BACKGROUNDS

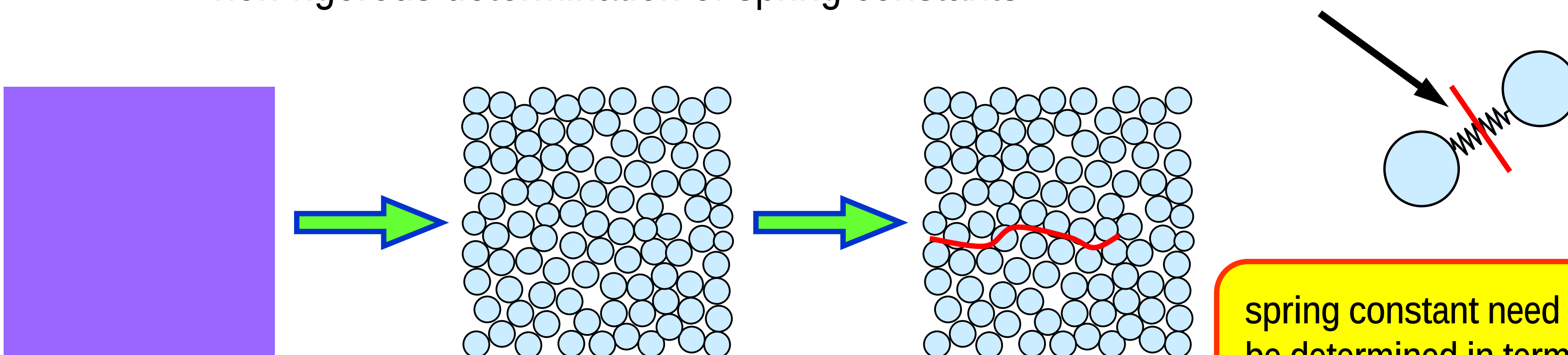
Two Models of Deformable Body

- continuum — deformation expressed in terms of field variables
- rigid-body spring — assembly of rigid-bodies connected by spring

Distinct Element Method (DEM)

- simple treatment of failure: breakage of spring
- non-rigorous determination of spring constants

breakage of spring



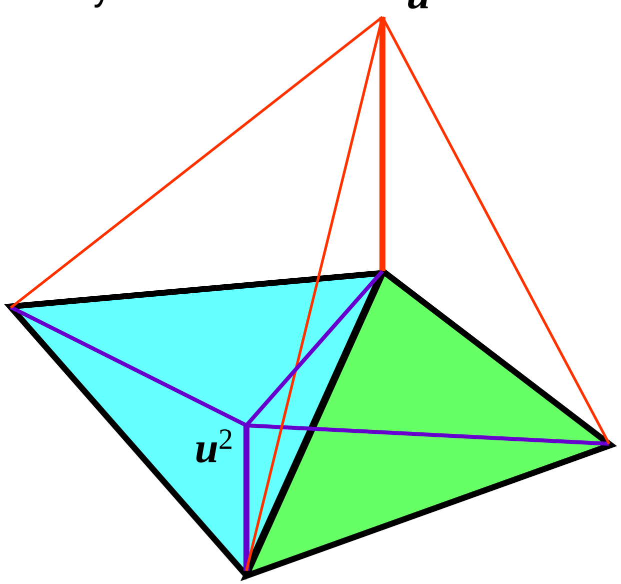
deformable body — particle modeling — DEM analysis

