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# A new light on arresting mechanism of dynamic earthquake faulting

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**Abstract.** Classic analyses have shown that dynamic growth of a shear crack cannot be arrested in a uniformly stressed elastic medium with homogeneously distributed fracture strengths. This leads to a general supposition that earthquake rupture growth is arrested by inhomogeneities in the distributions of strengths or stresses. We propose a novel idea for arresting mechanism of dynamic crack growth in the simulation with no constraints on the crack geometry. Our analysis shows that the arresting occurs spontaneously soon after crack bending even in the homogeneous medium and that inhomogeneities are indispensable not for stopping crack growth, but for its promotion.

## 1. Introduction

Classic approach based on linear fracture mechanics predicts that dynamic crack growth is never arrested in a uniformly stressed isotropic elastic medium with homogeneous strength distribution [Griffith, 1920; Irwin, 1958; Das and Aki, 1977a]. A planar-shaped crack is implicitly assumed there, and the fundamental model parameters are fracture strength of the medium and magnitude of shear traction released on the crack, which is generally referred to as stress drop in seismology. A static crack with positive stress drop generates high stress concentration at the crack tips. It is generally assumed in linear fracture mechanics that dynamic crack growth begins when the stress concentration level at the tips exceeds the fracture strength [Irwin, 1958; Das and Aki, 1977b]. Once dynamic growth begins, the crack tip velocity is shown to be monotonically accelerated up to an elastic wave speed level. The crack tip stress concentration is larger for a larger-size crack, so that the crack growth is never arrested in a medium with homogeneous distributions of stress drop and fracture strength.

Arresting of earthquake faulting is therefore supposed to occur by inhomogeneities in the spatial distributions of the stress drop or fracture strength in prevailing ideas. A possibility in the framework of this view is that a dynamic crack would stop growing when the tips come into insufficiently stressed zones. This might be the case for great earthquakes at plate subduction zones [Ruff and Kanamori, 1983]. In fact, the source regions of such earthquakes are hardly overlapped [Mogi, 1968]. Hence there is a possibility that an

earthquake rupture stops growing when its tips grow into regions where shear traction has already been released by former events. However the applicability of this mechanism is likely to be limited because there occur many smaller-size events even in a highly and broadly stressed region. The other possibility is barrier, which is generally referred to as region with locally high fracture strength in seismology [Das and Aki, 1977b]. Quantitative analysis, however, shows that unrealistically high fracture strength is required to arrest a dynamic earthquake rupture that has largely grown up [Husseini et al., 1975]. Dynamics responsible for arresting of dynamic earthquake faulting thus remains largely unknown.

We believe that the failure to understand the arresting mechanism of earthquake ruptures lies in an implicit assumption of a planar-shaped crack in the classic approach. In fact, actual faults are known to be very irregular [Tchalenko, 1975]. In addition, it has theoretically been pointed out that the direction of maximum stress shifts from the original crack plane beyond a critical crack velocity [Yoffe, 1951; Freund, 1990]. This suggests the tendency of rapidly growing crack to curve spontaneously. In spite of this tendency, a crack had been forced to grow straight in classic simulations of earthquake ruptures. Such simulations will lose the validity after a crack shows the tendency of curving.

Here we simulate dynamic earthquake rupture growth without any constraints on the crack geometry. It will be shown that crack shape is one of the key elements that controls the dynamics of ruptures including the arresting of rupture growth.

## 2. Method and Model

We have developed a new mathematical method for the analysis of spontaneous crack growth with arbitrary crack geometry using Boundary Integral Equation Method (BIEM) [Kame, 1998; Kame and Yamashita, 1999]. We have overcome the mathematical difficulty to evaluate hypersingular kernels appearing in BIEM by taking finite parts in the integrals. The crack trace is discretized in this approach, while radiated waves can be evaluated at arbitrary locations. Crack growth is modeled by adding new elements to the moving crack tips. The calculations are carried out with normalized physical quantities.

