Pure Appl. Geophys. 172 (2015), 2237–2246

DOI 10.1007/s00024-013-0744-0

Pure and Applied Geophysics

Earthquake Nucleation on Faults with a Revised Rate- and State-Dependent Friction Law

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Abstract—Recently NAGATA *et al.* (J Geophys Res 117:B02314, 2012) have proposed a new version of rate- and state-dependent friction law (RSF) that seems to have eventually resolved all the previously known discrepancies in the existing RSFs from laboratory observations. The values of *a* and *b*, empirical RSF parameters determined by fitting the same laboratory experiments, have been revised to be five times greater and a newly noticed weakening effect by shear stress with a coefficient *c* has been introduced. By using this revised RSF, we reinvestigated a problem of 2D quasi-static nucleation on faults. A crack-like nucleation-zone expansion known for the 'aging' version of RSF is not sustainable with the 'Nagata' law, which is understandable as the Nagata law does not produce a slip-weakening distance proportional to the involved strength reduction, an aging law's feature that contradicts laboratory observations. The later stage of Nagata-law nucleation shows localization of quasi-static slip within a limited spatial extent, but the localization is much milder than that predicted by the 'slip' version of RSF. With an appropriate *c* parameter of the Nagata law, the nucleation size seems to be reduced only by a factor from that of the aging law.

Key words: Earthquake, nucleation, rate- and state-dependent friction, preslip.

1. Introduction

Quasi-static nucleation is expected to occur on faults embedded in an elastic continuum before they rupture dynamically. Empirical friction laws called rate- and state-dependent friction (RSF) (e.g., DIETERICH 1979; RUINA 1983) have been extensively used to study earthquake nucleation since Dieterich's pioneering work (DIETERICH 1992), because RSF is based on laboratory rock friction tests and hence, is considered to be a 'realistic' friction law.

However, RSF laws actually contradict some well-established aspects of laboratory observations (e.g., MARONE 1995; KATO and TULLIS 2001; NAKATANI 2001; NAGATA *et al.* 2012), and different versions of RSF laws have been proposed to fix the problem. So far, consequences on nucleation have been examined for two popular versions of RSF (e.g., RUINA 1983; BEELER *et al.* 1994), both of which have their own shortcomings in reproducing laboratory observations, to reveal important differences. With the 'aging' version of RSF, nucleation initiates as slip localization to a patch of limited length, followed in some cases by a late-stage expansion of the accelerating patch to a fairly large size (DIETERICH 1992; RUBIN and AMPUERO 2005; FANG *et al.* 2010). RUBIN and AMPUERO (2005) have shown that this late-stage crack-like expansion is because of the aging law's (wrong) feature of producing a long slip-weakening distance proportional to the magnitude of involved strength reduction, which clearly contradicts laboratory observations (e.g., RUINA 1983; NAKATANI 2001). In contrast, with the 'slip' version of RSF, which predicts a fixed, slip-weakening distance independent of the magnitude of strength reduction (as observed in the laboratory tests), the later stage of nucleation proceeds as a narrow, migrating slip pulse, and the resulting quasi-static moment release is much less. AMPUERO and RUBIN (2008) have shown that this is because of the inability of fracture energy (proportional to the product of 'slip-weakening distance' and 'strength reduction') around the nucleation front to catch up with the increase of the energy release rate of the growing nucleation zone. Although the slip law has a clear flaw that it can not reproduce the laboratory-observed, time-dependent healing for zero or nearly-zero slip velocities (BEELER *et al.* 1994; NAKATANI and MOCHIZUKI 1996), AMPUERO and RUBIN (2008) proposed that predictions from the slip law

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might be more relevant because the later (and hence critical) stage of nucleation proceeds under somewhat raised levels of slip velocity, for which frictional healing is insignificant.

Recently, using a new rigorous approach in experimental data analysis, Nagata *et al.* (2012) have proposed a new revised version of RSF (called the 'Nagata' law hereafter), where the above-mentioned shortcomings of the slip and aging versions have been eliminated, as extended in Sect. 2. Kame *et al.* (2012a, b) have shown the consequences of the Nagata law on the behavior of a single degree of freedom elastic system. From the standpoint that earthquake nucleation should be reexamined with the friction law that best explains the laboratory observations, we here present numerical experiments of earthquake nucleation on faults embedded in an elastic continuum using the Nagata law, and discuss the phenomenological differences from nucleation on slip-law and aging-law faults.