spring constant need to be determined in terms of material parameters

MATHEMATICAL INTERPLETATION OF RIGID-BODY SPRING MODEL

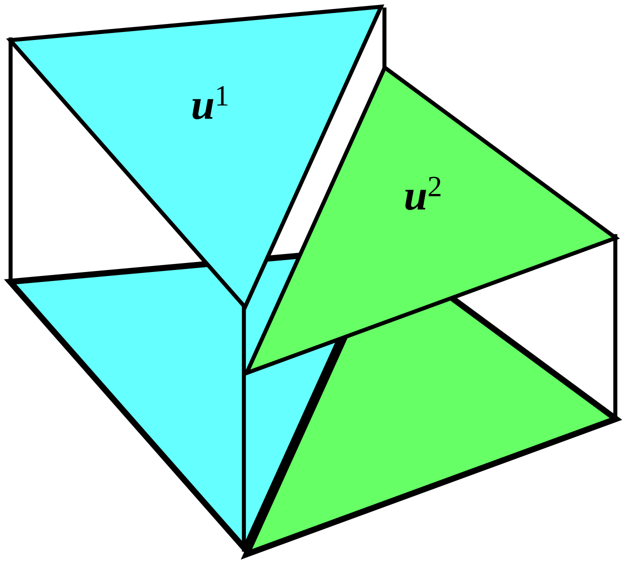
- Non-Overlapping Functions for Discretization

ordinary FEM



smooth but overlapping functions are used for discretization

DEM



characteristic functions of domain are used for discretization

PARTICLE DISCRETIZATION FOR FUNCTION AND DERIVATIVE

- Discretization of Function $f(\boldsymbol{x})$ in terms of $\{\varphi^\alpha(\boldsymbol{x})\}$

form of discretization

$$f^d(\boldsymbol{x}) = \sum_\alpha f^\alpha \varphi^\alpha(\boldsymbol{x})$$

$$E^f(\{f^\alpha\}) = \int_V (f(\boldsymbol{x}) - f^d(\boldsymbol{x}))^2 \, ds$$

minimize E^f

$$f^\alpha = \frac{1}{\int_V \varphi^\alpha \, ds} \int_V f \varphi^\alpha \, ds$$

- Discretization of Derivative $f_{,i}(\boldsymbol{x})$ in terms of $\{\psi^\alpha(\boldsymbol{x})\}$

different from $\{\varphi^\alpha\}$

form of discretization

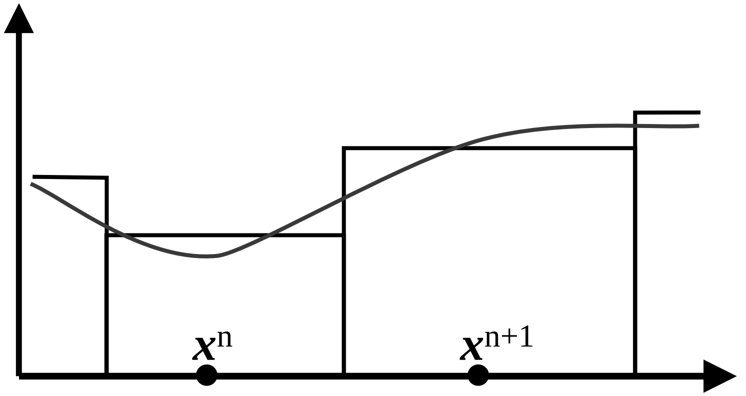
$$g_i^d(\boldsymbol{x}) = \sum_\alpha g_i^\alpha \psi^\alpha(\boldsymbol{x})$$

derivative of $\{\varphi^\alpha\}$ is not bounded but integratable

$$g_i^\alpha = \frac{1}{\int_V \psi^\alpha ds} \sum_\beta \left(\int_V \psi^\alpha \varphi^\beta_{,i} \, ds \right) f^\beta$$

optimal coefficients for given Voronoi blocks $\{\varphi^\alpha\}$ and Delaunay triangles $\{\psi^\alpha\}$

