2005 AGU Fall Meeting: Poster #S43A

Quasi-statically Self-chosen Faulting Path Modeling in Heterogeneous Medium - FEM-b Approach -

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1. Abstract

- We apply *FEM-β*, a newly proposed Finite Element Method (Hori, Oguni and Sakaguchi, JMPS, 2005), to quasi-statically *self-chosen faulting path modeling*.
- The method, *FEM-β*, is based on *particle discretization* of a displacement field with non-overlapping shape function and it provides an easy way to express *displacement discontinuities between any two adjacent nodes*: this is an advantage of FEM-β for self-chosen failure path modeling.
- *FEM-β*, originally developed for the analysis on tensile failure within a structual material containing local imperfection, is here tested for *earthquake shear faulting in strongly heterogeneous medium* to investigate the effect of elastic heterogeneity in the crust on *the formation of geometrically complex fault traces*.

2. MOTIVATION

- Non-planar fault traces in the heterogeneous crustal structure

The effect of *elastic heterogeneity* on *the formation of geometrically complex fault traces*
→ Quantitative approach by *self-chosen failure path modeling*

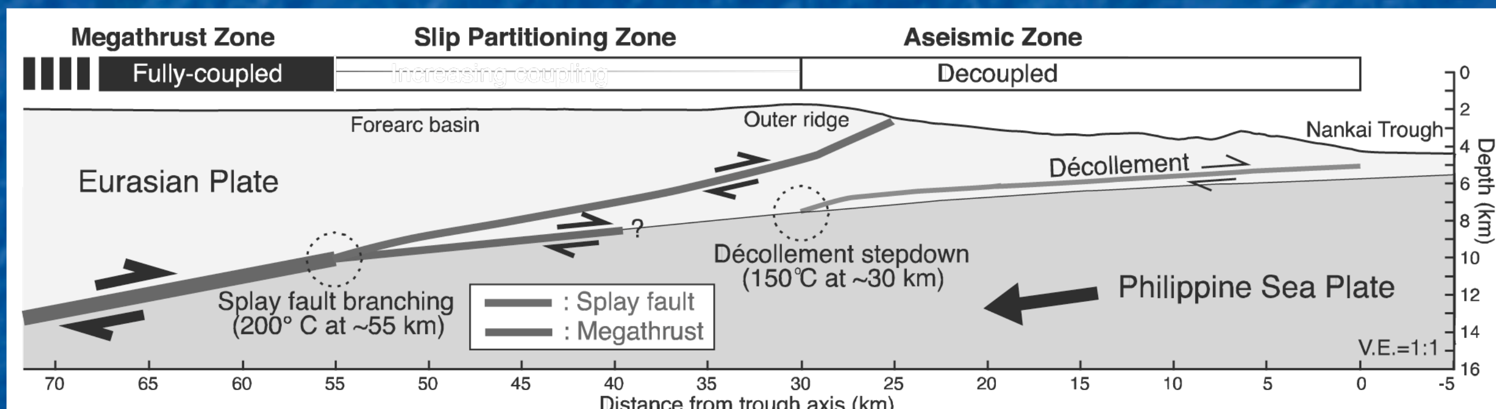


Fig.1: Schematic cross section of the updip portion of the Nankai subduction zone: splay fault branching (Park et al., Science, 2003)

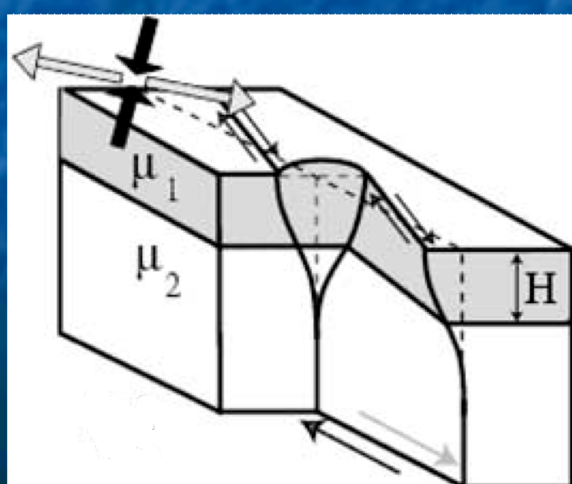


Fig.2: Non-planar fault surfaces and en-echelon surface ruptures in a layered medium (Bonafede et al., GJI, 2002)

3. Self-chosen faulting path modeling in heterogeneous medium

(3A) FEM-β : Finite Elemant Method with particle discritization (Hori et al., 2005)