2. *Different Versions of RSF Laws and Their Parameter Values*

2.1. *Formula for Three Versions of RSF*

RSF generally consists of two equations bearing logically separate roles (Nakatani 2001). One is the constitutive law, which describes the relationship between applied shear stress τ and slip rate V under the given physical state of the interface, specified by a state variable Φ. The other is the evolution law, which describes changes of the state Φ for various reasons. All the three versions of RSF presently discussed use the same constitutive formula

$$V = V_* \exp\left(\frac{\tau - \Phi}{a\sigma}\right), \quad \text{or} \quad \tau = \Phi + a\sigma \ln\left(\frac{V}{V_*}\right). \tag{1}$$

In this notation (Nakatani 2001), Φ is the stress required to cause slip at a reference velocity V_*. In this sense, Φ has an intuitive meaning as frictional strength analogous to the traditional concept of frictional strength as the threshold stress for slip to occur. Microscopically, Φ is often associated with the real contact area, which can be monitored independently by, for example, optical or acoustic methods (Dieterich and Kilgore 1996; Nagata *et al.* 2008, 2012). σ is the normal stress and a is an empirical constant called the direct effect coefficient, which may reflect the thermally-activated nature of the shear creep of frictional junctions (Heslot *et al.* 1994; Nakatani 2001). A smaller a means higher sensitivity of slip velocity to the applied stress, which makes the behavior closer to the traditional, strictly threshold-type friction. One important point in the revision of RSF (Nagata *et al.* 2012) is the much increased value of a by several times. When other factors are not considered, increased a may enhance ductile behavior such as quasi-static nucleation.

We now introduce three versions of the evolution laws presently discussed. Although many of the empirical constants appear repeatedly in different versions, their physical and/or phenomenological meanings are not necessarily the same between different versions.

The slip version of the evolution law is expressed as

$$\frac{d\Phi}{dt} = -\frac{V}{D_c}\left[\Phi - \Phi_* - b\sigma \ln\left(\frac{V_*}{V}\right)\right] \qquad \text{(Slip law)}, \tag{2}$$

where Φ_* is a reference state, and b and D_c are empirical constants. From Eq. (2), we can easily see that the steady-state value of Φ is given by

$$\Phi_{SS} = \Phi_* - b\sigma \ln(V/V_*). \tag{3}$$

Under a constant slip velocity, the slip law predicts an evolution of Φ that proceeds exponentially with slip, toward the steady-state value Φ_{SS}. The characteristic slip distance for the evolution is predicted to be D_c, an empirical constant appearing in Eq. (2), meaning that the evolution distance is not affected by the magnitude of the ongoing state change $\Delta\Phi$ (e.g., Plate1b of Nakatani 2001). Inserting Eq. (3) to Eq. (1), the shear stress τ_{SS} during the steady-state sliding is given by

$$\tau_{SS} = \Phi_* - (b - a)\ln(V/V_*). \tag{4}$$

Hence, $b - a$ is called a velocity-weakening parameter and a reference stress at $V = V_*$ is given by $\tau_* = \Phi_*$. Actually, these expressions for Φ_{SS} and τ_{SS} are common to all three versions of the RSF presently discussed. Therefore, the value of $b - a$, which is reliably constrained by the measurements of τ_{SS} at various velocities, must be the same for all the three RSF versions. It is readily seen that Φ does not change when $V = 0$, so the slip law contradicts the truly time-dependent healing observed at the zero shear-stress level (i.e., $V = 0$) (Nakatani and Mochizuki 1996). Likewise, the slip law grossly underpredicts the quasi-static healing at low (but non-zero) velocities (Beeler *et al*. 1994; Nakatani and Mochizuki 1996), which may be experienced during the interseismic period of an earthquake cycle.

The aging version of the evolution law is expressed as

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{b\sigma}{D_c} V_* \exp\left(-\frac{\Phi - \Phi_*}{b\sigma}\right) - \frac{b\sigma}{D_c} V \quad \text{(Aging law).} \tag{5}$$

The first term of Eq. (5) represents truly time-dependent healing. When $V = 0$, Φ increases by b per e-fold increase in time of stationary contact. The second term represents linear slip weakening at a constant rate b/D_c per unit slip. As mentioned earlier, expressions for steady-state Φ_{SS} and shear stress τ_{SS} become the same as those for slip law. When Φ is significantly larger than the steady state, as typically expected in the nucleation front, the state evolution under a constant V is dominated by the second linear slip-weakening term, and hence, the slip distance to complete the slip weakening is proportional to the magnitude of strength reduction $\Delta\Phi$ (e.g., Plate 1a of Nakatani 2001; Fig. 21a of Nagata 2012). This contradicts the laboratory observation that slip weakening is observed to complete within the same slip distance, independent of the magnitude of the involved strength reduction (e.g., Ruina 1983; Nakatani 2001).