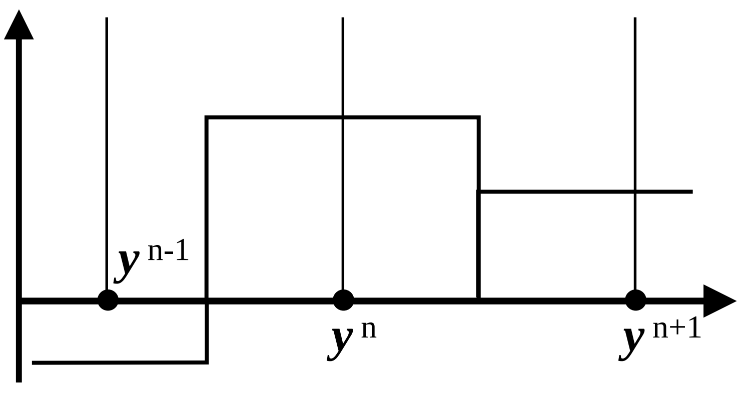
1-D PARTICLE DISCRETIZATION



mother points

$$f^{d}(\boldsymbol{x}) = \sum_{\alpha} f^{\alpha}\varphi^{\alpha}(\boldsymbol{x})$$

average value taken over interval for φ^{α} is coefficient of characteristic function φ^{α}

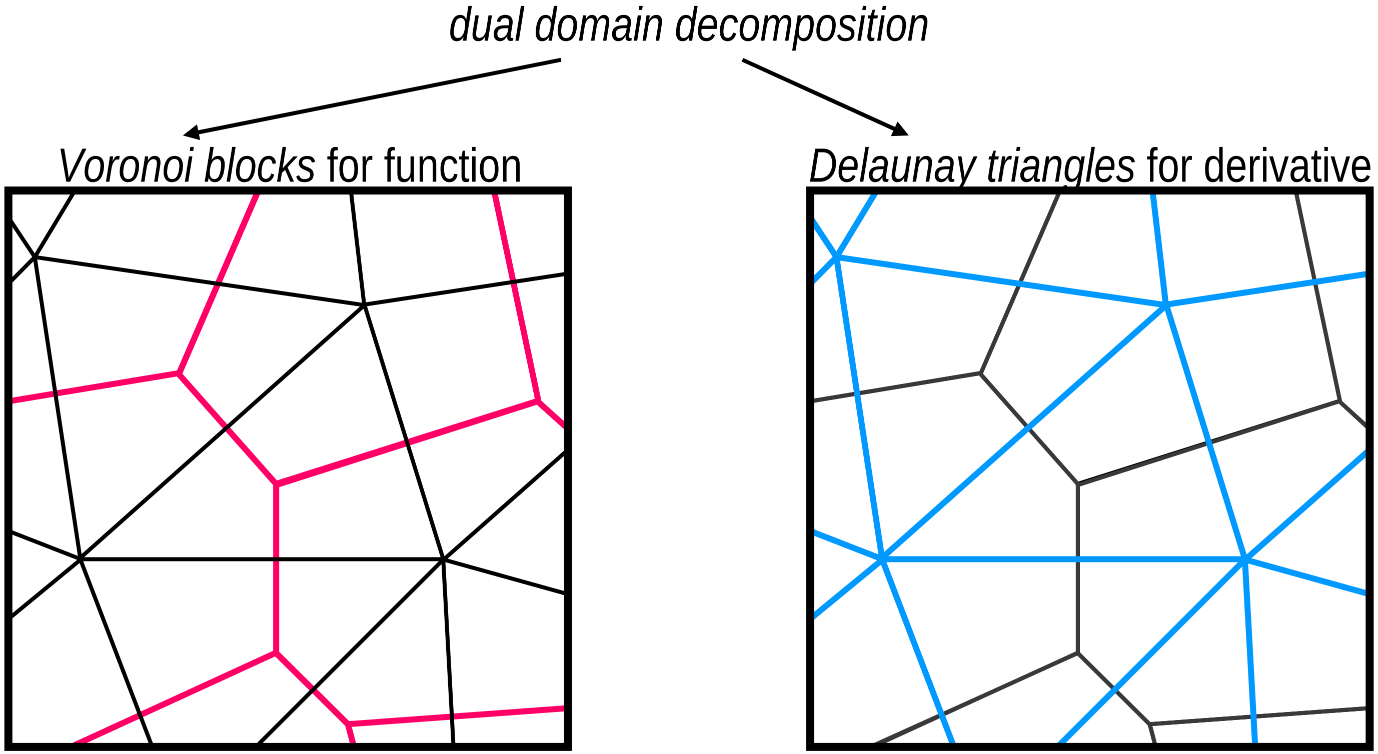


middle point of neighboring mother points

$$\frac{df}{dx}(\boldsymbol{x}) = g(\boldsymbol{x}) = \sum_{\alpha} g^{\alpha}\psi^{\alpha}(\boldsymbol{x})$$