Ordinary FEM

◎different elasticity

× discontinuity

FEM-β

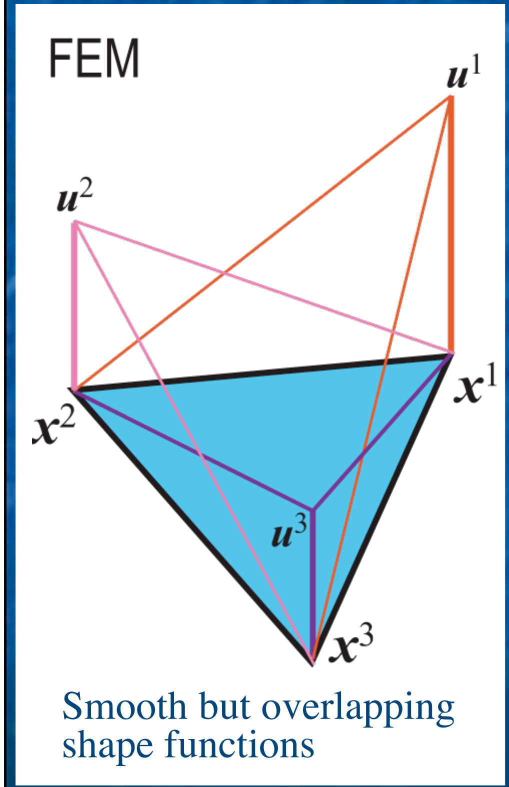
◎different elasticity

◎discontinuity

Displacement on Volonoi blocks (Ω^1, Ω^2, Ω^3,…)

→ Easy expression of failure as separation of two adjacent Voronoi blocks

→ Enabling self-chosen failure path modeling



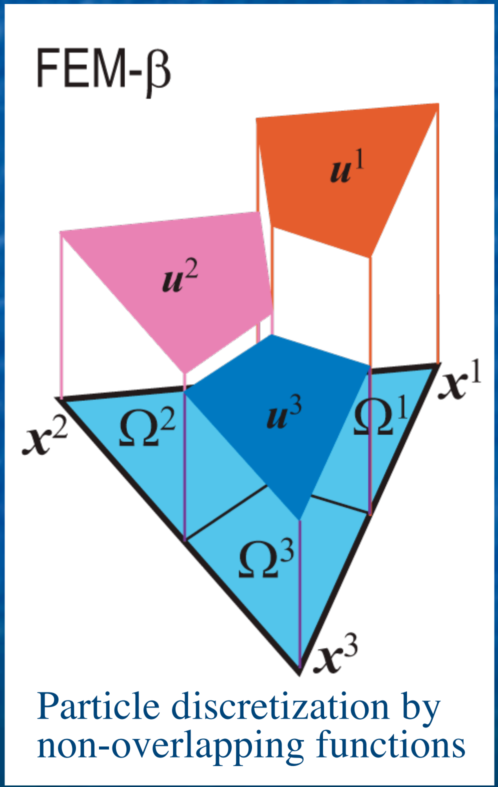


Fig.3. Comparison of discretization for ordinary FEM and FEM-β.

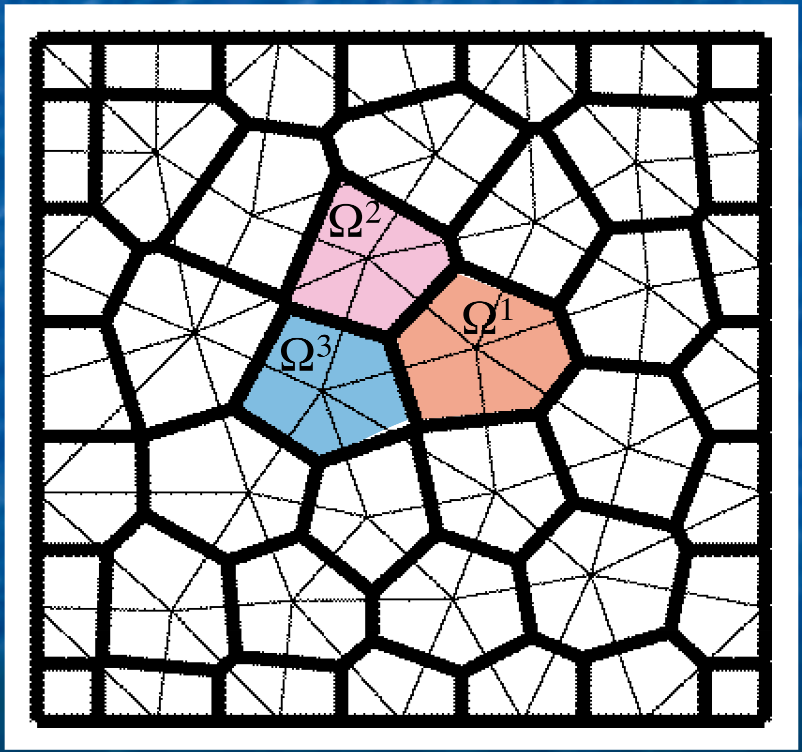


Fig.4. Voronoi (thick lines) and Delaunay (thin) tessellation for 2-D domain in FEM-β.

(3B) Interpretation of FEM-β as block-spring model

- B-matrix of FEM-β is equivalent to the ordinary FEM B-matrix.
- Particle discretization enables us to interpret FEM as Spring-Block System: Strain is represented by the displacement gap $\Delta^1=u^1-u^2$, Δ^2 *and* Δ^3.
- For a gap $\Delta^1=u^1-u^2$, three springs are responsible for the strain energy.

