Abstract. Spatio-temporal variation of earthquake activity is modeled assuming fluid migration in a narrow porous fault zone whose boundaries are impermeable. The duration of earthquake sequence is assumed to be much shorter than the recurrence period of characteristic events on the fault. Principle of the effective stress coupled to the Coulomb failure criterion introduces mechanical coupling between fault slip and pore fluid pressure. A linear relation is assumed in our simulations between the accumulated slip and fault zone width on the basis of laboratory and field observations. High complexity is observed in the rupture activity so long as an inhomogeneity is introduced in the spatial distribution of initial strength, which is defined as the fracture threshold stress before the intrusion of the fluid. Frequency-magnitude statistics of intermediate-size events obeys the Gutenberg-Richter relation for all the models in which spatial heterogeneity is introduced for the initial strength. The behavior of larger-size events seems to be rather model dependent. It is also observed that the rupture occurrence tends to be inactivated immediately before the occurrence of the largest event in a sequence. This never happens if a brittle rupture is assumed in an elastic medium with no mechanical effect of fluid. This inactivation will occur because it takes much time to build up fluid pressure to break a fault segment having high initial strength, whose rupture triggers the largest event in a sequence. Our calculations also show that a single predominant principal event cannot be observed in a sequence when both the variance and average value of the distributed initial strengths are large. This may explain a feature observed for earthquake swarm.

Introduction

It is widely believed that fluids play an important role in many aspects of earthquake faulting. As a typical mechanical effect, we can mention the lowering of fracture strength by fluid pressure on the basis of the principle of the effective stress coupled to the Coulomb failure criterion [Raleigh et al., 1976], which is thought to be applicable at least to the top few kilometers of the crust [Raleigh et al., 1976; Zoback and Hickman, 1982; Zoback and Healy, 1984; Hickman et al., 1995]. The role of pore fluid in reducing the effective value of the confining stress in bulk samples and the normal stress across frictional surfaces has been demonstrated in laboratory experiments [e.g., Brace and Martin, 1968; Byerlee and Brace, 1972]. Field evidence comes from earthquakes induced either through direct injection of fluids down boreholes or from the filling of large reservoirs with subsequent infiltration of water into the underlying rock mass [e.g., Healy et al., 1968; Raleigh et al., 1976]. Additional evidence of mechanical involvement of fluids in earthquake faulting comes from the substantial change in groundwater level, and surface discharge before and after some earthquakes in the shallow crust. For example, the variation in groundwater flow is observed at many locations before and after the 1995 Hyogokken-Nanbu (Kobe) earthquake; Tsunogai and Wakita [1993] report a steady increase in Cl\(^-\) and SO\(_4^{2-}\) concentrations with time from August 1994 to just before the earthquake at a site in the aftershock zone; these precursory changes are interpreted to be due to the intrusion of groundwater from deep fracture zone. It is also reported that large quantities of mineral water flowed out through numerous springs during the Matsushiro, Japan, earthquake swarm [Iijima, 1969]. This flow occurred mostly within the zone of fissuring associated with the fault [Tsuneishi and Nakamura, 1970].

Mechanical effects of fluids have been proposed in a number of theoretical studies as playing a role in a variety of earthquake rupture phenomena such as aftershocks [Nur and Booker, 1972], fault creep [Rice and Simons, 1976], earthquake precursory process [Rudnicki and Chen, 1988; Segall and Rice, 1995], and so on. Nur and Booker [1972] and Booker [1974] studied the effect of fluid migration on aftershock occurrence. However, they did not consider a fracture condition. For example, Nur and Booker [1972] instead assumed that the rate of aftershock occurrence is proportional to time derivative of the local pore pressure; Booker [1974] also made a similar assumption. However, the magnitude and distribution of fracture strength generally play a central role in the results of rupture simulation, so that fracture condition has to be explicitly taken into account in a more reasonable modeling. Sleep and Blanpied [1992] presented models of the earthquake cycle with transiently high pore fluid pressure. Segall and Rice [1995] extended their modeling and analyzed the conditions for unstable slip on a fluid-infiltrated fault using a rate and state dependent friction law including the effects of dilatancy and pore compaction. However, they could not consider the spatial migration of fluid or spatio-temporal change in earthquake
activity because of the assumption of a simple spring slider model.

In recent years much attention has been paid to the simulation of complexity of seismic activity [e.g., Bak and Tang, 1989; Carlson and Langer, 1989; Rice, 1993; Yamashita, 1993, 1995]. However, as far as the author knows, the effect of fluid migration on seismic activity has not been studied yet except in a recent study by Miller et al. [1996]. As stated above, the pore fluid pressure tends to reduce the effective stress, so that it facilitates earthquake ruptures, and the spatio-temporal variation of seismicity should generally be affected by the fluid migration. Our aim in this paper is to investigate the effect of fluid migration on the complexity of earthquake activity on a fault. While Miller et al. [1996] assumed the strain accumulation on a fault due to the monotonically increasing tectonic load, we only consider the sequence of activity whose duration time is much smaller than in their study, so that the remotely applied load is kept constant in our treatment.

