Fluctuation and a Many-Body Disk Model of Slip Phenomena

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A many-body disk system was investigated as a model of slip phenomena. A two-dimensional many-body disk system was used as a model of the boundary layer between slip surfaces. Frustrated states occurred in this system depending on the disk configuration. Experiments with this were carried out using a random packing configuration (packing fraction: 0.74 ~ 0.76). Acrylic resin disks were packed between a co-axial outer rotating cylinder and an inner fixed cylinder. The outer cylinder was rotated by a motor and the torque at the fixed inner cylinder was measured in a time series. Stick-slip and $1/\nu$ fluctuation were observed in the time series.

KEYWORDS: $1/\nu$ fluctuation, stick-slip, frustration, many-body disk, slip phenomena

Sliding friction, especially stick-slip phenomena, has been attracting a great deal of attention from physicists recently. A large earthquake is a typical example of stick-slip phenomena. In earthquake faults, there is a fault gouge between fault surfaces, which plays an important role in fault sliding. In general, there are some fragments generated by wearing that lie between slip surfaces. In experiments with granular materials, stick-slip occurs when a powder layer lies between slip surfaces. Stress fluctuation in granular materials under stress has also been reported, although the focus was not on stick-slip phenomena. In the present study, a many-body disk system was used as a model of the boundary layer between slip surfaces. The number of disks was much less than that in experiments with granular materials.

$1/\nu$ fluctuation is one of the most exciting topics in physics. It can be observed in a wide variety of different problems, such as $1/\nu$ noise in resistance including semiconductors and superconductors, and in other transport phenomena in quite different systems, e.g., traffic flow, the flow of the Nile river, and the luminosity of stars. $1/\nu$ fluctuation is not yet well understood, and there is no general model to explain the occurrence of $1/\nu$ fluctuation. In this letter, we show that a many-body disk model of slip phenomena exhibits $1/\nu$ fluctuation and may be a good prototype model for understanding it.

A schematic illustration of the many-body disk model of slip phenomena is shown in Fig. 1. A frustrated state is one characteristic of this model; a disk whose direction of rotation cannot be determined only by examining the directions of rotation of neighboring disks under shear stress is known as a frustrated disk (Fig. 1(b)). Experiments focusing on this many-body disk model of slip phenomena were performed. Figure 2 shows a schematic illustration of the experimental system employed. One hundred and forty-eight disks were packed between a co-axial inner cylinder 70 mm in diameter and an outer cylinder 140 mm in diameter. Each disk consisted of an acrylic resin cylinder with a diameter of 8 mm and a height of 10 mm. A rubber belt with a thickness of 6.0 mm was mounted on the inner cylinder in order to make contact with each disk. The presence of the rubber belt allowed each of the packed disks to change position relative to its neighboring disks. Therefore, a rear-
rangement of the disk configuration occurred during the experiment.

Let us consider the packing fraction in order to understand the configuration of the disks in the experiment. The area of the annular region between the inner cylinder and the outer cylinder was 10113 mm$^2$. The area occupied by the disks was $148 \times 4^2 \pi \approx 7439$ mm$^2$. In this experiment, the packing fraction of the disks was 0.74. This value is lower than that for random close packing (e.g., Bennett’s algorithm yields a value of $0.82^{16-18}$). The maximum packing fraction, $\pi/2\sqrt{3} \approx 0.907$, is achieved in the case of regular packing of disks in a triangular lattice. As for regular packing in a square lattice or a hexagonal lattice, the packing fraction is $\pi/4 \approx 0.785$ and $\pi/3\sqrt{3} \approx 0.605$, respectively. In these packing configurations, the disks support each other by being in close contact with their neighbors, so no disk can move freely. Although the disks in the present experiment were packed sparsely in comparison with random close packing, the packed disks supported each other.