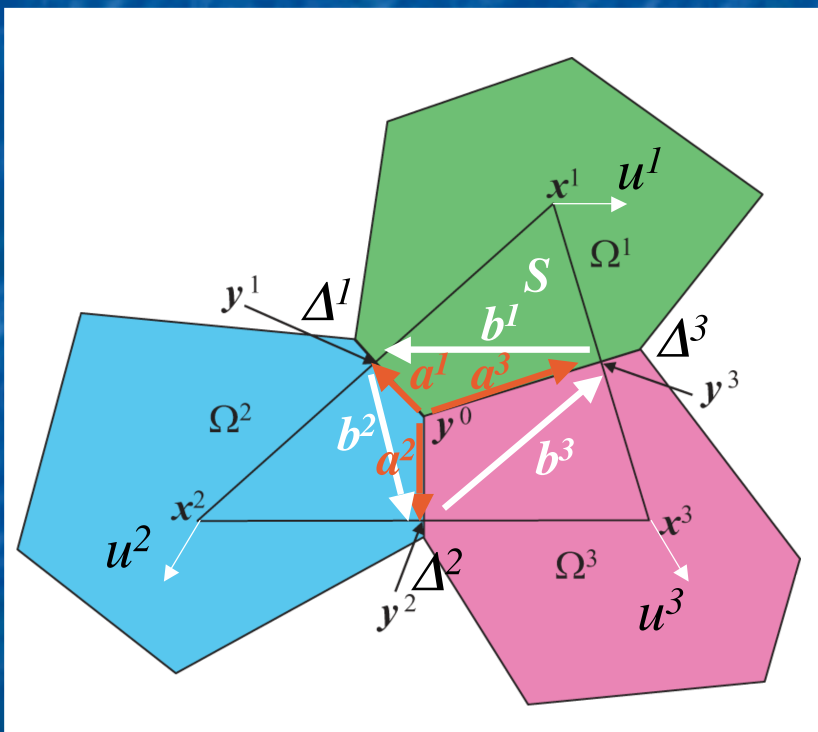


Fig.5. Strain average domain S in FEM-β

$$\begin{bmatrix} e_{11}\\ e_{22}\\ 2e_{12}\end{bmatrix} = \frac{1}{2S}\begin{bmatrix} b_y^1 & 0 & b_y^2 & 0 & b_y^3 & 0\\ 0 & -b_x^1 & 0 & -b_x^2 & 0 & -b_x^3 \\ -b_x^1 & b_y^1 & -b_x^2 & b_y^2 & -b_x^3 & b_y^3\end{bmatrix}\begin{bmatrix}u^1\\u^2\\u^3\end{bmatrix}$$

e = B u

$$= \frac{1}{2S}\begin{bmatrix} a_y^1-a_y^3 & 0 & a_y^2-a_y^1 & 0 & a_y^3-a_y^2 & 0\\ 0 & -(a_y^1-a_y^3) & 0 & -(a_y^2-a_y^1) & 0 & -(a_y^3-a_y^2)\\ -(a_y^1-a_y^3) & a_y^1-a_y^3 & -(a_y^2-a_y^1) & a_y^2-a_y^1 & -(a_y^3-a_y^2) & a_y^3-a_y^2\end{bmatrix}\begin{bmatrix}u^1\\u^2\\u^3\end{bmatrix}$$

= A Δ

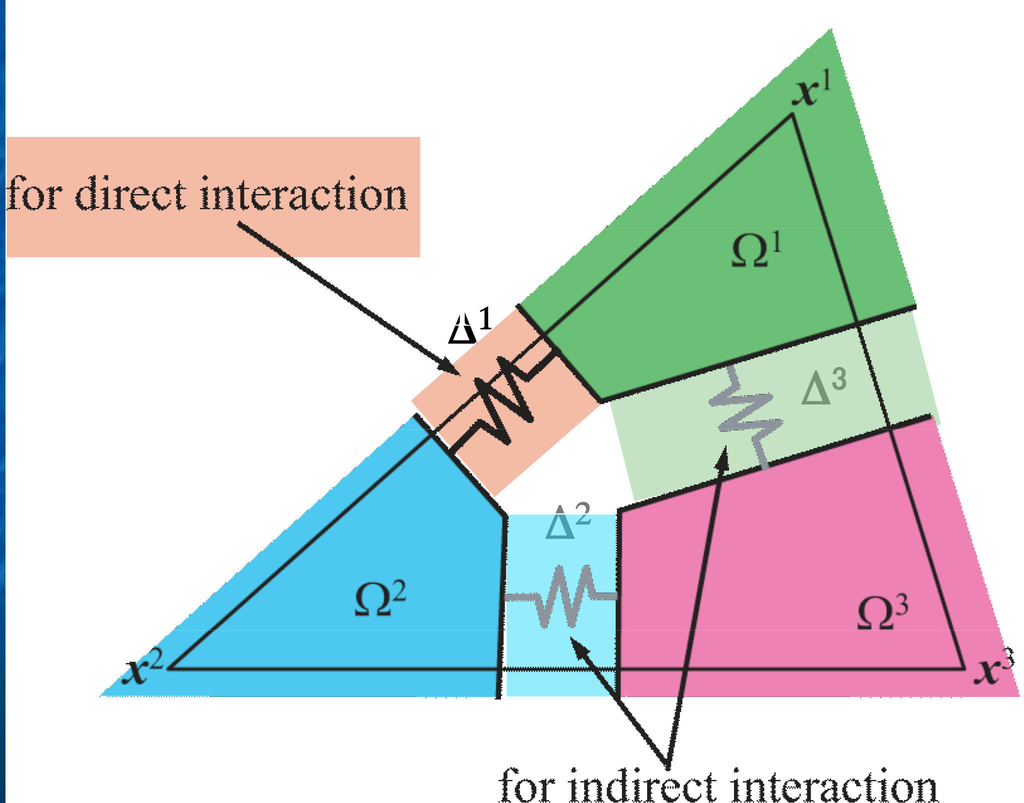
$$= \frac{1}{2S}\begin{bmatrix} a_y^1 & 0 & a_y^2 & 0 & a_y^3 & 0\\ 0 & -a_x^1 & 0 & -a_x^2 & 0 & -a_x^3 \\ -a_x^1 & a_y^1 & -a_x^2 & a_y^2 & -a_x^3 & a_y^3\end{bmatrix}\begin{bmatrix}\Delta^1\\ \Delta^2\\ \Delta^3\end{bmatrix}$$