Recently, Nagata *et al*. (2012) proposed a revised RSF on the basis of new rigorous methods of laboratory data analysis. Firstly, the direct effect coefficient a was constrained to be about 0.05, about five times larger than previously believed. The difference came from their new method to constrain a without using any evolution laws, contrasting to conventional methods involving the inference of the considerable state change during the imperfect velocity 'step' in real-world friction tests, for which some 'potentially wrong' evolution law had to be assumed. The larger a immediately led to similarly large b $\sim$0.05 because $b - a$ $\sim$0 was reliably constrained from velocity dependence of steady-state friction without any evolution laws. Secondly, a strong linear, negative dependence of $\mathrm{d}\Phi/\mathrm{d}t$ on $\mathrm{d}\tau/\mathrm{d}t$ was newly found from the misprediction analysis of Φ between the observed $\Phi(= \tau - a\sigma \ln(V/V_*))$ and the predicted Φ by the aging law. By adding a term representing this shear-stress effect to the traditional aging law, Nagata *et al*. (2012) proposed a new evolution law as

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{b\sigma}{D_c} V^* \exp\left(-\frac{\Phi - \Phi_*}{b\sigma}\right) - \frac{b\sigma}{D_c} V - c\frac{\mathrm{d}\tau}{\mathrm{d}t} \quad \text{(Nagata law),} \tag{6}$$

where c is an empirical constant representing the stress-weakening effect. From the above misprediction analysis, the value of c was determined to be about 2.0, which was supported by other lines of laboratory observations (Nagata *et al*. 2012) as well.

The most directly relevant consequence of the Nagata law to the present study is that it predicts an effectively constant slip-weakening distance. Although the slip-weakening term, the second term of Eq. (6), is itself the same between the Nagata law and the (problematic) aging law, the Nagata law as a whole does predict slip-weakening distances affected little by the magnitude of involved strength reduction (Figure 21b of Nagata *et al*. 2012). It is not that the Nagata law has a hard mechanism to ensure a constant slip-weakening distance like the slip law. Indeed, some positive dependence of slip-weakening distance on the magnitude of involved strength reduction is seen with the Nagata law, but the dependence is far less pronounced than the case with the aging law (Figure 21a of Nagata *et al*. 2012). According to the interpretation of different nucleation behaviors between slip-law faults and aging-law faults (Ampuero and Rubin 2008) mentioned in Sect. 1, we may expect that the nucleation of Nagata-law faults would be close to that of slip-law faults.

Likewise, the dependence of the evolution distance on the sign of involved strength change (Figure 20a of Nagata *et al*. 2012), another famous contradiction of the aging law to laboratory data (e.g., Marone 1995), has been effectively eliminated by the

Nagata law (Figure 20b of Nagata *et al.* 2012). Furthermore, Nagata *et al.* (2012) confirmed that the revised RSF could correctly reproduce both hold-slide and velocity-step tests with the same values of frictional parameters, which had never been achieved by the existing RSFs. The excellent match throughout the complex sequence of imposed loading history was not only the case with the shear stress history, but also the case with the history of Φ monitored acoustically, adding further confidence in the Nagata law as the best phenomenological description of laboratory friction experiments.

2.2. *RSF Parameter Values*

As extended in Sect. 2.1, physical/phenomenological meanings of the frictional parameters are not necessarily the same between different versions of RSF. From the standpoint of the present paper, that is, to explore earthquake nucleation expected from the friction 'as observed' in laboratory, comparison should not be done among the different RSF versions with the same values set to each parameter, as is usually done. Different RSF formulae are different approximations of the same laboratory observation, so we take the parameter sets that fit the same reference data with each formula as counterparts to be compared. As a data set showing typical behaviors of room-temperature rock friction tests, we use the velocity-step tests of Nagata *et al.* (2012) as a reference data set (called 'Data N' hereafter) and obtain different sets of parameters by the fitting of this same data with each formula. Further, noticing that setting $c = 0.0$ in the Nagata law leads to the aging law, we see that the parameter c tunes 'Nagataness'. Nagata *et al.* (2012) found that $c = 2.0$ is the optimum, but we prepare three sets of parameter values by fitting the same experimental data N with c fixed to 0.0, 2.0, or 4.0. The set with $c = 0.0$ (denoted N-0) is the pure aging law, $c = 2.0$ (set N-2) is the appropriately Nagatish set, and $c = 4.0$ (set N-4) is an overly Nagatish set. This last set N-4 was prepared in order to explore a systematic dependence of nucleation behavior on c. Also, since $c = 4.0$ can be certainly too large for the suite of experiments of Nagata *et al.* (2012), N-0 and N-4 can serve as bounds for the range of nucleation behavior depending on the adopted formula. Parameter values for these sets are shown in Table 1. Fittings of the data with each parameter set are shown in Fig. 2a of Kame *et al.* (2012a). Note that $b - a$ is 0.0055 for all of N-0, N-2, and N-4, whereas the value of a and b increases a lot when a larger c is assumed. Also, note that all the theoretical curves with N-0, N-2, and N-4 have effectively the same slip-weakening distance matching the data, though the D_c value is smaller for an increased c.