Formulation

We consider a narrow porous fault zone in an infinite two-dimensional isotropic homogeneous elastic medium (Figure 1). The rupture is assumed to occur only on the section $0 < x < g$. This fault section is simply referred to as fault in the following. The fault zone is assumed to behave as a fluid conduit. This seems to be widely accepted on the basis of petrologic studies of exhumed fault zones [e.g., Mc-Caig, 1988]. The remotely applied stress is assumed to be kept constant in a simulated rupture sequence; this implies that the assumed time range is negligibly small compared to the recurrence period of characteristic events on the fault. No earthquake is therefore expected to occur in this model unless fluid intrusion takes place. As given by Segall and Rice [1995], we neglect full poroelastic coupling in the fault zone. Continuity of fluid mass yields [e.g., Segall and Rice, 1995]

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_z}{\partial z} = \rho \phi \beta_\alpha \frac{\partial p}{\partial t}, \quad \beta = \beta_1 + \beta_\alpha,$$

where $q_x(x, z)$ is the fluid flux, $\rho$ is the fluid density, $\phi$ is the porosity, and $\beta_1$ and $\beta_\alpha$ are the fluid compressibility and plastic pore compressibility, respectively. Although Walder and Nur [1984] and Segall and Rice [1995] considered the effect of plastic pore deformation, we now neglect it mainly for mathematical simplicity: its importance may be relatively low in a time period much shorter than the recurrence period of characteristic events. Since Darcy’s law relates the fluid flux to pore fluid gradient through

$$q_x = -\frac{\kappa}{\eta} \frac{\partial p}{\partial x}, \quad x_1 = x, x_2 = z,$$

equation (1) can be reduced to a diffusion type equation, where $\eta$ and $\kappa$ are the viscosity and permeability of the fluid, respectively.

We assume that there is no fluid transport across the fault zone boundaries on the basis of laboratory experiments of Blanpied et al. [1992]. In shearing experiments on granite gouge sandwiched between granite forcing blocks, they showed that redistribution of material in solution can reduce the granite permeability, causing a self-generated impermeable seal along the fault boundaries. This seal formation can lead to overpressure in the fault zone. Sleep and Blanpied [1992] discussed field evidence for low-permeability seals.

In dealing with problems where the horizontal length scale is much greater than the thickness of the fluid conduit, one can simplify (1) by assuming the vertical flow is small compared to the horizontal flow [Liggett and Liu, 1983]. The result is a reduction in the dimensionality of the equation, which is obtained by the integration of (1) in the direction normal to the fault. The integration of (1) over the fault zone width gives

$$\frac{\partial}{\partial x} \int_0^b \rho \phi \beta_\alpha \frac{\partial p}{\partial t} dz = \int_0^b \rho \phi \beta_\alpha \frac{\partial p}{\partial t} dz - 2 \rho \phi \beta_\alpha \frac{\partial p}{\partial t} b,$$

if we assume the condition of no fluid communication across the fault zone boundaries:

$$q_x(x, s, t) \frac{\partial}{\partial x} = q_x(x, s, t),$$

$$q_x(x, r, t) \frac{\partial}{\partial x} = q_x(x, r, t),$$

where the bar stands for a value averaged over the fault zone width, and $z = r(x)$ and $s(x)$ denote the upper and lower boundaries of the fault zone. If we only consider the fluid flow averaged over the fault zone width, (3) is reduced to

$$\frac{\partial}{\partial x} \int_0^b \rho \phi \beta_\alpha \frac{\partial p}{\partial t} dz = \int_0^b \rho \phi \beta_\alpha \frac{\partial p}{\partial t} dz$$

where $b(x) = r(x) - s(x)$ is the fault zone width at $x$ and $\tilde{c}$ is the hydraulic diffusivity averaged over the fault zone width. We assume that the pore fluid pressure $p$ in (5) denotes the perturbation from the initial state, so that the initial condition of (5) is described as

$$p = 0 \quad 0 < x < g \quad t = 0.$$  

We assume high-pressure fluid at $x < 0$, and it is assumed to flow into the fault for $t > 0$ (Figure 1). The fault is assumed to be impermeable at the right end, so that we have

$$-\rho b \frac{\partial p}{\partial x} = \gamma(t), \quad x = 0,$$

$$\frac{\partial p}{\partial x} = 0 \quad x = g,$$

for $t > 0$, where $\gamma(t)$ is the given fluid flux at $x = 0$.  