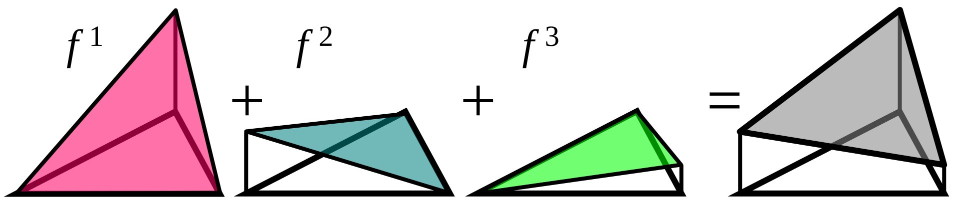
average slope of interval for ψ^{α} is coefficient of characteristic function ψ^{α}

2-D PARTICLE DISCRETIZATION

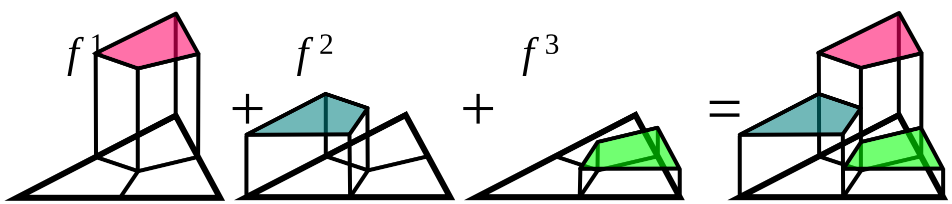


function and derivative are discretized in terms of sets of non-overlapping characteristic functions, such that function and derivative are uniform in Voronoi blocks and Delaunay triangles,

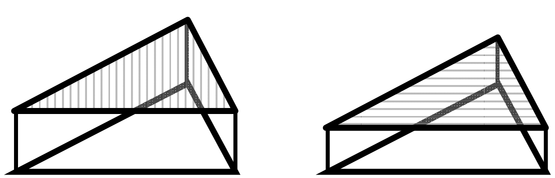
COMPARISON OF PARTICLE DISCRETIZATION WITH ORDINARY DISCRETIZATION



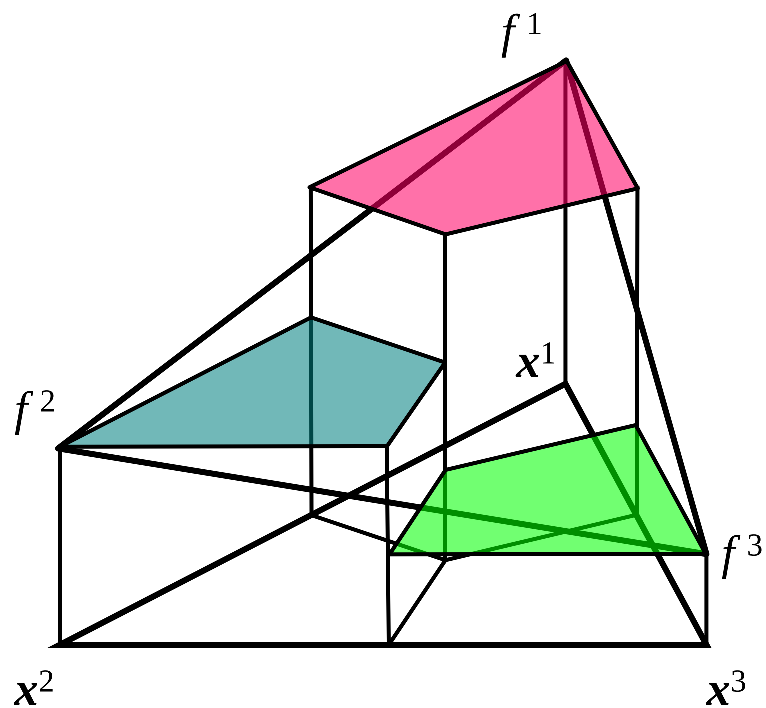
ordinary discretization with linear function



particle discretization



particle discretization of derivative



derivative of particle discretization coincides with slope of plane which is formed by linearly connecting Voronoi mother points