Fig.6. Strain ***e*** in terms of ***u*** with **B**-matrix and in terms of ***Δ*** with **A**-matrix in FEM-β.

(3C) Expressing Shear failure in FEM-b

FEM-β, originally developed for tensile failure, is here tested for shear faulting.

- Tensile failure as a displacement gap Δ^1: subtract B-matrix components corresponding to cutting of the three springs.
- Shear failure: subtract B-matrix components responsible only for the shear strain along a failure plane (no change in the normal strain).



$$J(e) = \frac{1}{2}\begin{bmatrix}u^1 & u^2 & u^3\end{bmatrix} \underbrace{B^T D\, B}_{=K} \begin{bmatrix}u^1\\ u^2\\ u^3\end{bmatrix}$$

J(*e*): strain energy
K: element stiffness matrix
D: elasticity matrix

$$= \frac{1}{2}\begin{bmatrix}\Delta^1 & \Delta^2 & \Delta^3\end{bmatrix} A^T D^A A \begin{bmatrix}\Delta^1\\ \Delta^2\\ \Delta^3\end{bmatrix}$$

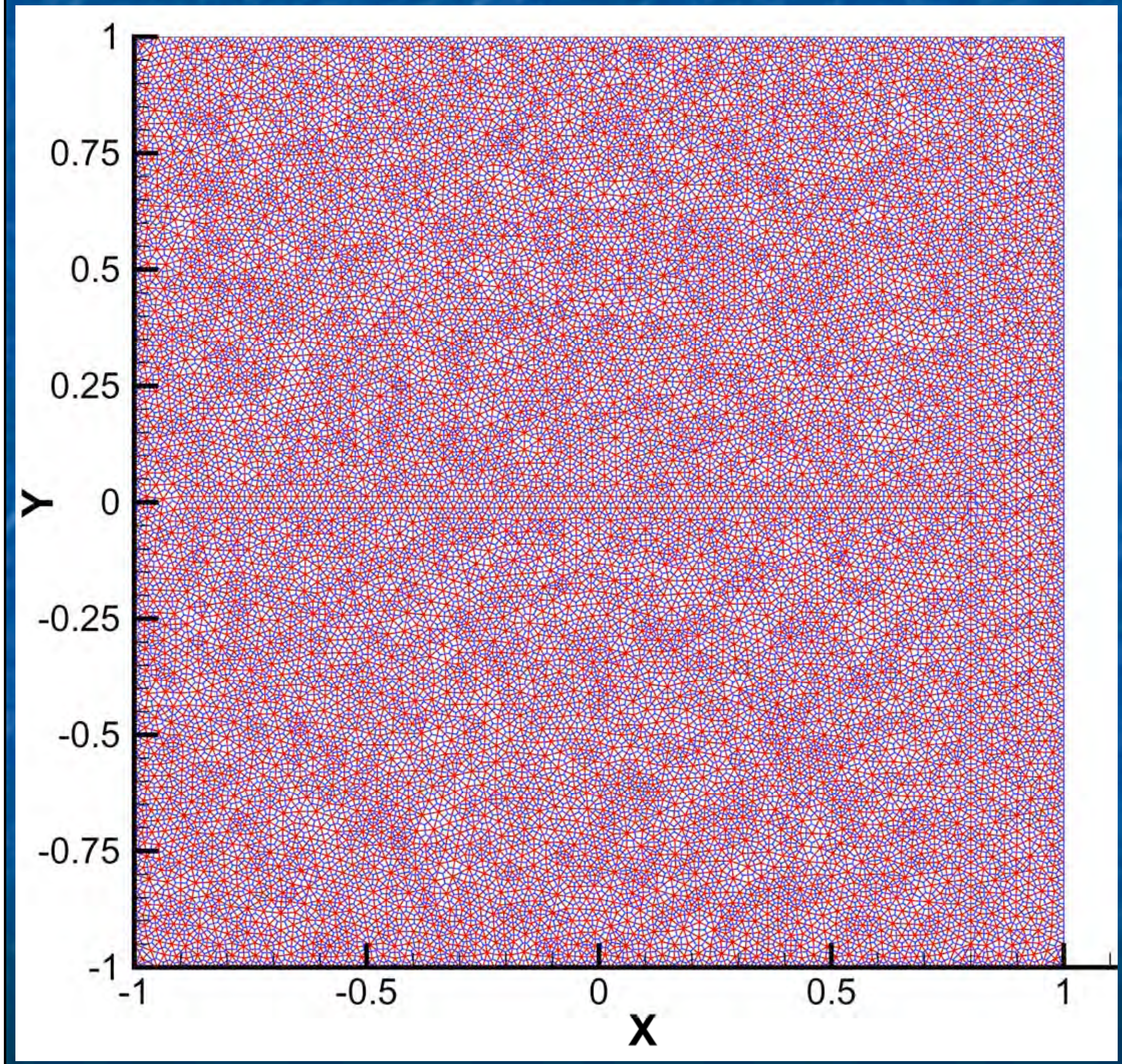
$$J(\Delta^1) = \frac{1}{2}(A^{1^T} D^A A^1)\Delta^1\Delta^1$$

$$+\frac{1}{2}(A^{2^T} D^A A^1)\Delta^2\Delta^1 + \frac{1}{2}(A^{3^T} D^A A^1)\Delta^3\Delta^1$$