The width of the annular region between the inner fixed cylinder including the 6.0 mm rubber layer and the outer rotating cylinder was 29.0 mm, which was narrower than that of four layers of 8 mm disks. However, the width of the rigid part of the annular region without the rubber layer was 35 mm, which was wider than that of four layers of disks. Even in the tightly packed situation, this setting of the experiment permitted a disk to get over the disk layer due to elastic deformation of the rubber layer, allowing rearrangement of the disks in the annular region during the experiment. The packing fraction value of 0.74 indicated that the packing in the experiment was not so tight, and this facilitated the rearrangement of disks.

The outer cylinder was rotated by a motor (Yokogawa Precision (DR1150A)) that was controlled in units of $1.0 \times 10^{-3}$ degrees. The accuracy of the motor was $\pm 1.0 \times 10^{-3}$ degrees per unit rotation. The experiments were carried out at a constant rotation rate of 1/80 Hz (angular frequency/$2\pi$). The outer cylinder was rotated at a very low constant rotation rate; thus, the system was considered to be in a quasi-static condition. The torque at the inner fixed cylinder was measured. The maximum response of the torque sensor and its amplifier (Kubota Torkducer KB22-015) was 3 kHz. The linearity of the torque measurement was $\pm 1\%$. The output of the torque measurement system was fed into a digital voltmeter (HP3458a) connected to a PC via a GP-IB. The sampling frequency of the digital voltmeter was 10 Hz.

The results obtained from measurement of torque at the inner fixed cylinder in a time series are shown in Fig. 3. In this figure, the scale of the horizontal axis is magnified from top to bottom in order to see stick-slip events (sawtooth behavior) more clearly. The total experimental time was 5000 s. In Fig. 3(c), the main stick-slip events are indicated by half-tone lines. The maximum period of one stick-slip event is shorter than 5 s.

Spectral analysis was carried out for the time series of torque and the power spectra were calculated. In Fig. 4, the power spectrum, $S(f)$, is plotted against frequency ($f$) on a log-log scale. The log-log plot of $S(f)$ vs. $f$ shows a linear relationship indicative of the power law $S(f) \sim 1/f^\alpha$. However, there is a breaking point (crossover point) at around 0.1 $\sim$ 0.2 Hz; there are two different slopes in the graph. The regression curves, indicated by solid lines in Fig. 4, over the low- and high-frequency ranges were determined by the least square method. The values of $\alpha$ estimated for the low- and high-frequency ranges are 0.82$\pm$0.05 and 2.46$\pm$0.02, respectively. Over the frequency range of 0.0002 $\sim$ 0.2 Hz, the $1/f$ spectrum, $S(f) \sim 1/f$, is obtained.

The experiment at a different rotation rate was performed. The breaking point at around 0.1 $\sim$ 0.2 Hz coincided with the maximum period of one stick-slip event (~5 s.). When the experiment was carried out at a higher rotation rate, the breaking point shifted to higher frequencies. The time series of the torque and the log-log plot of $S(f)$ vs. $f$ for the experiment performed at a constant rotation rate of 1/8 Hz (10 times faster than the previous experiment) are shown in Fig. 5. The maximum period for one stick-slip event is about 1 s., and the breaking point is at around 1 Hz, which is about 10 times higher than the previous experiment.

The disks packed in the annular region were movable.
and underwent random migration in the course of the experiment. The migration of the disks was monitored by means of a video camera (Sony DCR-TRV900). Thirty of the one hundred and forty-eight disks were marked for convenience in order to monitor the movement of the disks. Disk mobility was estimated by tracing the thirty marked disks. The average migration of the disks per unit outer cylinder rotation was about $2/3 \times 360^\circ$. The annular region was divided into twelve bins at $30^\circ$ intervals. Initially, the marked disks were gathered together; the marked disks were in only three bins. Upon rotating the outer cylinder, the marked disks diffused throughout all of the bins. The unbiased variance $V(t)$, 

$$V(t) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i(t) - \bar{x})^2,$$

was calculated from the experimental data obtained at a rotation rate of $1/80$ Hz. Here, $x_i(t)$ is the number of marked disks in the $i$-th bin at time $t$, $n$ is the number of bins (=12), and $\bar{x}$ (=2.5) is the average of $x_i$. The variance $V(t)$ is plotted against the number of rotations of the outer cylinder in Fig. 6. After about 20 rotations, $V(t)$ becomes constant.