PARTICLE DISCRETIZATION TO CONTINUUM MECHANICS PROBLEM

- Conjugate Functional

$$J(u,\sigma) = \int_V \sigma_{ij}\varepsilon_{ij} - \tfrac{1}{2}\sigma_{ij}c_{ijkl}\sigma_{kl}\, ds$$

$$u_i = \sum u_i^\alpha \varphi^\alpha$$

hypo-Voronoi for displacement

$$\sigma_{ij} = \sum \sigma_{ij}^\alpha \psi^\alpha$$

Delaunay for stress:
coefficients are analytically obtained by stationarizing J

- FEM-β: FEM with Particle Discretization
  - stiffness matrix of FEM-β coincides with stiffness matrix of FEM with uniform triangular element
  - including rigid-body-rotation, FEM-β gives accurate and efficient computation for field with singularity

FAILURE ANALYSIS OF FEM-β

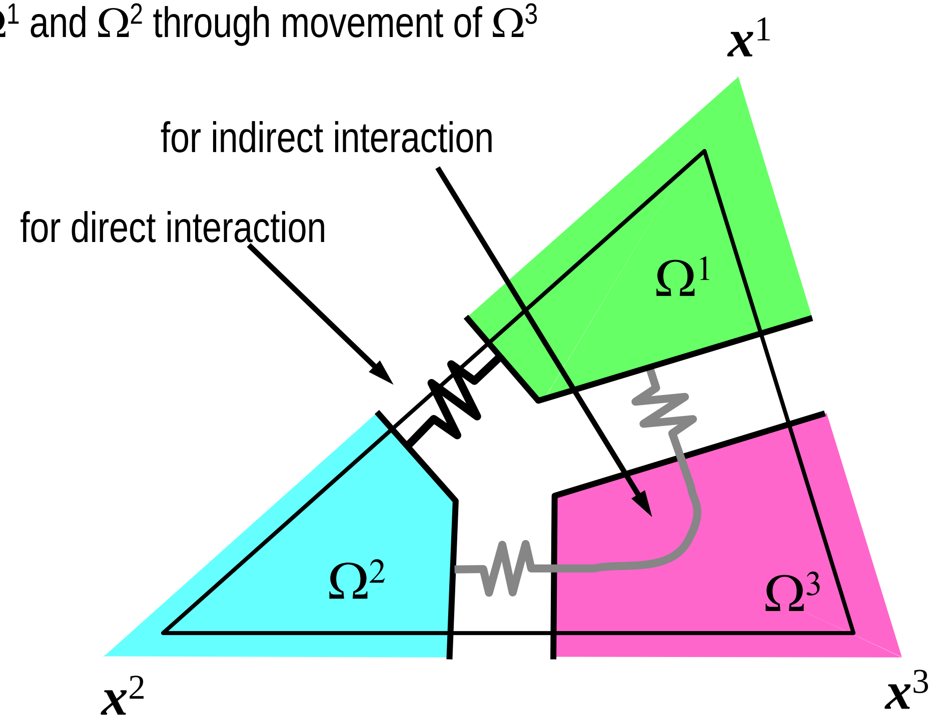
stiffness matrix of FEM-β

$$\begin{bmatrix} [k_{11}] & \boxed{[k_{12}]} & [k_{13}] \\ [k_{21}] & [k_{22}] & [k_{23}] \\ [k_{31}] & [k_{32}] & [k_{33}] \end{bmatrix}$$