Table 1

RSF Parameters (a, b, c, D_c)

RSF#	a	b	c	D_c (μm)
N-0	0.017	0.0225	0.0	0.62
N-2	0.051	0.0565	2.0	0.33
N-4	0.085	0.0905	4.0	0.20
A-0	0.034	0.0395	0.0	0.62
A-2	0.102	0.1075	2.0	0.33
A-4	0.170	0.1755	4.0	0.20
B-0	0.017	0.035	0.0	0.62
B-2	0.051	0.069	2.0	0.33
B-4	0.085	0.103	4.0	0.20

Of course, frictional properties of faults should have some variations, even within a range of conditions relevant to earthquake nucleation. Hence, we prepare two more sets of data representing somewhat different frictional properties, following Kame *et al.* (2012a). We first generate 'Data A' to represent a surface having a twice as large of a direct effect a but the same velocity-weakening parameter $b - a$ (= 0.0055) as N-2, by simulating a velocity-step test using the Nagata law with a parameter set of $(a, b, c, L) = (0.102, 0.1075, 2.0, 0.33\ \mu\text{m})$. Then, we obtain parameter sets A-0 and A-4 by fitting this Data A with $c = 0.0$ and 4.0 assumed, respectively. Parameter set A-2 is of course exactly what we set to generate Data A. On the other hand, Data B is generated with another parameter set B-2 of $(a, b, c, L) = (0.051, 0.069, 2.0, 0.33\ \mu\text{m})$. This set, where $b - a = 0.018$, represents a surface exhibiting stronger velocity weakening. Parameter set B-0 and B-4 are made with the fitting of Data B, with $c = 0.0$ and 4.0 assumed, respectively. All the parameter values are listed in Table 1 and corresponding velocity-step responses are shown in Fig. 2 of Kame *et al.* (2012a).

3. *Simulation Method*

Following Dieterich (1992), quasi-static nucleation is modeled here by adopting the Nagata law. A fault is divided into n equally-spaced segments with a length Δs and loaded by a constant stressing rate $\dot{\tau}_r$:

$$\tau_i = \tau_i^0 + \dot{\tau}_r t + \Delta\tau_i \qquad (i = 1, 2, \ldots, n), \qquad (7)$$

where τ_i is the shear stress, τ_i^0 is the initial stress, $\Delta\tau_i (= \sum S_{ij}\delta_j)$ is the change of stress resulting from the slip δ_j over the fault, and S_{ij} is the stress kernel obtained from elastic dislocation solutions. By equating Eq. (7) with Eq. (1) and by substituting the Nagata law of Eq. (6) to eliminate Φ, we obtain a couple of nondimensional, differential equations as

$$d\tau_i'/dt' = \dot{\tau}_r' + S_{ij}' V_j', \qquad (8)$$

$$d(\ln V_i')/dt' = (a/b)^{-1}\{(1+c)d\tau_i'/dt' - \exp[-(\tau_i' - (a/b)\ln V_i')] + V_i'\}, \qquad (9)$$

where $\tau_i' = (\tau_i - \tau_*)/(b\sigma), V_i' = V_i/V_*, t' = t/(D_c/V_*), \dot{\tau}_r' = \dot{\tau}_r/(b\sigma/(D_c/V_*)), S_{ij}' = S_{ij}/(b\sigma/D_c)$. A factor $(1 + c)$ resulted from the shear-stress-dependent term, which reduces to the aging law case when $c = 0.0$.

For aging law faults, Rubin and Ampuero (2005) have shown that spatial discretization has to be fine enough to resolve a length scale

$$L_b \equiv \frac{\mu_* D_c}{b\sigma}, \qquad (10)$$

where $\mu_* = \mu/(1 - \nu)$ is the stiffness of a medium for edge dislocation, μ is the rigidity, and ν is the Poisson's ratio. Nucleation length varies from a few to hundreds of L_b (Rubin and Ampuero 2005; Ampuero and Rubin 2008). Slip-law faults require a further finer resolution by a ln V' factor (Ampuero and Rubin 2008). Because the Nagata law is more similar to the slip law, we settle on the segment length $\Delta s = L_b/20$. We have confirmed that further two- or four-times finer spacing, comparable to those employed for the slip law in Ampuero and Rubin (2008), does not affect the results. A whole fault is represented by $n = 2{,}400$ segments (i.e., fault extent is $0 \le x/L_b \le 120$) with pinned ends.

It has to be noted that b and D_c appear in the above normalization, whereas they are different for different RSF formulae, even for the same frictional response (i.e., the same laboratory data). From this view point, meaningful comparison must be done among results with different formulae intended to fit the same reference data. Such comparison should be meaningful not only in the qualitative but also in the quantitative sense. For this reason, the normalizing factor for each plot axis is set to be the same between presentations of results with different RSF versions. Specifically, D_c, $b\sigma$, and L_b from the N-0 set are used as the normalizing factors for slip, stress, and spatial coordinates in presentations of all the N-0, N-2, and N-4 results. Comparison is done in the same way for cases with Data A and Data B, by using the same normalization scales based on A-0 and B-0, respectively. Of course, presentation without normalization is another sensible choice, but we do normalize to facilitate quantitative comparison with the slip-law nucleation of Figures 2 and 7 of Ampuero and Rubin (2008). The only exception is the metric velocity axes employed in Ampuero and Rubin (2008). In the following, $V_* = 10^{-9}$ m/s is chosen for direct comparison of our results with the figures of Ampuero and Rubin (2008).