Fig.7. Three springs conecting Ω^1 and Ω^2 in Δ^{123}

4. Results for Mode II crack

- Deformation: *2-D* plane strain. Simple shear loading up to $\varepsilon_{12}=0.05$.
- B.C. fixed displacement on $y=1,-1$, free surface on $x=1, -1$.



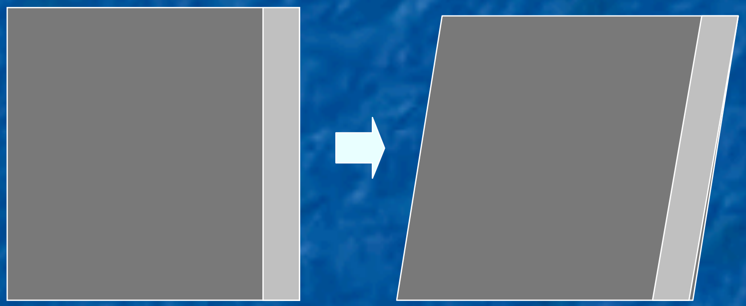


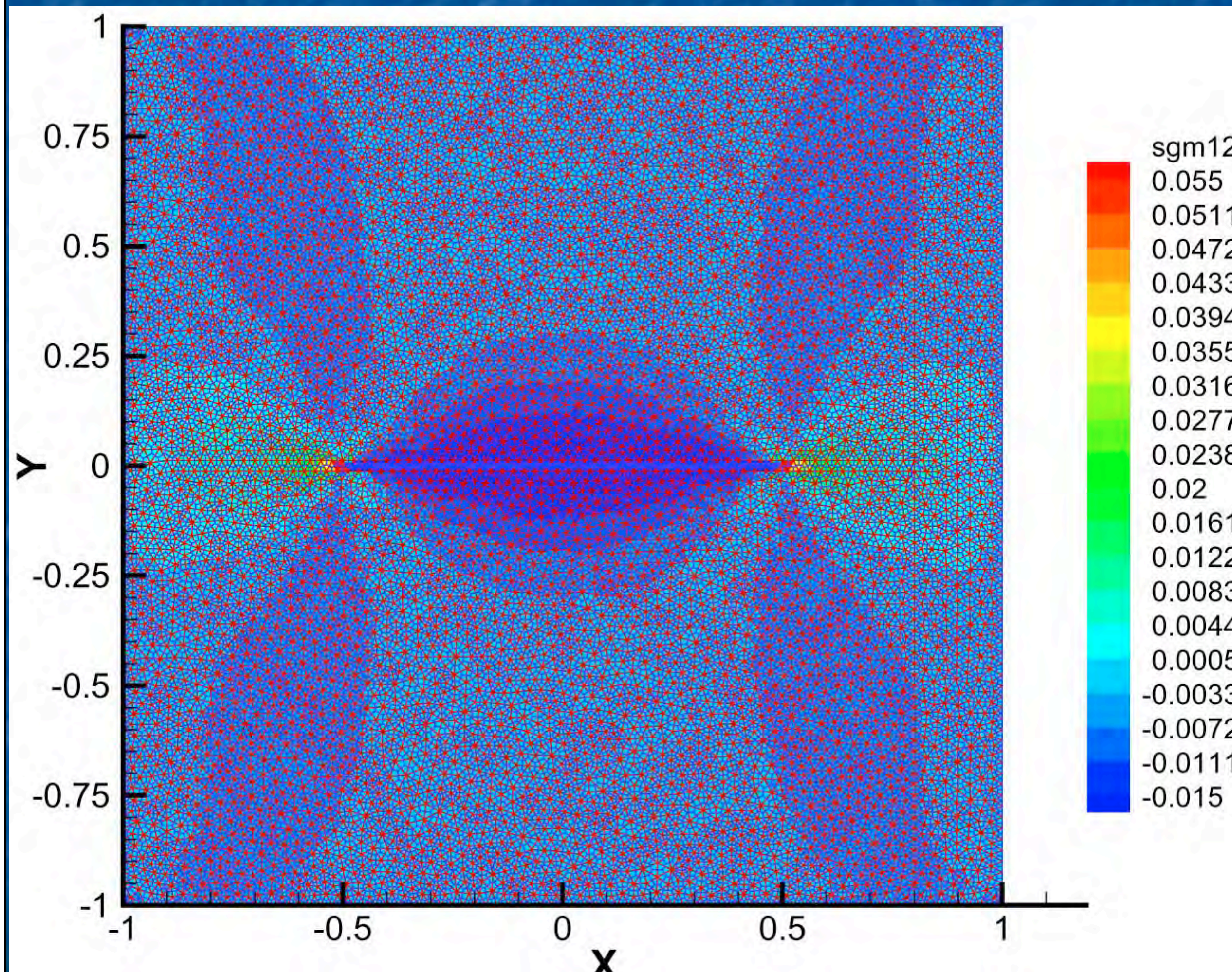
Fig.8. FEM-β mesh used in the analyses. 12918 elements & 6620 blocks.