$$\left([k_{ji}]^T = [k_{ij}]\right)$$

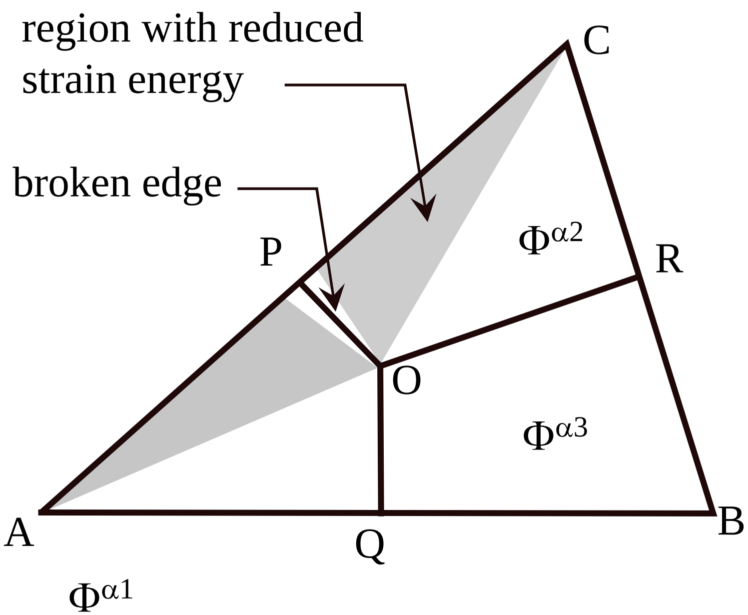
$$[k_{12}] = [k_{12}^{\text{direct}}] + [k_{12}^{\text{indirect}}]$$

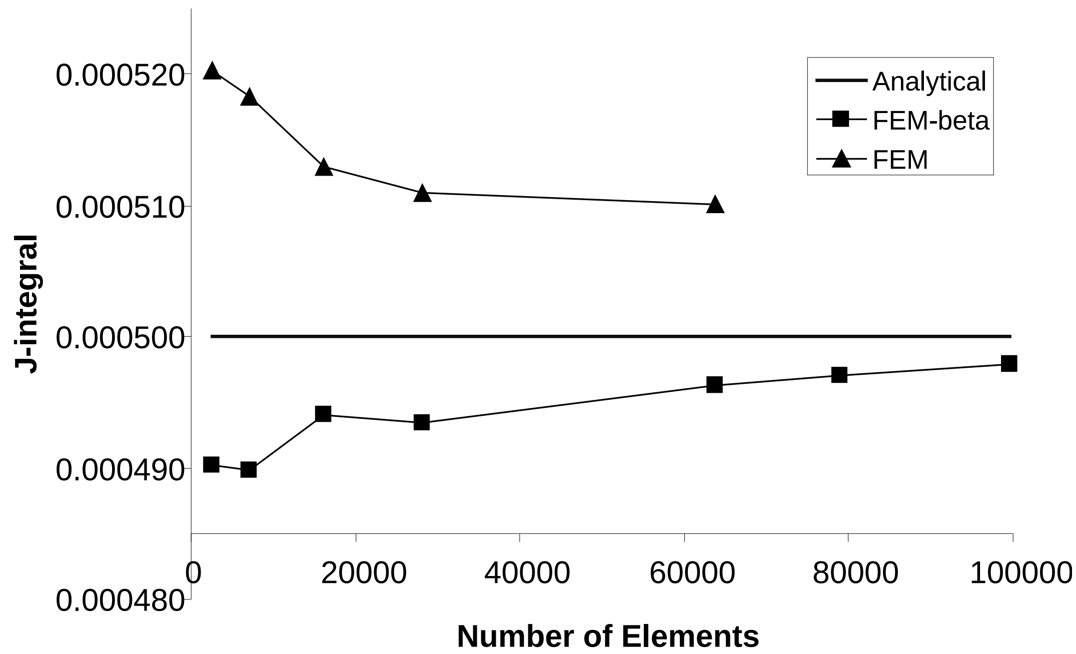
strain energy due to relative deformation of Ω^1 and Ω^2 through movement of Ω^3



cut two springs of direct and indirect interaction together or separately, according to certain failure criterion of continuum