Figure 1. Geometry of the fault, which runs straight along the $x$ axis. The fault zone width is given by $b(x)$, which is assumed to be homogeneous over the fault before the rupture occurrence.
As a plausible mechanism for locally elevated fluid pressure, we can mention the shear-induced pore compaction in a sealed fault. Byerlee [1993] proposed that the formation of sealed fluid compartments of various sizes in a fault zone and their compaction can lead to locally elevated fluid pressure. The formation of the seal is considered to be due to the redistribution of materials in solution as discussed by Sleep and Blanpied [1992, 1994]. The rupture of one of the strongest impermeable seals separating highly compacted fluid compartments can lead to an earthquake in this model. The fault valve model proposed by Sibson [1992] also seems to be partly based on the idea of pore compaction. He considered that transition from hydrostatic to suprahydrostatic regime occurs at some depth across a discrete permeability barrier. Below the barrier, the fluid pressure may become suprahydrostatic due to porosity reduction; the porosity reduction occurs only at depth where pore space is disconnected. The rupture of the barrier leads to an earthquake in this model. In our model we assume a strong impermeable seal at \( x = 0 \) separating high-pressure and low-pressure fluid compartments whose rupture at \( t = 0 \) triggers a sequence of earthquakes; the high pressure may be caused by the pore compaction and the existence of the strong seal. This seal may correspond to the impermeable barrier according to the fault valve model of Sibson [1992], or one of the strongest impermeable seals in a fault zone if we assume the model of Byerlee [1993].

The deformation is assumed to be plane strain in the country rock. We also assume that the coupling between the shear stress \( p_{zy} \) on the fault zone boundary in the country rock and the fluid migration is caused only through the fracture occurrence. In other words, the shear stress is not caused only by the quasi-static fluid flow. This will be allowed if the width of the fluid conduit is narrow enough as assumed in the present paper. The fracture condition is given by

\[
\tau_\varepsilon = a_0 + m_\varepsilon (\sigma_n - p)
\]

(8)

according to the principle of the effective stress, where \( a_0 \) is a constant, \( \tau_\varepsilon \) is the static shear traction at fracture, \( m_\varepsilon \) is the coefficient of static friction, \( \sigma_n \) is the total normal traction on the fault and \( p \) is the fluid pressure. The contribution from the fault slip to the normal traction at \( y = 0 \) is negligible because of the assumptions of the narrow straight fault zone and the occurrence of shear slip only. Hence the normal stress can be assumed to be constant, and (8) is rewritten as

\[
\tau_\varepsilon = a_\varepsilon - m_\varepsilon p \quad a_\varepsilon > 0,
\]

(9)

where \( a_\varepsilon \) is constant. Since \( a_\varepsilon \) denotes the strength before the fluid intrusion, it is referred to as initial strength in this paper. We consider only the perturbation from the initial equilibrium state in this paper, so that the traction can be assumed in the form

\[
\tau_I = -\alpha_f - m_f p \quad a_f > 0
\]

(10)

on the slipping fault surface, where \( a_f \) is constant and \( m_f \) is the coefficient of sliding friction. Laboratory experiments have shown that the two coefficients are in the range \( 0.5 < m_\varepsilon, m_f < 1.0 \) [Byerlee, 1978; Wong, 1986] at seismogenic depths and there is little difference between the two [Wong, 1986], so that we simply assume \( m_\varepsilon = m_f = 0.7 \).

Since the coupling between the shear stress \( p_{zy} \) on the fault zone boundary in the country rock and the fluid migration is assumed to take place only through the rupture occurrence as stated above, the shear stress is written as [e.g., Yamashita, 1993]

\[
p_{zy}(x) = -\frac{\mu}{2(1-\nu)} P \int_{-\infty}^{\infty} \frac{1}{x-s} d\Delta u(s) ds
\]

(11)

terms of the relative slip \( \Delta u(s) \) on the fault, where \( \nu \) and \( \mu \) are Poisson's ratio and the rigidity of the country rock and \( P \) denotes the Cauchy principal value. In this paper, catastrophic rupture is assumed to be instantaneous, so that quasi-static analysis as shown in (5) and (11) is allowed. Equation (11) is an integral equation, and the relative slip is to be derived, using the condition (10). Hence the problem is reduced to solving (5) and (11) under the boundary conditions (7) and (10) and the initial condition (6). Equations (5) and (11) are coupled through the fracture condition (9) and boundary condition (10).