There are various mechanisms that resist shear stress in this system. In a certain situation, the rolling friction of the disk is the only mechanism that resists shear stress; in this case, the torque measured at the inner fixed cylinder will be very small. In such a case, the rotation of the disks is transferred via contact with neighboring disks without loss. However, when shear stress is applied to the system in which the frustrated state is generated with respect to the release of shear stress by each disk rotation, shear stress cannot be released by the disks’ rotations only. In that case, dissolution of the frustrated state due to breaking disk contact is needed to release shear stress. When the shear stress is increased, the frustrated state is disrupted and the stick-slip event occurs. The rearrangement of disk configuration occurs during this process, due to which fluctuation of the measured torque will appear as a $1/f$ fluctuation.

The frustrated state of the disks depends on the disk contact network which is defined as a network generated by connecting the centers of the disks that contact each other with lines. Let us consider the relationship between the behavior of individual disks and the disk contact network. One of the ideal disk contact networks is a regular lattice: a triangular, square, or hexagonal lattice. For simplicity, the individual disks are assumed to have two behavioral states: clockwise and anticlockwise rotations.
When the disk contact network is a triangular lattice, all disks are in the frustrated state. This is equivalent to a system of Ising spins with anti-ferromagnetic interaction in a triangular lattice. In this case, the many-body disk system behaves like a solid under shear stress. On the other hand, in the case of a square lattice, there is no frustrated disk, so the many-body disk system behaves like a fluid under shear stress. Thus, odd path loops contribute to the production of a frustrated state: a triangular lattice consists of odd three-path loops, and a square lattice consists of even four-path loops. In general, the contact disk networks generated by random packing consist of both even and odd path loops. From the viewpoint of spin systems, a many-body disk model of slip phenomena is one of the systems of Ising spins with anti-ferromagnetic interaction in irregular lattices. In light of the frustration, the frustrated disks, whose number depends on the topology of the disk contact network, govern the resistibility to shear stress. The amplitude of stick-slip is related to the resistibility of the many-body disk system to shear stress. Although the above discussion may be oversimplified, the viewpoint of frustration gives us a clue for understanding our results.

Does the detailed disk configuration affect the generation of 1/f fluctuation? Experiments at a constant rotation rate of 1/80 Hz were carried out by changing a packing fraction. The experimental results obtained at packing fractions of 0.75 and 0.76 (one hundred and fifty and one hundred and fifty-two disks were packed into the annular region, respectively) also show a 1/f fluctuation ($\alpha = 0.78 \pm 0.05$ and $0.89 \pm 0.05$, respectively) over the frequency range of 0.0002 – 0.2 Hz. As for the value of $\alpha$, 0.82 $\pm$ 0.05 and 1.15 $\pm$ 0.03 were obtained from experiments on a packing density of 0.74 at rotation rates of 1/80 Hz and 1/8 Hz, respectively. The value of $\alpha$ of 0.99 $\pm$ 0.07 was also obtained by another experiment of the packing fraction of 0.74 at a rotation rate of 1/80 Hz. Thus, although the value of $\alpha$ ranges from 0.8 to 1.2, the detailed disk configuration does not affect the generation of 1/f fluctuation.

The many-body disk model of slip phenomena is not only a successful slip model that can produce stick-slip phenomena; it is also a good candidate for a prototype model for understanding the generation of 1/f fluctuation. As an elementary process, the behavior of each disk can be monitored using a video system during the experiments. The 1/f fluctuation of torque measured at the inner fixed cylinder is a macroscopic behavior of the system. In this model, the torque which reflects the macroscopic behavior of the system, as well as the motion of individual disks which provides information on microscopic elementary processes of the system, can be measured simultaneously. Therefore, 1/f fluctuation as a macroscopic phenomenon can be investigated based on the microscopic elementary processes in the many-body disk model, which will give us a clearer understanding of the production of 1/f fluctuation in nature.

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