The configuration of the model is illustrated in Figure 1. A static seed crack is assumed for the nucleation of dynamic rupture. The critical stress fracture criterion is assumed in the calculations for the extension of crack tip [Das and Aki, 1977a; Koller et al., 1992]. The crack tip is assumed to extend in the direction where the shear traction around the tip takes the local maximum [Koller et al., 1992]. The crack begins dynamic growth at  $t = +0$  by a slight increase in the remotely applied load. Note that only the

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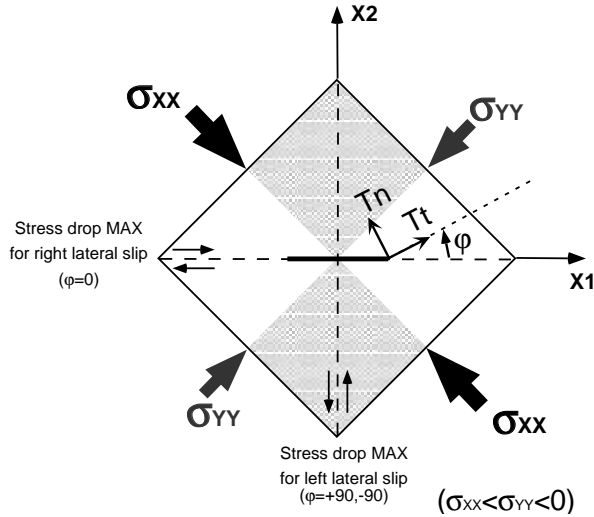
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shear traction is considered here for the fracture criterion at the tips. Coulomb's fracture criterion, which is given by the combination of normal and shear tractions, is sometimes assumed in the interpretation of results of fracture experiments with rocks [Mogi, 1974]. However, we have to note that Coulomb's criterion is a macroscopic fracture criterion, which is not appropriate in our case since local extension of crack tip is investigated.

### 3. Results

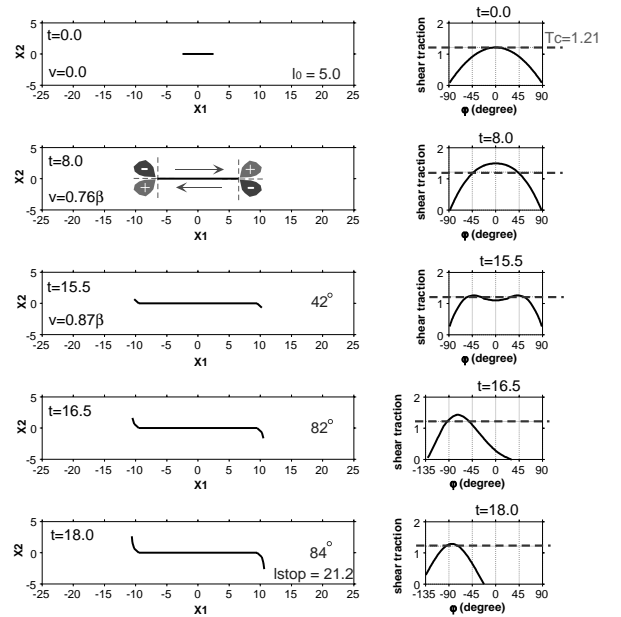
We first investigate a model in which all the model parameters are homogeneously distributed. This enables us to elicit the dynamics of spontaneously propagating crack. We assume the fracture strength  $T_c = 1.21$  over the medium, which is slightly larger than the shear traction at the tips of the seed crack. Figure 2 shows the snap shots of the dynamic crack growth. The shear traction takes the maximum in the direction of the original crack plane at  $t = +0$ , that is, the crack begins its growth along the original crack plane. The crack growth is accelerated soon after the nucleation, and the crack velocity attains  $0.76\beta$  at  $t = 8.0$ , where  $\beta$  denotes the shear wave velocity of the medium. The maximum shear traction axis remains directed to the original crack plane, so that the crack continues to grow straight. At  $t = 15.5$ , the