We assume a linear relation between the fault zone width and the accumulated slip on the basis of laboratory and field observations [e.g., Engelder, 1974; Teufel, 1981; Chester et al., 1993]. For example, Chester et al. [1993] observed that thickness of internal damaged zone of the San Andreas fault scales with fault slip. Teufel [1981] showed that width of fault gouge increases with increasing fault slip in his drained, triaxial compression, pore pressure experiments on precut rocks. Although Engelder [1974] and Teufel [1981] suggested a possibility of the existence of upper critical gouge zone width, we assume a linear relation \( b(x) = \xi \sum_m \Delta u^m(x) \) with no upper critical width for mathematical simplicity, where \( \Delta u^m(x) \) denotes the slip caused at location \( x \) by the \( m \)th catastrophic event. This assumption may imply that our analysis can be applied only to underdeveloped faults.

In our formulation, only the fault zone width is assumed to depend on the slip among the model parameters to affect the fluid migration. This is of course a rough idealization even for a narrow fault zone model. For example, changes in the porosity and permeability are also expected to result from the slip occurrence. In fact, Teufel [1981] observed that microfracture density increases with increasing slip in his laboratory experiment, which may suggest a possibility that the porosity is proportional to the slip. In addition, the permeability should be proportional to the porosity in some way [e.g., Probstein, 1994, p.100]. However, we do not have simple quantitative relations between porosity, permeability, and slip that are applicable to earthquake fault zone, so that we confine ourselves to our present idealized modeling.

**Numerical Procedure**

Qualitatively, the sequence of the events occurs as follows: the fluid begins to flow into the fault at time \( t = 0 \). The fluid migration is described by (5). The fracture strength (equation (9)) decreases with the accumulation of the fluid pressure, and the fault slip occurs if the fracture condition (9) is satisfied after attaining sufficient fluid pressure. With the slip occurrence the shear stress suddenly drops to the residual stress level (equation (10)) on the slipping region. The amount of relative slip can be calculated by solving integral equation (11) with condition...
(10); the seismic moment $M_0$ released by the catastrophic event is given from the calculated relative slip in the form $M_0 = \mu \int \Delta u(s) ds$. In general, quasi-static slip occurs on surface where catastrophic slips have already occurred because of temporal variation of the sliding frictional stress; however, our calculations show that the quasi-static slip is negligible compared to the catastrophic one. We do not consider healing on ruptured surface because of the assumption of relatively short time period. The fault zone width expands at a location where the slip occurs; the fault zone expansion implies the formation of new fault zone boundary outside the original one. We assume that new seals are formed at the new fault zone boundary soon after a catastrophic event due to the redistribution of material in solution accelerated by the catastrophic rupture.

All the equations are discretized, and the fault is assumed to comprise a computational grid where space and time evolution of stress, slip, and fluid pressure fields are calculated. We fix the number of fault segments at $N = 100$, and unbreakable barriers are assumed at both ends of the fault. All the calculations are carried out in terms of nondimensional quantities. Refer to the Appendix for the discretization of the equations and normalization of the physical quantities. According to Rice [1993], our fault model is classified as an inherently discrete model, which contrasts a continuum model. While fault segments can rupture individually in the former model, they cannot in the latter. Motivation of the inherently discrete model was discussed by Ben-Zion and Rice [1995]. We carry out the calculation until all the segments are ruptured. This implies that the total amount of strain energy release is almost the same in each sequence since the remotely applied load is kept constant in the calculations and negligible amount of energy is released by quasi-static slips on ruptured surface. The fluid flux $\gamma$ given at $x = 0$ is assumed to be time independent for simplicity. The model magnitude $M$ is given by the logarithm of the released moment in the form $M = \log_{10} \sum_k D_k^c$, where $D_k^c$ is the catastrophic nondimensional slip on the $k$th segment. The relation between the nondimensional fault zone width $Z$ and accumulated slip

is rewritten as $Z = \xi \sum_m D_m$ in the nondimensional form, where $\xi = \mu \xi/2\pi (1 - \nu)\alpha_s$ and $D_m$ is the nondimensional slip caused by the $m$th event. We arbitrarily assume $0.01 < \xi < 0.5$ in the calculations. We assume that the fault zone width is negligibly narrow at $t = 0$ compared to the width after the rupture of the entire fault; in all the calculations we arbitrarily assume $Z_{i,0} = 0.1$ ($i = 1, ..., N$), where $Z_{i,j}$ is the nondimensional fault zone width at the $i$th segment and at the $j$th time step. This value is shown to be negligibly small compared to the final fault zone width in each simulation. Since even an inactive fault seems to have high permeability compared to the country rock [e.g., Chester and Logan, 1986], it will be reasonable to assume a narrow fluid conduit in the fault zone before rupture excitation.