Red lines:
Delaunay triangulation of the domain for the stress and strain.

Blue lines:
Voronoi block boundaries on which displacement gap (=failure) is allowed.

(4a) Homogeneous medium

Fig.9. FEM-β: Stress change in σ_{12} due to a crack in homogeneous medium (*mode II*)



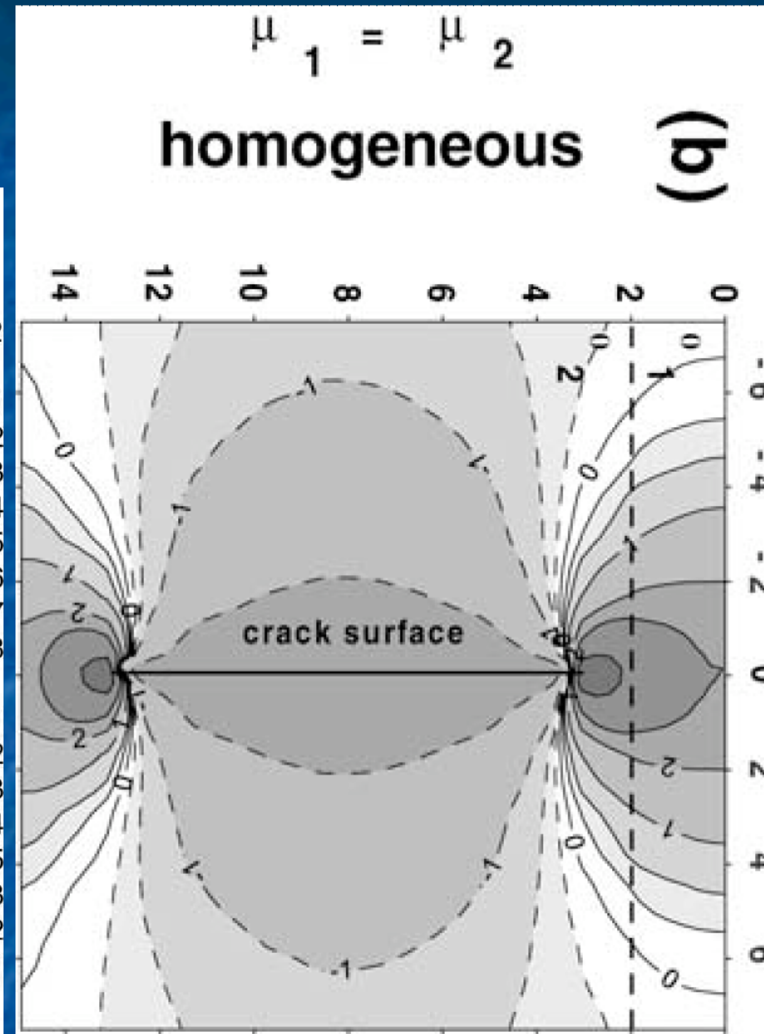
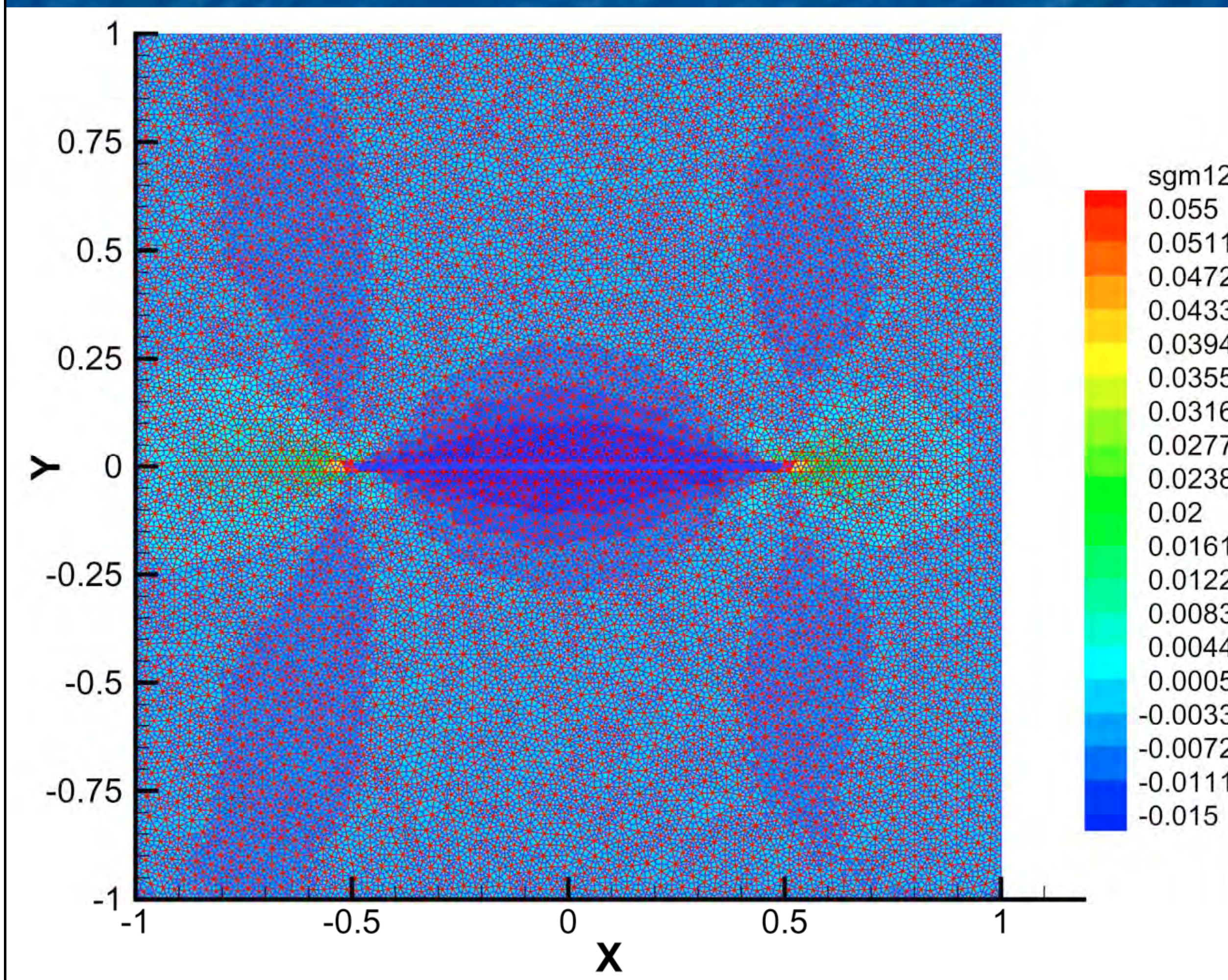