**Figure 1.** The configuration of the model. We assume homogeneous isotropic elastic medium with infinite extent. The uniform stress state caused by the remotely applied compressive stresses  $\sigma_{XX}$  and  $\sigma_{YY}$  is assumed as a reference state, and the relative change from this is analyzed below. Shear traction is assumed to be completely released on the crack surface. The stress drop on the crack inclined at angle  $\varphi$  with the  $X1$  axis is given by  $\Delta\sigma(\varphi) = \tau_0 \cos 2\varphi$ , where  $\tau_0 = (\sigma_{YY} - \sigma_{XX})/2$ . The static seed crack with right lateral slip is assumed on the plane  $\varphi = 0$ , where the maximum stress drop occurs. Stress drop is negative for cracks whose inclination angle  $\varphi$  is in the ranges  $45^\circ < |\varphi| < 135^\circ$  (gray shaded zones) because left lateral slip is expected to release the shear traction there. At each time step, the stress concentrations around the tips are calculated and compared with the fracture strength. If the maximum shear traction  $Tt(\varphi')$  exceeds the fracture strength  $T_c$ , the tip is assumed to extend by 1 element in the maximum direction  $\varphi'$ .

velocity attains  $0.87\beta$  and the maximum shear traction exceeds the fracture strength not on the original crack plane for the first time. The directions deviate from the original crack plane by  $\pm 42$  degrees; the angle is measure from the  $X1$  axis (see Figure 1). Here the crack is assumed to bend in the direction of  $-42$  degree, taking account of the effect of friction virtually. The crack generates compressive and tensile stresses in the regions marked with plus and minus symbols, respectively, in Figure 2. It is expected that slip occurs more easily in the tensile stress regime than in the compressive one. It will therefore be reasonable to assume the crack growth in the direction of  $-42$  degree. The crack bending increases with the growth; see the shots at  $t = 16.5$  and  $18.0$ . After  $t = 18.0$  the maximum shear traction never exceeds the fracture strength, so that the dynamic crack growth is arrested at this instant. The final crack length is  $l_{stop} = 21.24$ , which is about four times as large as the initial length;  $l_{stop}$  denotes the length of the final crack projected onto the  $X1$  axis. Contrary to one's intuition, the crack growth is spontaneously arrested even under homogeneous conditions.

The spontaneous crack arresting can be interpreted as follows; the crack continues to bend with growth and the tips finally enter regions where the shear traction to be released on the crack is negative (see Figure 1). This causes the significant reduction of stress concentration at the tips and the crack tip stress becomes less than the fracture strength. Our simulation accords with the observation that active faults in Japan usually have bends at their tips [Matsuda, 1967; Kanamori, 1972].



**Figure 2.** Snap shots of spontaneous crack growth with no constraints on the crack geometry. The model parameters ( $\tau_0$  and  $T_c$ ) are homogeneously distributed. The normalized parameters  $\tau_0 = 1.0$  and  $T_c = 1.21$  are used in the homogeneous model. The normalized initial crack length is  $l_0 = 5.0$ . The right column represents the angular distribution of shear traction  $Tt(\varphi)$  corresponding to the left column. Here  $\varphi$  does not represent the crack tip angle, but represents the angle of the inclined plane on which  $Tt$  acts (see Figure 1).

The above analysis indicates that dynamic growth of in-plane shear crack is prone to be arrested soon after the nucleation under compressive stresses. This can naturally explain the fact that a great majority of earthquakes do not grow to large ones. We, however, face a new problem, that is, the mechanism to drive an earthquake rupture to grow to a larger one. It is now shown that spatial inhomogeneities in the model parameters significantly affect the final crack length. We first investigate the effect of spatial inhomogeneity in the  $X1$  direction, and assume four typical examples, that is, models A, B, C and D (Figure 3). Around the seed crack, we assume the values for the fracture strength and the stress drop same as in the homogeneous model. Larger or smaller values are assumed for those quantities outside that zone. For each model, we calculate spontaneous crack growth and compare the final crack geometry with that obtained for the homogeneous model. It is observed for models A and D that the final crack length is smaller than that in the homogeneous model. Models A and D have, respectively, lower strength and higher stress drop outside the nucleation zone. These have been believed to be factors to accelerate the crack growth in prevailing ideas in which crack is implicitly assumed to grow straight. Hence our results apparently contradict intuitive expectation from such prevailing ideas. However we have to note that higher crack acceleration leads to sooner crack bending, so that the crack tip enters a region sooner where the stress released on the crack is negative. Models B and C have respectively higher strength and lower stress drop outside the nucleation zone. In these cases, low acceleration retards the initiation of crack bending, so that the crack grows much more than in the homogeneous model.