Reference Simulation for Homogeneous Models

In this section we carry out reference simulations for models in which distributions of the model parameters are spatially homogeneous on the fault. Such models are referred to as homogeneous models in the following. The effect of inhomogeneously distributed model parameters can be understood by reference to the homogeneous model. We specifically investigate the effect of the initial strength $A(=\alpha_s/\alpha_f)$, given flow flux $\Gamma(=nq/\rho \propto)$, and constant $\xi(=Z/\sum_m D_m)$. These will be important model parameters to determine the rate of fluid migration and the rupture occurrence.

Examples of the spatio-temporal variations of pore fluid pressure and relative slip are illustrated in Figure 2. It is observed that the system entirely ruptures before the fluid migrates to the other end of the fault section. This occurs because the stress at the crack tip exceeds the threshold for the rupture of the entire system. Note that the crack growth excited by the fluid pressure buildup is arrested after breaking only one segment in an earlier stage of the rupture sequence: the stress at the crack tip is still smaller than the above threshold value. However, the crack tip stress monotonically increases with the crack growth, and it can exceed the threshold sooner or later, leading to the rupture of the

![Figure 2](image-url)

**Figure 2.** An example of the calculation for the homogeneous model; $\Gamma = 1.0$, $A = 5.0$, and $\xi = 0.1$ are assumed. (a) Spatio-temporal evolution of pore-fluid pressure $P$ in the fault zone. (b) Spatio-temporal evolution of relative slip $D$ on the fault.
entire fault. It is understood from Figure 2b that the maximum fault zone width is about 11.0 (note the assumption \( \xi = 0.1 \)), which suggests that the fault zone width is considerably large, after the rupture of the entire fault, compared to the initial width 0.1. As is typically shown in Figure 2b, each catastrophic event ruptures only one fault segment in all the calculations for the homogeneous models except for the largest event occurring lastly in a sequence.

The dependence on the fluid flux given at \( x = 0 \) is illustrated in Figure 3 for some values of \( \Gamma \), which denotes that a larger fluid flux yields earlier occurrence of ruptures and shorter duration of rupture sequence. This will be because fluid buildup occurs earlier for a larger fluid flux. Other features are almost independent of \( \Gamma \) in the assumed range of its value.

The effect of \( \xi \) is investigated in Figure 4. This figure shows that catastrophic events occur earlier, and the duration of a sequence is shorter for a larger value of \( \xi \). This will be because the fluid can migrate into the fault zone more easily when \( \xi \) is larger. Other phenomena are almost independent of \( \xi \), so that the effect of \( \xi \) seems to be qualitatively similar to that of \( \Gamma \).

We next investigate the effect of the initial strength, \( A \), which will be shown to have more diverse effects on the rupture sequence than the other parameters. The effect on the temporal variation of ruptures is illustrated in Figure 5. We observe in this figure that the duration of sequence is longer and the magnitude of the largest event in a sequence is smaller when the initial strength is larger. The longer duration will occur because the critical crack length for the rupture of the entire fault is larger when the initial strength is larger. This implies that the number of unbroken segments should be smaller at the instant of attaining the critical crack length when the initial strength is larger, so that the magnitude of the largest event in a sequence should be smaller for larger initial strength. Figure 6 illustrates the time interval between two successive events, which shows that the rupture occurrence rate is much accelerated immediately before the occurrence of the largest event in each sequence. This is commonly observed in any sequence of the homogeneous models. The rupture occurrence is shown to be almost periodic at the earlier part of a sequence except for the beginning part; the periodic rupture occurrence will be totally governed by the almost steady fluid migration.

**Inhomogeneous Models**

The consideration in the preceding section suggests that the initial strength, \( A \), has a more significant effect on the rupture sequence than the other model parameters have. We therefore direct our attention only to the effect of the initial strength in the investigation of the effect of the spatial inhomogeneity in the distribution of model parameters; \( \Gamma \) and \( \xi \) are fixed at 1.0 and 0.1, respectively, in all the calculations. A model in which the initial strength is spatially
inhomogeneous on a fault is referred to as an inhomogeneous model.

At present, we have no experimental or seismologically derived information that permits us to specify the distribution of $A$. Therefore we assume one of the simplest statistical distributions, a homogeneous distribution; in other words, the probability density is assumed to be homogeneous in a specified range, $A_{\text{min}} < A < A_{\text{max}}$. We assume three models for the distribution of $A$; model I ($5 < A < 10$), model II ($2.5 < A < 7.5$), and model III ($6.5 < A < 8.5$). The effect of the extent of the distribution can be studied in comparison between models I and III. We can investigate the effect of the mean value by comparing model I with II. In each of the three models we carry out 10 numerical experiments with a different set of values of $A$. The computation is carried out up to the time when a fault is entirely ruptured.