Fig.10. Analytic solution (*mode III*) of strress change in σ_{23} in homogeneous medium (Bonafede *et al.*, 2002)

(4b) Hoterogeneous (Layered) medium: $\mu_1=0.1\mu_2$

Fig.11. FEM-β: Stress change in σ_{12} due to a crack in layerd medium (*mode II*).



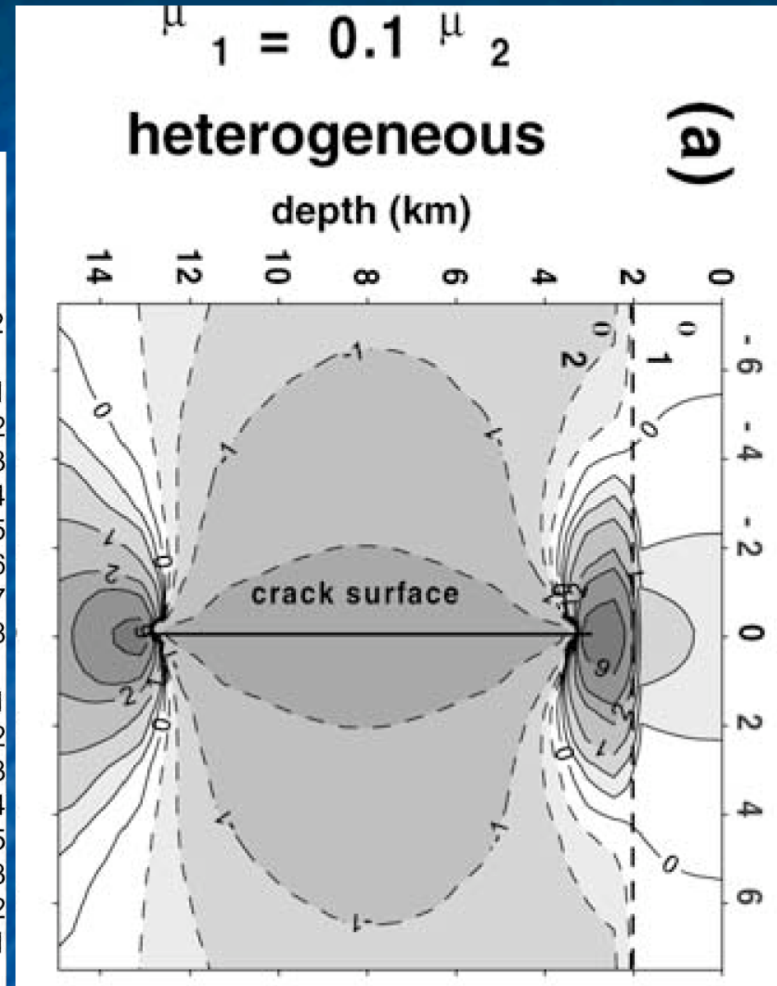


Fig.12. Analytic solution (*mode III*) of stress change in σ_{23} in layered medium (Bonafede *et al*., 2002)