We now investigate the effect of fault zone structure, which is believed to have lower fracture strength than the surrounding crustal rocks. The fault zone is assumed as a singular plane with zero thickness (Figure 3E). This idealization is valid only when the crack length is much larger than the fault zone thickness. In this model, the outer high-strength regions are expected to impede the crack bending, so that much higher crack tip velocity must be attained for the beginning of bending than in the homogeneous model. The crack growth therefore lasts longer, and stops finally at the length of  $l_{stop} = 87.84$ . As we see in model E, fault zone structure has a property that helps a crack growth to continue longer. In such a planar preexisting zone, the final length will be longer for the greater strength contrast.

#### 4. Summary

Quantitative understanding of the arresting of dynamic earthquake faulting has crucial importance because it is related to the prediction of the size of future earthquakes. In this paper, we showed a spontaneous arresting mechanism by which we can predict the final fracture size deterministically. We, however, have to take account of more realistic situation to simulate actual earthquakes. For example, dynamic coalescence among cracks and constitutive friction law on crack faces may be crucial elements to be considered in such simulations. We also have to know the spatial distributions of model parameters precisely enough. Multi-disciplinary approach will be necessary for the purpose.

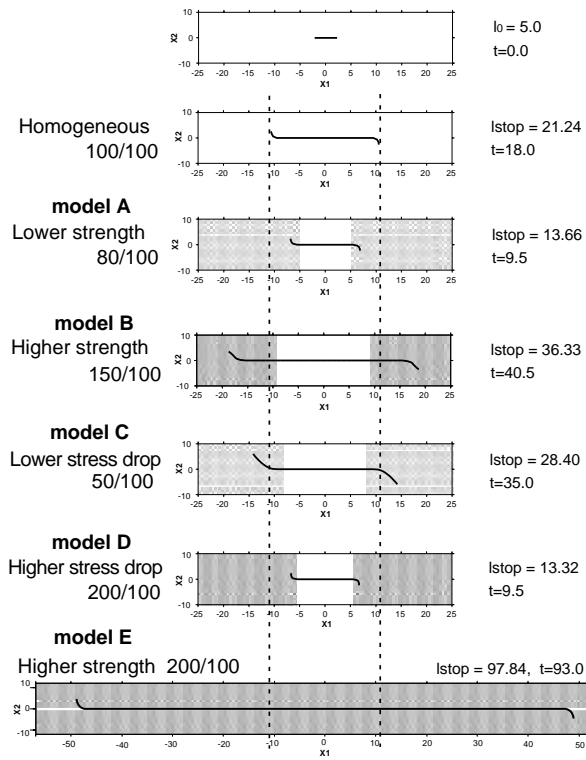
**Acknowledgments.** We are grateful to L. Knopoff for his constructive review. We are also grateful to H. Fujiwara for advising the finite part technique in evaluating the hypersingularities. N.K. was supported by a Research Fellowship of the Japan Society for the Promotion of Science for Young Scientists. T.Y. was

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**Figure 3.** Effects of inhomogeneities on the final crack lengths. Four types of spatial inhomogeneities in the  $X_1$  direction are assumed in the models A, B, C and D. The same parameters as in the homogeneous model are assumed in the white regions, while larger or smaller values are assumed in the gray regions. Model E has a fault zone structure that has strength inhomogeneity in the  $X_2$  direction. The degree of the inhomogeneity is represented by percentage.

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### Method and Model

We have developed a new mathematical method for the analysis of spontaneous crack growth with arbitrary crack geometry using Boundary Integral Equation Method (BIEM) [Kame, 1998; Kame and Yamashita, 1999]. We have overcome the mathematical difficulty to evaluate hypersingular kernels appearing in BIEM by