**Temporal Variation of Rupture Activity for Inhomogeneous Models**

A typical example of the rupture sequence of model III is illustrated in Figure 7 together with an example of the homogeneous model having $A = 7.5$, which is equal to the mean value of $A$ assumed in model III. Although the deviation from the homogeneous model is smallest in model III among the three inhomogeneous models, we find significant differences between the two sequences. We first notice that the duration of the sequence is considerably longer in the example of model III. In fact, as shown in Figure 8, all the sequences in the inhomogeneous models have much longer duration than in the homogeneous models. Another notable finding in Figure 7 is that the time interval between two successive events does not necessarily decrease with time near the occurrence time of the largest event in the sequence of model III. The sequence of model III illustrated in Figure 7 rather has a long quiescence period immediately before the occurrence of the largest event; this feature is generally observed in all the sequences of the inhomogeneous model. We study in the following what the above differences can be attributed to.

We first investigate causes leading to longer duration of sequences in the inhomogeneous models. Figure 9 illustrates the rupture occurrence times of 50 segments nearest to the fluid source at $x = 0$ for the sequences shown in Figure 7. One of the most characteristic features is that several neighboring segments tend to rupture simultaneously in the inhomogeneous model, while in the homogeneous model each segment ruptures individually. This feature is also observed in the other inhomogeneous models. It is characteristic in Figure 9 that relatively long quiescent period exists before and after the simultaneous rupture of several neighboring segments in the sequence of the inhomogeneous model. It therefore seems that long duration of the sequence is associated with the simultaneous rupture of neighboring segments and/or the existence of quiescent period before and after such rupture.

We now study why simultaneous ruptures tend to occur and why there occur relatively long quiescent periods in the inhomogeneous models. Figure 10 shows the spatial distribution of the initial strength, $A$, on the 15th to 40th segments for the example of model III illustrated in Figures 7 and 9; our calculation shows that neighboring segments surrounded by a single broken ellipse ruptures simultaneously. Figure 10 suggests that a rupture of segment having local maximum strength tends to induce rupture of nearest segments ahead with lower strengths. The above observation implies that two factors contribute to the lengthening of sequence duration. It generally takes much time to build up the fluid pressure to rupture a locally strong segment. Hence the rupture of such segment is relatively delayed, which can be a factor to lengthen a sequence in the inhomogeneous models. The other factor is related to the simultaneous rupture of several segments ahead triggered by the rupture of the locally strong segment. Our calculations show that some amount of fluid penetrates ahead of the rupture tip during the build up of the fluid pressure at the locally strong segment. Thus will contribute to the simultaneous rupture of several segments. If such simultaneous ruptures occur and the rupture tip is suddenly advanced, then it takes considerable time for the high-pressure fluid to migrate to the rupture tip again and to contribute to the catastrophic growth of the rupture tip. These two factors will form the activity quiescence before and after the simultaneous rupture of neighboring segments and will contribute to the lengthening of the sequence duration.

Figure 8 denotes that model I generally has the longest duration of sequence among the three inhomogeneous models. This observation can also be understood in terms of the two factors stated in the preceding paragraph. Since model I has the largest $A_{\text{max}}$ value among the three inhomogeneous models, it generally takes more time to rupture a

![Figure 4](image-url)  
**Figure 4.** Effect of $\xi$ on the rupture sequence; $A = 5.0$ and $\Gamma = 1.0$ are assumed.

![Figure 5](image-url)  
**Figure 5.** Dependence of the rupture sequence on the initial strength; $\Gamma = 1.0$ and $\xi = 0.1$ are assumed.
locally strong segment. On the other hand, model I is also characterized by having the largest value of $A_{\text{max}} - A_{\text{min}}$ in the three models, so that it will be easy for the rupture of such strong segment to induce rupture of neighboring segments ahead. This will retard the remigration of the fluid to the rupture tip in model I.

As typically shown in Figure 7, our calculations for the inhomogeneous models generally show that the time interval between two successive events does not necessarily decrease with time near the occurrence time of the largest event in a sequence. The relative inactivation of rupture occurrence before the occurrence of the largest event in a sequence will occur because the largest event is triggered by the rupture of segment with a locally maximum initial strength; as discussed before, the existence of a locally high-strength segment retards the fluid migration and temporarily inactivates the rupture occurrence.

**Frequency Distribution of Event Sizes for Inhomogeneous Models**

We now study how the frequency distribution of event sizes is influenced by the spatial distribution of the initial strength, $A$, on a fault. Figure 11 shows cumulative frequency-magnitude statistics for each of models I, II, and III; the results of 10 sequences are superimposed in each model. The frequency-magnitude statistics of Figure 11 show three event groups for each model: small events with almost constant level for $M < 1.6$, intermediate-size events approximately obeying the Gutenberg-Richter (GR) relation, and large events deviating from the GR relation for $M > 3.6$.