(4c) Strains and stresses for different crack positions against the soft layer

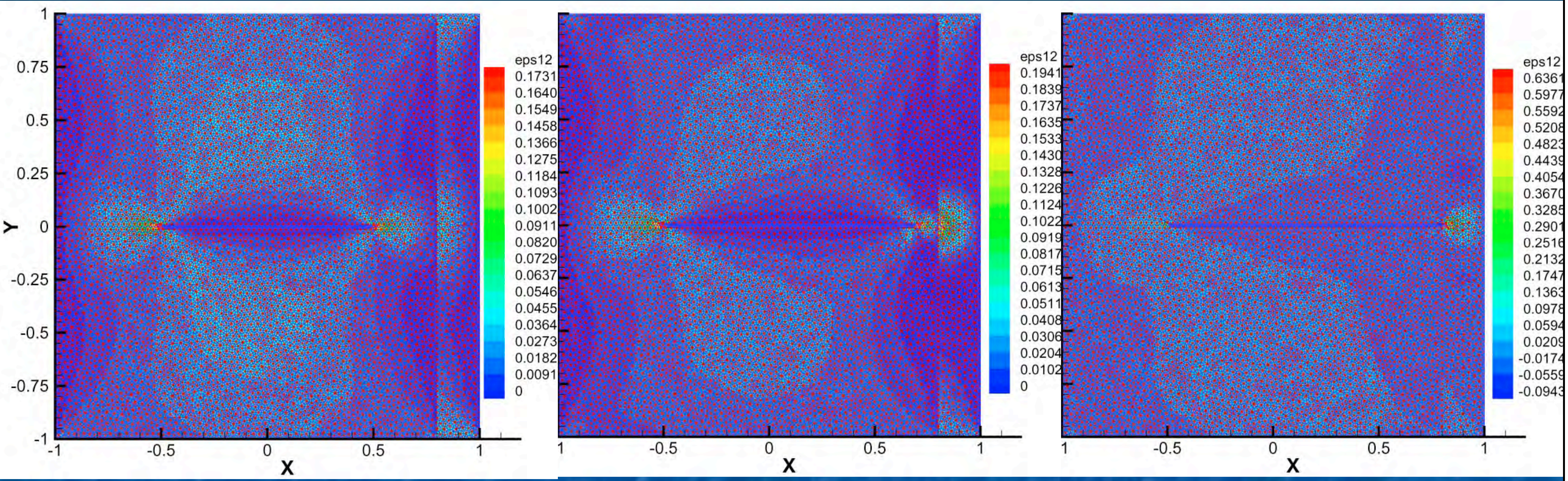


Fig.13. FEM-β: Strain (ε_{12}) due to crack in layerd medium (*mode II*).

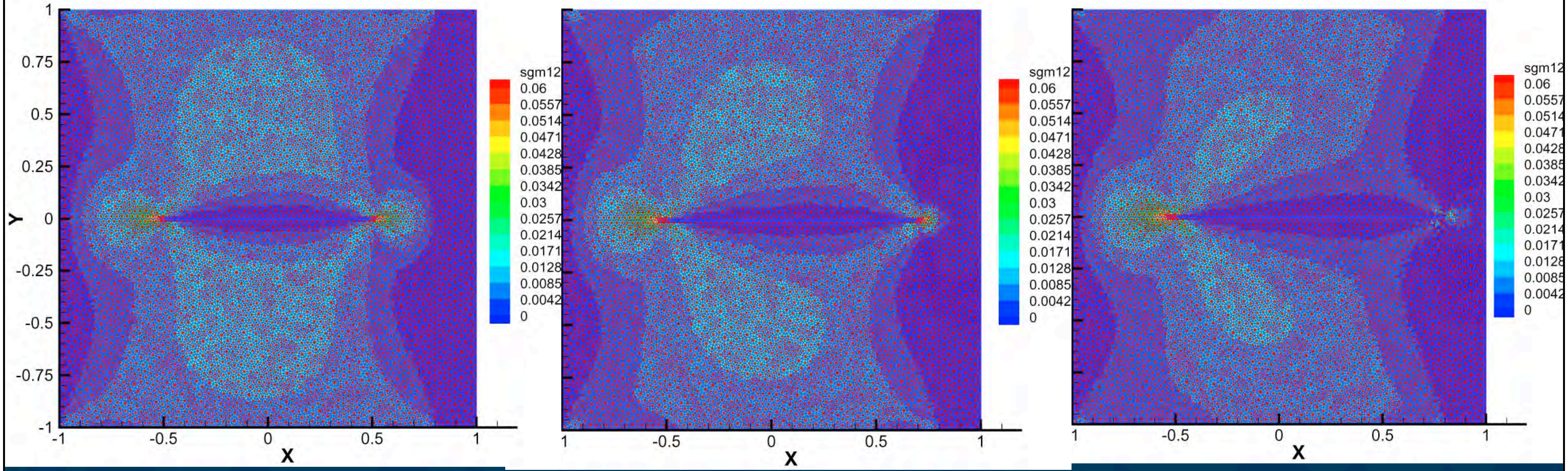


Fig.14. FEM-β: Stress (σ_{12}) due to crack in layerd medium (*mode II*).

5. Summary

- *Self-chosen faulting path modeling in heterogeneous medium is considered.*
- *FEM-β, developed for tensile failure, is here tested for shear crack growth.*
- *The stress field in layered medium containing a crack is analyzed.*

Further studies:

- *Analysis of quasi-static crack growth toward stress concentration direction*
- *Criteria for rupture growth in heterogeneous medium.*
- *Extension of FEM-β to dynamic analysis*