FAILURE MODELING BY BREAKING SPRINGS



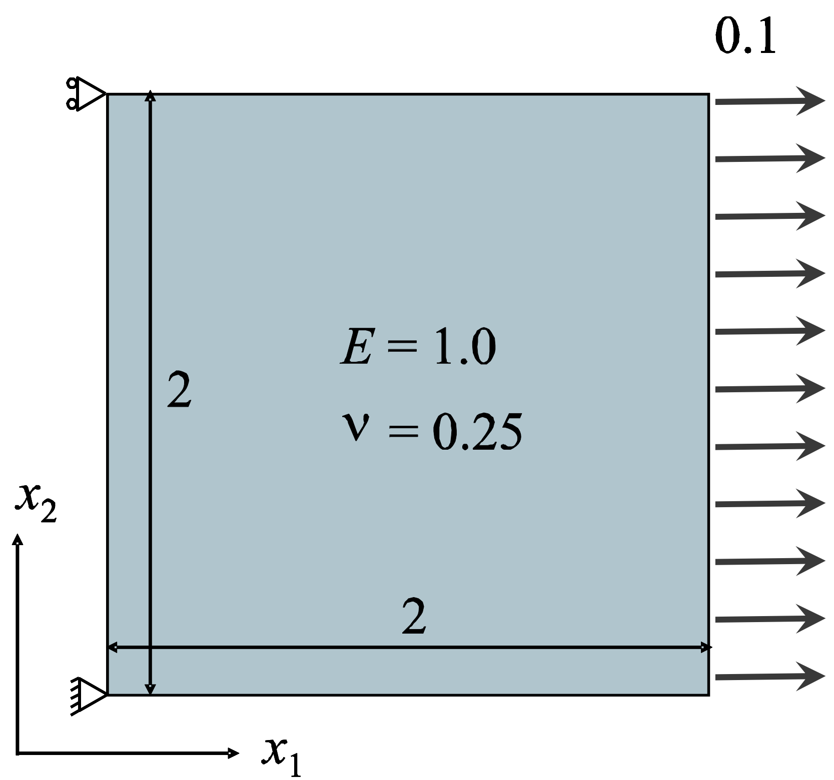


relative error of J-integral is around a few percents

EXAMPLE PROBLEM

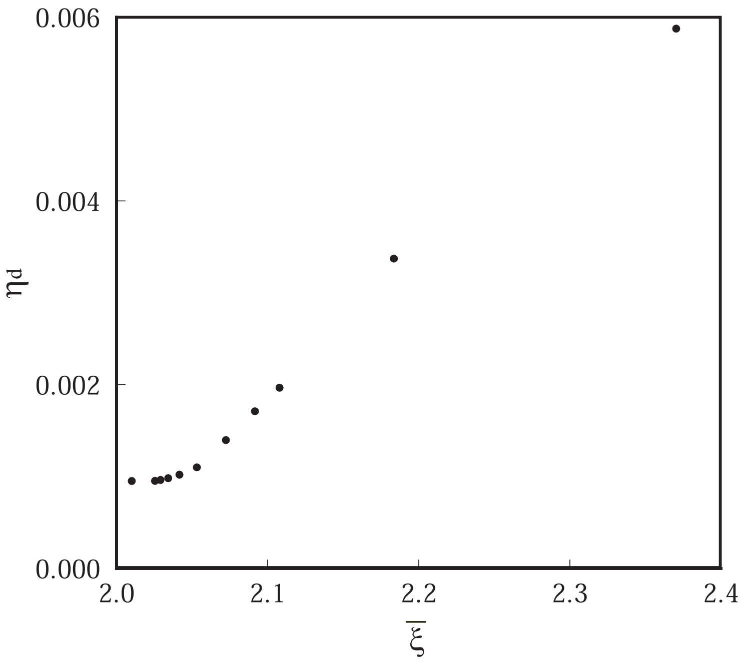
- Simulation of Crack Growth
- Check of Convergence
  - displacement
  - strain/strain energy
- Pattern of Crack Growth

  can small difference in initial configuration cause large difference in crack growth?

