Monte-Carlo Simulation of Failure Phenomena using Particle Discretization

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Motivation

Deformation

Failure

Phenomenon \equiv \text{Behavior of Homogeneous body}

Phenomenon \not\equiv \text{Behavior of Homogeneous body}

Accurate solution for homogeneous body with expensive discretization

Utter Significance

Almost Meaningless
Effect of local heterogeneity
- Behavior of ideally homogeneous body \(\neq\) What really happens
  \(\text{(extensive, expensive analysis on ideally homogeneous body \cdots ?)}\)
- Convergence in local sense needed?
  \(\text{(Failure phenomena do not converge in local sense)}\)
- Methods with wide variety of failure patterns depending on local heterogeneity
  \(\text{(Which could be called mesh dependence)}\)

Number of DOF
- “Fine Mesh \(\Rightarrow\) High Accuracy“ does not always hold
- Proper order of discretization depending on the scale of local heterogeneity
Objectives

- Numerical analysis on failure behavior of bodies with local heterogeneity
- See the difference between the failure behavior of ideally homogeneous body and that of locally heterogeneous bodies
- Examine the applicability of Particle Discretization Scheme (FEM-β) to analysis of failure behavior of bodies with local heterogeneity
Example Problem

Initial Cracks

uniform tension

<table>
<thead>
<tr>
<th>Young's modulus</th>
<th>1.0</th>
</tr>
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<tbody>
<tr>
<td>Poisson's ratio</td>
<td>0.25</td>
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<td>Disp. B.C.</td>
<td>0.1 (vertical)</td>
</tr>
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Mesh: very fine, incremental crack growth
Direction: based on the energy release rate for virtual extension of the crack

The only one pair of crack path is obtained

Kamaya and Totsuka, Corrosion Science, 2002
Body with Local Heterogeneity --- What we expect

Different crack paths depending on local heterogeneity?

Do they converge to homogeneous solution?
Stochastic Treatment

Brute force:
Monte-Carlo simulation using models with different distribution of material properties (stiffness, strength etc.)

but…
- Meshless related methods: sophisticated discretization requires relatively high computational cost
- Adaptive mesh: re-mesh at each step costs a lot

We need
- a method with i) less computational cost
- ii) simple treatment of failure
- iii) no change in configuration
Easy Treatment of Failure in FEM-b

stiffness matrix of FEM-β

\[
\begin{bmatrix}
[k_{11}] & [k_{12}] & [k_{13}] \\
[k_{21}] & [k_{22}] & [k_{23}] \\
[k_{31}] & [k_{32}] & [k_{33}]
\end{bmatrix}
\]

\[[k_{12}] = [k_{12}^\text{direct}] + [k_{12}^\text{indirect}]\]

Spring properties are rigorously determined with material properties; \(E\) and \(ν\)

for indirect interaction

for direct interaction

Appropriately change the components of stiffness matrix/spring constants, according to a suitable failure criterion
Monte-Carlo Simulation of Crack Propagation in Locally Heterogeneous Body

Initial Cracks

uniform tension

1000 models with different mesh geometry

1000 models with different distribution of material strength

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Example of Crack Path

Let’s have a look at animations
Example of Crack Path
Example of Crack Path
Example of Crack Path
Source of the Difference
Significant variance in crack paths due to local heterogeneity
Quantitative discussion
We are going to look at PDF of crack paths.
Probability Density Function of Crack Path

Please click it.

Initial Crack
Convergence of PDF

Initial Crack

Crack location on each check lines

PDF

at x=0.0

at x=0.1

at x=0.2

Initial Crack

Convergence of PDF
Summary

- Importance of local heterogeneity in analysis of failure phenomena
- Monte-Carlo simulation of crack propagation in heterogeneous bodies with different distribution of material strength
- Easy treatment of failure is needed --- FEM-\( \beta \)
- Wide variety of crack paths, PDF for crack paths
What we got…rigorous formulation + easy treatment of failure

- Simple treatment of failure like DEM (Strength of Material)
- No change in Geometry/Configuration
- Particle physics type simulation ⇒ suitable for parallel, massive computation
- Easy treatment of local heterogeneity

What we sacrifice…fracture mechanics, local convergence

- Candidate for crack path is pre-determined when a mesh is made
- (Crack surface = Cavity) ⇒ Blunt Crack
- Solution does not converge to the exact solution for the problem of crack growth in ideally homogeneous body