A bump is observed near $M \approx 3.6$ for models II and III. Each of the events near this bump stands for the largest event in a sequence rupturing all the unbroken segments near $x = g$. Hence the events near $M \approx 3.6$ will be strongly influenced by the finiteness of the system size in models II and III. Such large-scale ruptures are forced to be arrested at the barrier assumed at the ends of the fault, so that there will be an upper threshold for the magnitude of the event; this will form a bump near $M \approx 3.6$. By contrast, no such bump is observed for model I. This will be because the largest event in a sequence is not much influenced by the finiteness of the system size in model I. In fact, the magnitude of the largest event in a sequence is relatively small in model I; for example, compare Figures 7 and 12. In addition, the largest event does not rupture all the unbroken segments on a fault in some sequences of model I. This will occur because the rupture growth is easily arrested due to large variance and large average value of the initial strengths.

Our calculations show that each of small events for $M < 1.6$ ruptures only a single fault segment, so that the effect of interactions among fault segments is relatively weak for these events. Hence we can conclude like many authors [e.g., Yamashita, 1993] that the Gutenberg-Richter law observed for intermediate events in Figure 11 is due to strong local interactions among fault segments; larger events for $M > 3.6$ are influenced by the finiteness of the system size.

**Discussion and Conclusions**

We do not intend to explain real seismological behavior quantitatively in this paper. Our study is rather for qualitative understanding of the effect of the model parameters, so that some idealized assumptions are made to isolate the effect of each model parameter. For example, a fault...
with one-dimensional extent is a highly idealized assumption, which can, however, be useful to abstract the effect of complexity in fluid path on a two-dimensional fault. The assumption of constant fluid flux at $x = 0$ can abstract the details of the fluid source. The assumption of unbreakable barrier at $x = 0$ is also for removing the details of the fluid source. However, if the coexistence of the unbreakable barrier and fluid source near $x = 0$ is interpreted literally, it implies the existence of high-pressure fluid localized near an end of a fault. The results obtained in this study can be a basis for more realistic three-dimensional treatment. We assumed an impermeable seal along the fault zone boundary for the idealized treatment. However, seals for actual faults will not be perfect and fluid needs not be trapped for geological times in a fault zone. A slightly leaky seal will allow the average pressure during an earthquake cycle to be roughly hydrostatic as noted by Sleep and Blanpied [1992].

It was shown in this paper that rupture occurrence is not necessarily activated near the occurrence time of the largest event in a sequence of the inhomogeneous model. This never happens so long as a brittle rupture is assumed in an elastic medium with no mechanical effect of fluid; in fact, the rate of rupture occurrence is shown to increase as the time of the largest event approaches [Yamashita and Knopoff, 1989; Yamashita, 1993]. Since the inactivation of the rupture occurrence cannot be observed in the homogeneous models, the inactivation must be related to both fluid migration and spatial inhomogeneity in the distribution of the initial strength. Seismological observations have shown that foreshocks do not always precede large shallow earthquakes. In a systematic study, Jones and Molnar [1979] examined teleseismically located events before major earthquakes globally from 1914 to 1973 and found that roughly 40% of the main shocks with $M > 7$ were preceded by one or more foreshocks. Mogi [1963] found that only about 4% of major events were preceded by foreshocks by investigating earthquakes in and near Japan from 1926 to 1956. It is therefore suggested on the basis of our analysis that nucleations of the majority of large shallow earthquakes are possibly affected by fluid migration.

While only monotonic changes are observed in spatio-temporal variation of rupture activity in the homogeneous models, the activity is observed to be significantly complex in an individual sequence of the inhomogeneous models. However, some statistical regularities are observed even in the inhomogeneous models. For example, we observe that the duration of rupture sequence is longer when the average value and/or variance of the initial strength distribution are larger for fixed fluid flux given at the end of the fault. It is also shown that the frequency-magnitude statistics obey well the Gutenberg-Richter relation for intermediate-size events for all the inhomogeneous models, while the $b$ value appears to be slightly model dependent. The occurrence of larger events deviating from the GR relation is also shown to be model dependent. In fact, it is observed for models II and III that a greater frequency of occurrence of the larger-size events than would be expected from lesser-size events. On the other hand, model I shows a lesser frequency of occurrence of the larger events.

It is sometimes suggested that occurrence of earthquake swarm is associated with fluid migration [e.g., Tsuneishi and Nakamura, 1970]; earthquake swarm is generally characterized as a sequence of earthquakes in which there is no single predominant principal event. Our calculations show that this feature is observed in model I. In fact, no predominantly large event is observed in a sequence of model I, which is implied by the frequency-magnitude curve shown in Figure 11. It is more clearly shown by a typical example of a sequence of model I illustrated in Figure 12. Hence it can be inferred that earthquake swarm takes place when the average initial strength, $A$, is comparatively large and the variance of its distribution is also large on a fault. Mogi [1963] showed that earthquake swarm tends to occur at remarkably fractured regions by investigating earthquake activity in and near Japan. Our analysis together with Mogi's finding suggests that highly fractured regions in the Earth's crust are characterized by high average value and large variance of the distribution of the initial strength.