All the simulations are done by using normalized variables. Following Rubin and Ampuero (2005), frictional properties and initial velocity $(V_i'^0 = 1.0(= V_*'))$ are assumed to be uniformly distributed, whereas initial stress $\tau_i'^0$ is assumed to be randomly distributed between $[-1,\ 0]$ that is significantly lower than $\tau_{SS}'(V_*) = 1.0(= \tau_*' = \Phi_*')$. Initially, healing occurs and the velocity decreases down to $\sim 10^{-12}$–10^{-11} m/s, which realizes a strongly locked fault. As the applied stress increases with time, nucleation starts. $\dot{\tau}_r' = 0.1$ and $\mu'_* (= \mu_*/(b\sigma)) = 11.56 \times 10^3$ are chosen the same as in Ampuero and Rubin (2008). Time integration is numerically done by using the Runge-Kutta method (Press *et al.* 1992) until $V'_{\max} = 10^9 V'_*$.

The pinned boundary may affect the location of nucleation. If a uniform $\tau_i'^0$ is assumed, nucleation necessarily occurs at the center. Under the random initial conditions described above, however, nucleation location varied widely, losing particular preference with respect to the pinned ends, suggesting that the boundary effect is limited.

4. Results and Interpretations

We start with the results for N-0, N-2, and N-4 (Fig. 1). Data N (the laboratory data of Nagata *et al*. 2012) showing modest velocity weakening ($b - a = 0.0055$) may be typical at room temperature. As mentioned earlier, each parameter set was obtained by the fit of Data N with different degrees of stress-weakening (represented by parameter c) assumed. The Nagata law with the parameter set N-0 is nothing more than the classical aging law and Fig. 1a shows the nucleation proceeds in a crack-like expansion mode as expected from the theoretical prediction of Rubin and Ampuero (2005) for the aging-law faults with $1 > a/b > 0.5$ ($a/b = 0.755$ in our N-0 case). In the initial stage where the elastic stress concentration at the front of the slipping patch is minor (4th stress profile in Fig. 1a), the nucleation is a self-accelerating small patch of a half-length of

$$L_v = 1.3774 L_b, \tag{11}$$

which is derived from the balance of the localization effect due to slip/velocity-weakening friction and the expansion effect due to elasticity (Rubin and Ampuero 2005). When frontal stress concentration becomes significant (5–6th stress profiles in Fig. 1a), crack-like expansion commences. Rubin and Ampuero (2005) showed that the transition to a crack-like expansion was related to the return to steady-state within the accelerating patch. The velocity profile remains fairly flat and significant slip proceeds throughout the expanding crack. This 'crack-like' expansion is distinguished from another mode of nucleation style called 'propagating pulse', where significant ongoing slip is restricted to a narrow region at the edge of the already slipped part (Fig. 2 of Ampuero and Rubin 2008), which is expected for slip-law faults (Ampuero and Rubin 2008). With the aging law, the crack-like expansion of a quasi-static slip region can grow up to a limiting size, whose half-length L_∞ is given by

$$L_\infty = \left(\frac{b}{b-a}\right)^2 \frac{L_b}{\pi} \qquad (b - a > 0). \tag{12}$$

It is derived from the balance of the crack's energy release rate and fracture energy at the nucleation front (Rubin and Ampuero 2005), and it can be very large when a/b is close to (but less than) unity as often seen in laboratory results. Also, Rubin and Ampuero (2005) have pointed out that velocity-neutral friction may be expected at the bottom of a seismogenic layer, where large earthquakes tend to nucleate. Rubin and Ampuero (2005) have semi-analytically examined the behavior of the involved equations and concluded that such is a consequence of the aging law's (wrong) feature that slip-weakening distance and, hence, fracture energy strongly increase with the logarithmic velocity jump at the front of the nucleation zone, which increases as the nucleation proceeds (velocity profile in Fig. 1a).

Figure 1b shows the result with the N-2 parameter set derived by the fitting of the same data N with stress-weakening factor c assumed to be 2.0, the value considered to be the most likely from various lines of evidence (Nagata *et al*. 2012). This N-2 result is the central result of the present paper. As explained earlier, the values of a, b, and D_c are all different from those of the N-0 set. Hence, the value of a/b, which is an important controlling parameter for nucleation with aging and slip laws (Rubin and Ampuero 2005; Ampuero and Rubin 2008), is different from that of N-0.

