L \), width \( W \)) for a large earthquake to be simulated as shown in Fig.1. The fault plane is divided into \( 1 \times 1 \) element. The element size is taken to match the fault size of a small earthquake used as empirical.

Fig.1. Schematic source model for synthesis.

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Green's function. After this, we call the event to be simulated 'target earthquake' and the event used as empirical Green's function 'element earthquake'. The synthetic motion \( U \) for target earthquake is given using the observed record \( u \) for element earthquake as

\[
U(x, t) = \sum_{i,j} \sum_{k=1}^{m} \sum_{l=1}^{n} \frac{r_{ij}}{r_{ij}} \cdot \hat{u}(x, t - t_{k} - t_{d}),
\]

where \( t_{k} = r_{ij}/v_{s} \) and \( t_{d} = (k - 1)\tau/n \). \( F_{ij}^{s} \) and \( r_{ij} \) are radiation pattern and focal distance for \((i,j)\) element, \( \tau \) is rise time of target earthquake.

The parameters \( k \) such like \( l, m, n \) are determined from the scaling relations by Kanamori and Anderson (1975). For instance, when the moment ratio of target earthquake to element one is given \( N \), the parameters \( l, m, n \) should be equal to \( N \) if the self-similar scaling is ideally applicable.

The approach (1) has some problems in use. Because of including the term of uniform time shift, \( t_{d} \), to match the dislocation time function of target earthquake, the synthetic has an artificial periodicity of \( \tau/n \). Some revised methods have been attempted to avoid the periodicity. One approach is to shift the periodicity to a higher frequency by subdividing \( \tau/n \) into smaller units. Then the summation with respect to \( k \) in (1) become

\[
\sum_{n} \hat{u}(x_{0}, t - t_{k} - (k - 1)\tau/n). \quad (2)
\]

The synthetic using (1) and (2) gave a good fit to the observed in the low frequency motions up to 1/\( \tau \). However, this method underestimates the high frequency motions, because it generates an \( w^{-2} \) high frequency fall-off for \( u \) with flat spectra as discussed in detail by Irikura (1983).

The other approach is to assume random time shifts. That is, the term of \( t_{d} \) in (1) is replaced by \((k - a_{k})\tau/n\), where \( a_{k} \)'s are uniform random numbers between 0 and 1 (Muramoto and Ohnuma, 1985 Yoshikawa et al., 1985). Then the summation with respect to \( k \) in (1) become

\[
\sum_{k=1}^{n} \hat{u}(x_{0}, t - t_{k} - (k - a_{k})\tau/n). \quad (3)
\]

Assuming \( l = m = n = N \), the synthetic in (1) and (3) leads to the summation of \( N^{2} \) records with a random phase lag. Resultantly, the high frequency spectral level of the synthetic becomes \( N^{-1/2} \) as large as that of element earthquake. On the other hand, if both spectra of the target earthquake and the element one obey \( w^{-2} \) scaling model, the ratio of the high-frequency spectral levels between them should be \( N \) for the moment ratio of \( N^{2} \). Therefore the synthetic using random time shifts overestimate high frequency motions, compared with the \( w^{-2} \) model. Similar discussions were made by Heaton and Hartzell (1986).

In order to obtain the synthetic which is consistent with spectral scaling at high frequencies, only \( N^{2} \) records should be summed randomly for the case with the moment ratio of \( N^{2} \). An application to synthesize the acceleration motion was presented by Tanaka et al (1982) in which \( N^{2} \) records were summed up by dropping the summation with respect to \( k \) in (1). Although any randomness were not directly considered in their summation, the phase term \( t_{k} \) in (1) might become nearly random in a near-distance region from a rectangular fault plane. Their synthetic results gave a good fit to the peak acceleration and high frequency spectral levels of the observed motions, although naturally they fail to fit the low frequency motions.

We propose a revised method for synthesis of acceleration motions, combining (1) and (2) with (3). The revised expression is given by

\[
U(x, t) = \sum_{i,j=1}^{m} \sum_{k=1}^{n} \frac{r_{ij}}{r_{ij}} \cdot \hat{u}(x_{0}, t - t_{k}) + \sum_{i,j=1}^{m} \sum_{k=1}^{n} \frac{r_{ij}}{r_{ij}} \cdot \hat{u}(x_{0}, t - t_{k} - (k - a_{k})\tau/n). \quad (4)
\]
Equation (4) is consistent with the summation of $N^3$ records and then matches the moment of target earthquake. The first term produces an $w^{-2}$ high frequency spectral fall off, while the second term produces an $w^{-1}$ fall off. Therefore, high frequency motions come from the first term with the summation of $N^2$ records, assuming $l = m = n = N$. Resultantly, the synthetic using (4) have the spectral content predicted from $w^{-2}$ model, if both spectra of target earthquake and element one obey $w^{-2}$ scaling.