Appendix: Discretization of Equations and Numerical Calculations

The relative slip $\Delta u$, fluid pressure $p$, and shear stress $p_x$ are calculated at the center of each segment when solving (5) and (11). Equation (11) is therefore discretized as

\[
\frac{\partial p}{\partial t} = \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right) - \lambda \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} - \lambda \frac{\partial u}{\partial y} \frac{\partial u}{\partial z} - \lambda \frac{\partial u}{\partial z} \frac{\partial u}{\partial x}.
\]

Figure 9. Rupture occurrence times at the 1st to 50th fault segments. Calculated example of model III is compared with that of the homogeneous model. $\Gamma = 1.0$ and $\xi = 0.1$ are assumed in both examples. We assume $A = 7.5$ in the calculation of the homogeneous model; note that the average value of $A$ is 7.5 in model III. Temporal variation of event sizes is shown in Figure 7 for these two examples.

Figure 10. The initial strengths, $A$, on the 15th to 40th fault segments for the example of model III shown in Figures 7 and 9. See the text about the broken ellipses.
I. Figure 12. An example of the rupture sequence of model for each inhomogeneous model.

Figure 11. Cumulative frequency-magnitude distribution for each inhomogeneous model.

\[ S_i = \sum_{j=1}^{N} D_j \frac{1}{(i-j)^2 - 1/4}, \]  
\[ (A1) \]

where \( N \) is the number of the fault segments, and \( S_i (\equiv p_e/\alpha_f) \) and \( D_i (\equiv \mu \Delta u/2kh(1-\nu)\alpha_f) \) are the nondimensional shear stress and slip, respectively, on the \( i \)th segment at \((i-1)h < x < ih\), where \( h \) is the size of each fault segment. Equation (5) is written in the finite difference scheme

\[ P_{i,j+1} = P_{i,j} + \frac{C \Delta T}{Z_{i,j}} \left( \frac{Z_{i+1,j} - Z_{i-1,j} + 4Z_{i,j}}{4} \right) \]
\[ \quad - 2Z_{i,j}P_{i,j} - \frac{Z_{i+1,j} - Z_{i-1,j} - 4Z_{i,j}}{4} P_{i+1,j} \]
\[ (A2) \]

In the above, \( \Delta T \) is the nondimensional time increment, \( C = \bar{c}/c_0 \), where \( c_0 \) is a constant having the same dimension as that of \( \bar{c} \). We arbitrarily assume \( C = 0.5 \) and \( \Delta T = 0.5 \times 10^{-3} \) in all the calculations. The nondimensional time \( T \) is defined as \( T = \bar{c}t/h^2 \), and \( P_{i,j} (\equiv p/\alpha_f) \) is the nondimensional pore fluid pressure at the center of the \( i \)th segment and at the \( j \)th time step \( T = j \Delta T; Z_{i,j} (\equiv b/h) \) denotes the nondimensional fault zone width at the \( i \)th segment and at the \( j \)th time step. The boundary condition (7a) yields

\[ P_{0,j} = P_{2,j} + 2\Gamma_j/Z_{1,j} \]  
\[ (A3) \]

by introducing the fictitious segment at \( i = 0 \), where \( \Gamma_j \) is the nondimensional fluid flux given at the \( j \)th time step, that is, \( \Gamma_j = \eta \alpha_f (\bar{T}_j)/\rho_k \). We obtain from (A2) and (A3)

\[ P_{i,j+1} = P_{i,j} + C \Delta T \left( 2(P_{i,j} - P_{i+1,j}) \right) \]
\[ \quad - \Gamma_j (Z_{i+1,j} - 3Z_{i,j}/(Z_{i,j})^2) \]  
\[ (A4) \]

as the boundary condition at the segment \( i = 1 \) by assuming \( i = 1 \) in (A2) and a smooth change in the fault zone width near \( X = 0 \), that is, \( Z_{1,j} - Z_{2,j} = Z_{0,j} - Z_{1,j} \). Our calculations reveal that there is little change in the computation results if we make another simple assumption, that is, no spatial change \( Z_{0,j} = Z_{1,j} \) near \( X = 0 \). The condition (7b) combined with (A2) yields

\[ P_{N,j} = P_{N,j} + 2C \Delta T (P_{N-1,j} - P_{N,j}) \]  
\[ (A5) \]

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