The nucleation in the N-2 case grows from a small patch and then turns (e.g., the 8th velocity profile in Fig. 1b) to an expanding crack with significant stress concentration at both ends. Up to this point, it is very similar to the N-0 case. However, after some crack-like expansion (e.g., the 11th velocity profile in Fig. 1b), the slip near the right-hand front stops accelerating, whereas the remaining part accelerates further and the leftward expansion of the nucleation zone still proceeds in a similar manner to the N-0 case. The N-2 nucleation after the 11th profile may be called a propagating pulse, as in the slip-law nucleation (Ampuero and Rubin 2008), but we note that its velocity profile behind the propagating tip remains fairly flat and broad, contrasting to the sharp and narrow pulse obtained with the slip law (Figs. 2 and 7 of Ampuero and Rubin 2008). This N-2 result is in between the aging-law nucleation and the slip-law nucleation. The displacement profile of N-2 shows that the leftmost two-thirds of the nucleation zone remained active throughout the process, earning significant moment release in this final stage after the

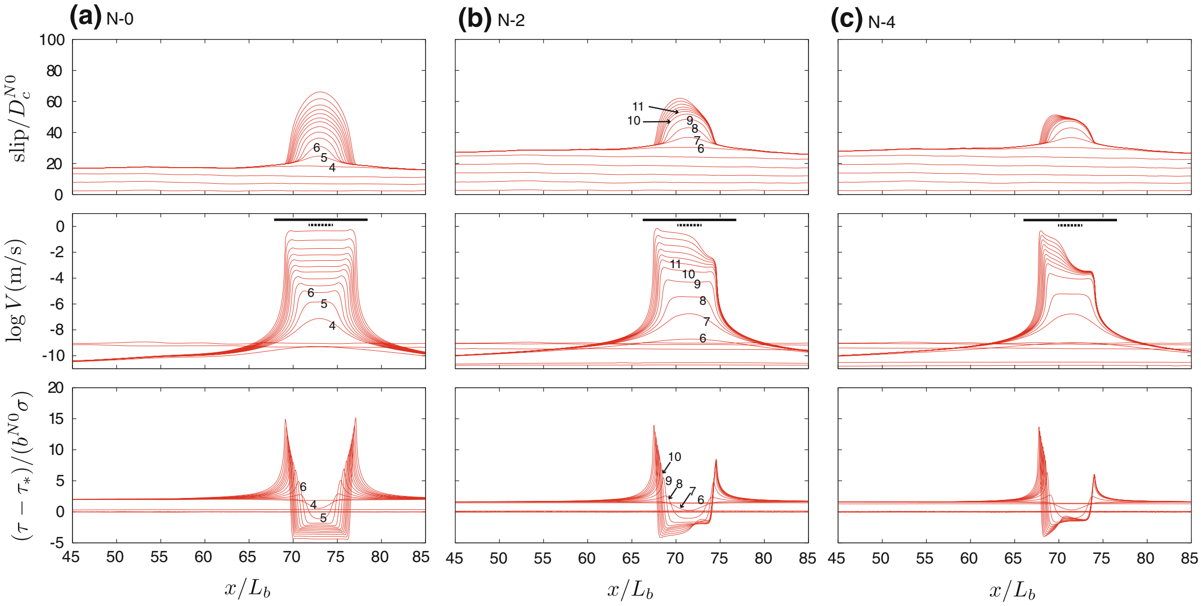

Figure 1

Three simulation results with RSF parameter sets fitted to the same reference Data N: **a** N-0, **b** N-2, and **c** N-4. Top, middle, and bottom panels represent snapshots of normalized slip, slip velocity and normalized shear stress, respectively. Lengths of $2L_\infty$ (*solid bar*) and $2L_v$ (*dashed bar*) for the aging law are plotted for reference

10th profile. The total moment release since the start of significant localization (4th slip profile in N-0, 6th slip profile in N-2) is reduced only by a factor compared to the N-0 case.

The overly Nagatish N-4 nucleation (Fig. 1c) shows stronger localization than that of the N-2, suggesting that a larger c value somehow pushes the Nagata-law nucleation closer to the slip-law nucleation. However, the localization in the N-4 case is still considerably weaker than slip-law nucleation. Since $c = 4$ is certainly too extreme compared with laboratory observations, we would argue that earthquake nucleation expected for friction 'as observed in laboratory tests' is in between those with the aging and slip laws.

The slip-weakening distance predicted by the Nagata law is similar to that predicted by the slip law, nearly independent of the imposed velocity jump or involved strength reduction, as mentioned in our subsection 2.1 and shown in Fig. 21 of Nagata *et al.* (2012). Therefore, an expanding crack should not be a sustainable mode of quasi-static nucleation, as Ampuero and Rubin (2008) realized for slip-law nucleation. On the other hand, we do not really know why the 'pulse' in the later stage of Nagata-law nucleation shows much weaker localization than in the slip-law nucleation, given that operation of truly time-dependent healing (which is present in the Nagata law, but not present in the slip law) would not affect the process at this later stage at considerable slip velocities, as Ampuero and Rubin (2008) argued. As mentioned earlier, our intention is to investigate how nucleation occurs if we use the 'correct' description (i.e., the Nagata law) of typical laboratory friction tests, and mathematical origins underlying the nucleation process are beyond the scope of the present study. It is worth mentioning that very recently, Bhattacharya and Rubin (2012) presented a mathematical insight into this problem supported by analytical results and numerical simulations.