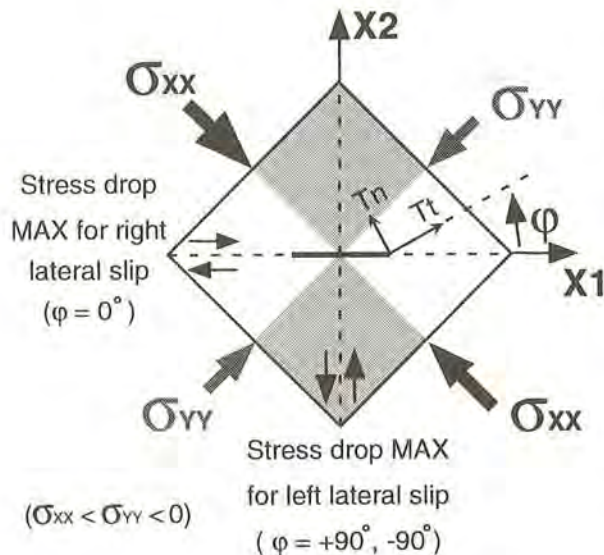
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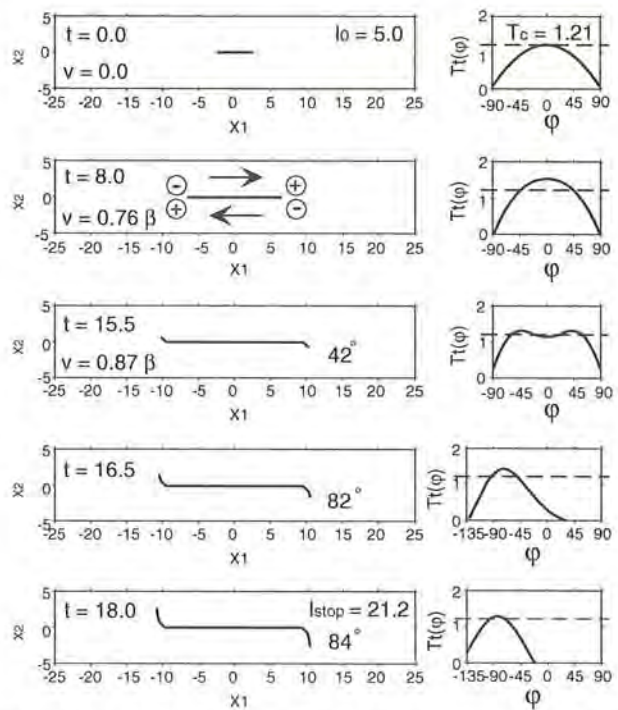
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**Figure 2.** Snap shots of spontaneous crack growth with no constraints on the crack geometry. The model parameters ( $\tau_0$  and  $Tc$ ) are homogeneously distributed. The normalized parameters  $\tau_0 = 1.0$  and  $Tc = 1.21$  are used in the homogeneous model. The normalized initial crack length is  $l_0 = 5.0$ . The right column represents the angular distribution of shear traction  $Tt(\varphi)$  corresponding to the left column. Here  $\varphi$  does not represent the crack tip angle, but represents the angle of the inclined plane on which  $Tt$  acts (see Figure 1).

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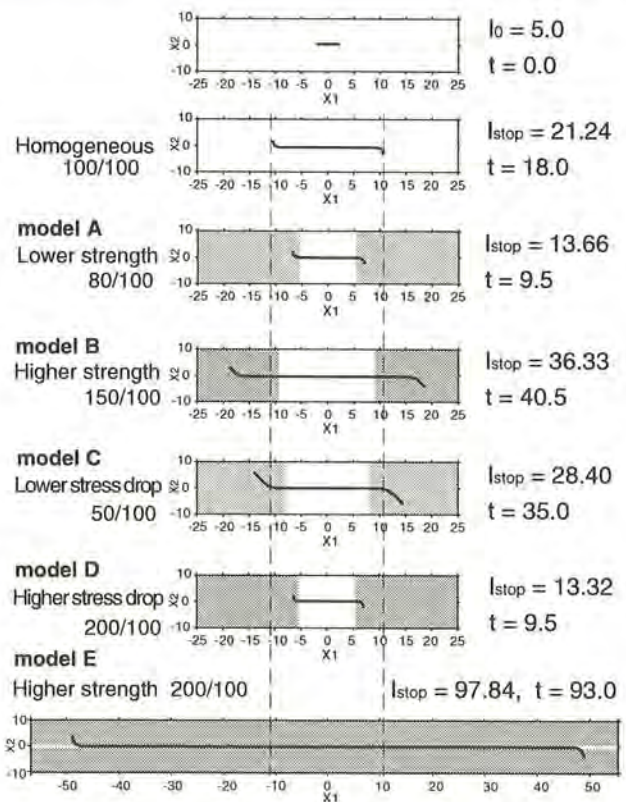
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## Summary

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