We need to pay attention to stress drop estimated from high frequency spectral level and fault size (Madariaga, 1977), because $w^{-2}$ model based on the assumption of constant stress drop. If the stress drop of element earthquake, $\sigma_e$, is different from that of target one, $\sigma$, the record of element one should be multiplied by a factor of $\sigma / \sigma_e$ to match stress drop before used as empirical Green's function.

SPECTRAL RATIO

We examine the scaling law of the source spectra for strong motion records of the sequence of 1983 Japan-Sea earthquake (the mainshock magnitude 7.7 by JMA). The locations of the observation stations and the mainshock epicenter and aftershock distributions are shown in Fig. 2.

The second largest aftershock ($M=6.1$) located south of the mainshock fault zone has its own aftershocks. The spectral ratios for pairs of earthquakes ($M=6.1$)/($M=5.0$) and ($M=5.0$)/($M=4.0$) at three stations, TUC, TEG and FKR are shown in Fig. 3. The theoretical ratio from $w^{-2}$ model give a good fit to the observed ones.

The spectral ratio of the mainshock ($M=7.7$) to the second largest aftershock ($M=6.1$) at TUC is shown in Fig. 4 and compared with the theoretical ones. The observed does not fit the theoretical from $w^{-2}$ model in the high frequency

Fig. 2. Location of the strong motion stations and the epicentral area of the aftershocks.

Fig. 3. Observed spectral ratios for pairs of earthquakes having nearly the same epicenter but different size.

Fig. 4. Observed spectral ratio of mainshock ($M=7.7$) to the second largest aftershock ($M=6.1$).
range. Assuming the specific barrier model with 6 cracks for the mainshock source, the observed fits well the theoretical in the high frequency range.

SYNTHETIC EXAMPLE 1
(case of $w^{-2}$ scaling model)

Fig. 5. Synthetic velocity seismogram for M 6.1 earthquake using smoothing model.

Fig. 6. Synthetic acceleration seismogram for M6.1 earthquake using the revised model.
M 6.1 earthquake and its aftershock with M 5.0 are considered to obey $w^{-2}$ model as shown in Fig.2. We attempt to synthesize the ground motions for M 6.1 earthquake using the records of M 5.0 earthquake. First, the synthetic velocity motion calculated by (1) and (2) (smoothing method) is compared with the observed one in Fig. 5. The synthetic waveform seems to give a good fit to the observed. However, the spectrum of the synthetic shows a rapid fall off in the high frequency range, compared with that of the observed. Next, we used the revised method, (4), for synthesis of acceleration motions. The synthetic acceleration motion is compared with the observed one in Fig.6. The waveform envelope and the spectral amplitude level of the synthesized give a good fit to those of the observed.

SYNTHETIC EXAMPLE 2
(case of the specific barrier model)

The $w^{-2}$ model fails to fit the high frequency amplitudes of the mainshock (M=7.7). The source spectrum of the mainshock as well as the largest aftershock (M=7.1) is considered to have rather the spectral content predicted by the specific barrier model. Now we attempt to make a synthesis of the mainshock motions. This is more general method than the case of $w^{-2}$ model.

The synthesis is made by the following step as an extension of (4). I. The fault plane of the mainshock is divided to a mesh of the patch size (corresponding to the crack size). II. Empirical Green's function should be the records of small earthquakes which have the fault area matching the patch size of target earthquake. In the case when we have no records of small earthquakes meeting the above condition, we construct the empirical Green's function by superposing the records of smaller earthquakes to match the patch size using (4). III. The ground motions for the mainshock are synthesized by the delay-and-summation of the empirical Green's function with lag time corresponding to rupture propagation over the faulting area.

An example of the synthetic ground motions for the mainshock is shown in Fig.8 using the records of M 6.1 earthquakes following the above steps, to meet the spectral structure from the specific barrier model in Fig.7. The synthetic gives a good fit to the observed.

CONCLUSION

A new formulation is derived to synthesize high frequency motions using empirical Green's function technique. If the spectrum of small earthquake motion used as empirical Green's function obey spectral scaling, the synthetic motion using this equation have also the spectral content predicted from $w^{-2}$ model. Therefore, in the case which both large and small earthquakes have the source spectra predicted from $w^{-2}$ model, the spectrum of the synthetic always fits that of the observed. Finally a more general method is attempted to synthesize large earthquake motions which has spectral content predicted from the specific barrier model. The strong acceleration motions for the mainshock (M=7.7) and the second largest aftershock (M=6.1) of 1983 Japan-Sea earthquake were successfully synthesized using smaller aftershocks as empirical
Green's functions.

REFERENCES


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