In Fig. 1b and c, expansion stopped at the side closer to the pinned end at $x/L_b = 120$, while expansion at the other side continued. This may appear to be a boundary effect of the pinned end because a pinned end tends to suppress the slip, resulting in migration away from the closer pinned end. However, among different realizations of random initial conditions (not shown here), migration

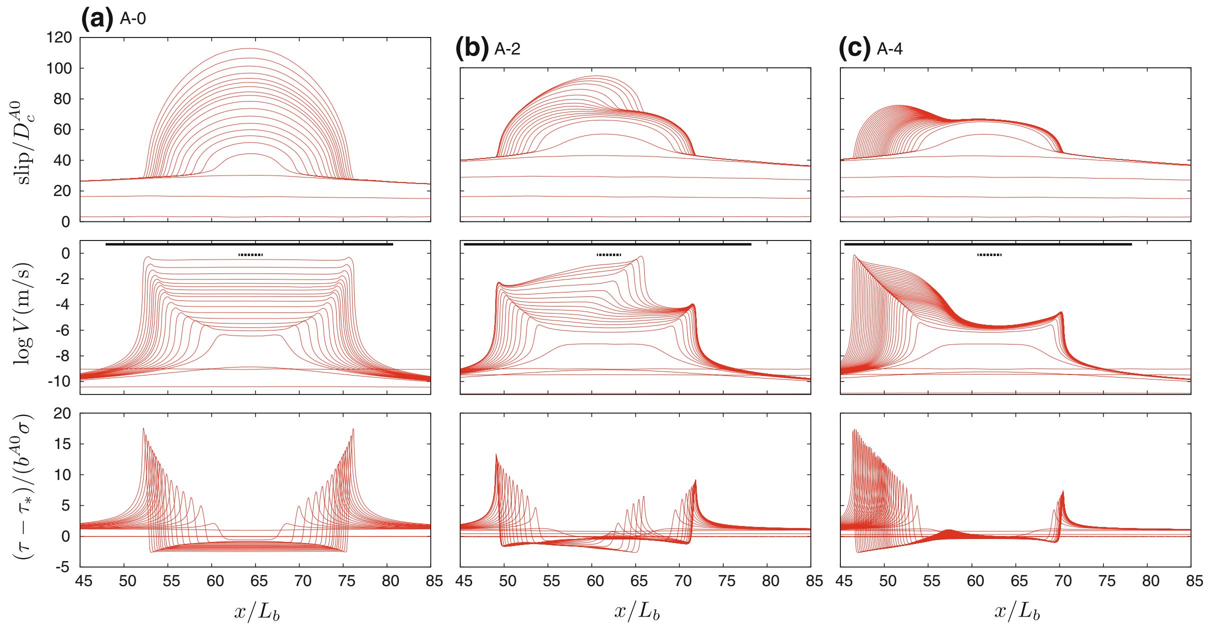

Figure 2

Three simulation results with RSF parameter sets fitted to the same reference Data A: **a** A-0, **b** A-2, and **c** A-4. Top, middle, and bottom panels represent snapshots of normalized slip, slip velocity, and normalized shear stress, respectively. Lengths of $2L_\infty$ (*solid bar*) and $2L_\mathrm{v}$ (*dashed bar*) for the aging law are plotted for reference

toward the closer pinned end occured as often as migration away from it.

Now we try other parameter sets A-0, A-2, and A-4 (Fig. 2). Each set corresponds to an (imaginary) data for a surface with the same extent of velocity weakening $b - a = 0.0055$, but showing a twice-greater direct effect. The A-0 (aging-interpretation) parameters yield the a/b ratio of 0.86, closer to unity than $a/b = 0.78$ in the N-0 set. As seen from Eq. (12), L_∞ for A-0 is about three times greater than that for N-0, so the A-0 nucleation can grow much larger than N-0 nucleation, as confirmed from Figs. 1a and 2a.

Figure 2b shows the result with the parameter set A-2 (i.e., appropriately Nagatish interpretation), where c is assumed to be 2.0. All the features of Nagata-law nucleation suggested by the comparison of N-2 with N-0 are replicated here. The nucleation style changes from an expanding crack to a weakly localized pulse where the actively slipping zone remains fairly large, measuring about the two-thirds of the large nucleation zone of the A-0 (aging-law) case. From the displacement profiles, we see that the total moment release is only reduced by a factor from the A-0 case, and is larger by an order of magnitude than the slip-law nucleation ($a/b = 0.8$ and 0.9 cases in Fig. 7 of Ampuero and Rubin 2008). As a result, the strong divergence of nucleation size as a/b approaches to unity, known for aging-law nucleation (as compared N-0 and A-0), is still relevant in the Nagata-law case, as confirmed by a comparison of N-2 and A-2.

Figure 2c shows the result with the parameter set A-4. As expected, the A-4 case yields a narrower pulse, but again the localization is considerably weaker than in the slip-law nucleation ($a/b = 0.8$ and 0.9 cases in Fig. 7 of Ampuero and Rubin 2008).

Finally, we briefly report the results for B-0, B-2, and B-4 (Fig. 3). Each parameter set corresponds to a strongly velocity-weakening case, where a/b for B-0 (aging law fitting) is 0.49. In all cases, the result was the acceleration of a fixed-length patch of a few times of L_b, as already known for both aging and slip laws with $a/b < 0.5$, for which the patch half-length is $L_\mathrm{v} = 1.3774 L_b$ for the aging law and slightly smaller for the slip law (Rubin and Ampuero 2005; Ampuero and Rubin 2008).

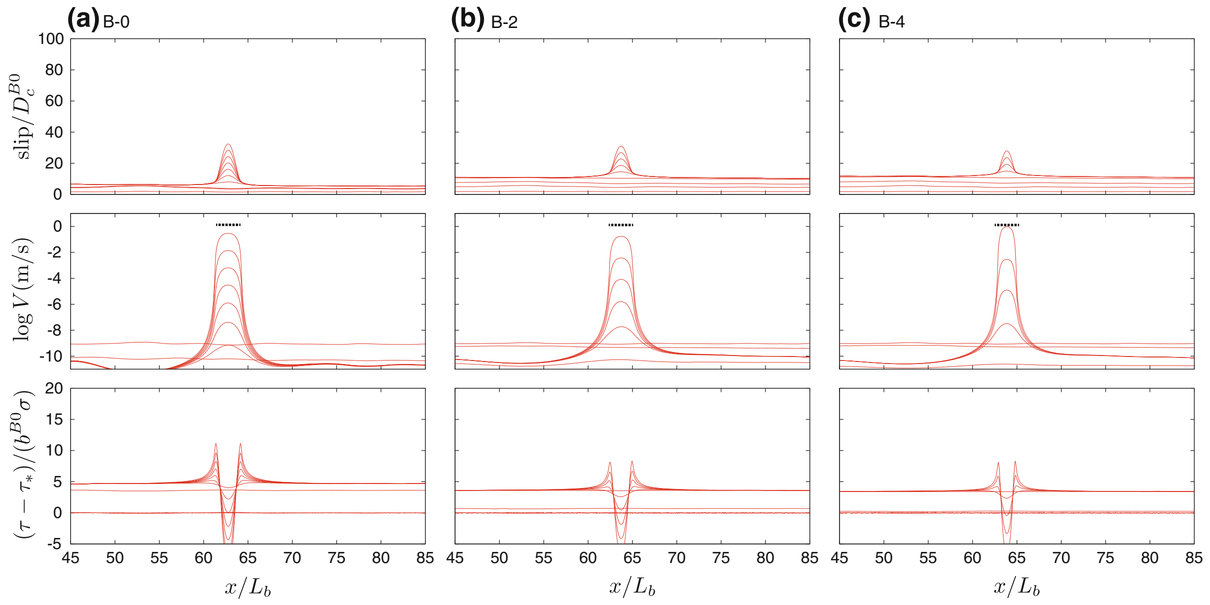

Figure 3

Three simulation results with RSF parameter sets fitted to the same reference Data B: **a** B-0, **b** B-2, and **c** B-4. *Top, middle*, and *bottom panels* represent snapshots of normalized slip, slip velocity, and normalized shear stress, respectively. Length of $2L_\nu$ (dashed bar) for the aging law is plotted for reference

5. Conclusion

By numerical experiments on a frictional fault embedded in an elastic continuum, we have confirmed that the crack-like nucleation-zone expansion, which can grow to a fairly large size for faults obeying the aging law (RUBIN and AMPUERO 2005), is not sustainable for faults obeying the Nagata law. This agrees with the theory attributing such a large quasi-static growth to the aging law's (wrong) feature of slip-weakening distance increasing linearly with the magnitude of the involved strength drop, a feature not shared by the Nagata law. However, deviation from the crack-like expansion is less dramatic with the Nagata law, compared with the strongly localized slip-pulse mode of nucleation predicted by the slip law. Nagata-law nucleation in its later stage takes the form of a mildly localized slip pulse. The resultant quasi-static moment release in Nagata-law nucleation is considerably smaller than that expected from the aging law, but it is still much larger than that expected from the slip law. The strong increase of nucleation-zone size with a decrease of the extent of velocity weakening, expected on aging-law faults, seems to be relevant to Nagata-law faults as well, though FANG *et al.* (2010) have pointed out that the very large limiting size expected of the aging law is not necessarily reached during an earthquake cycle.

Acknowledgments

The comments by two anonymous reviewers are greatly appreciated. Naoyuki Kato provided his numerical code. N.K. was supported by MEXT KAKENHI 21107007. This study was also supported by MEXT, under its Observation and Research Program for Prediction of Earthquakes and Volcanic Eruptions.

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(Received March 11, 2013, revised August 25, 2013, accepted November 19, 2013, Published online December 6, 2013)