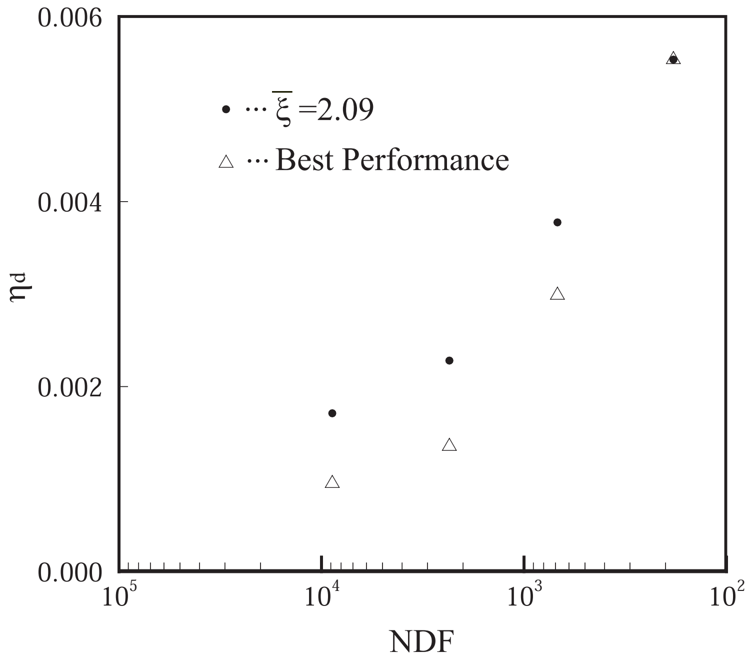


uniform tension

CONVERGENCE OF SOLUTION



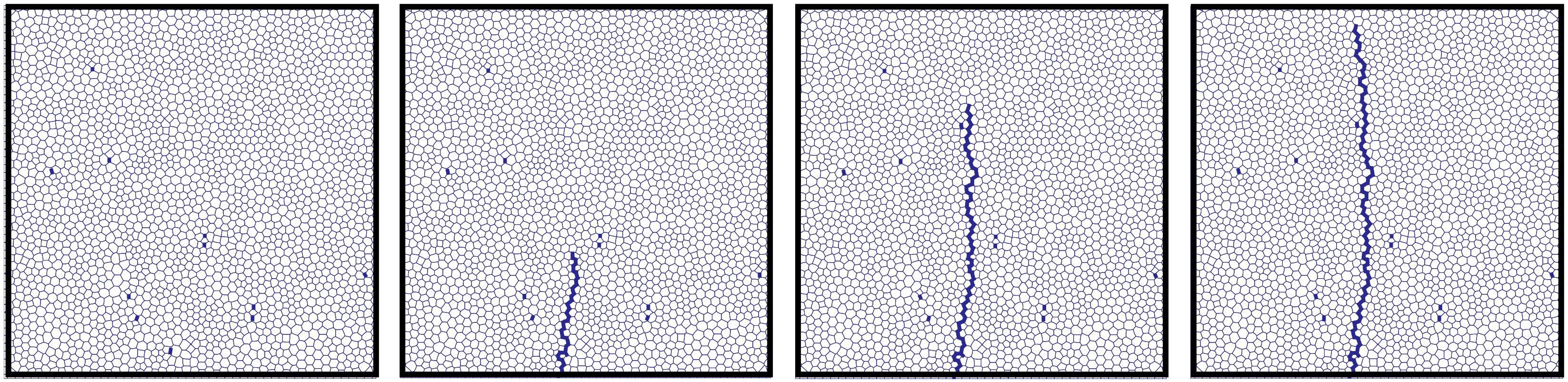
displacement norm w.r.t. mesh quality



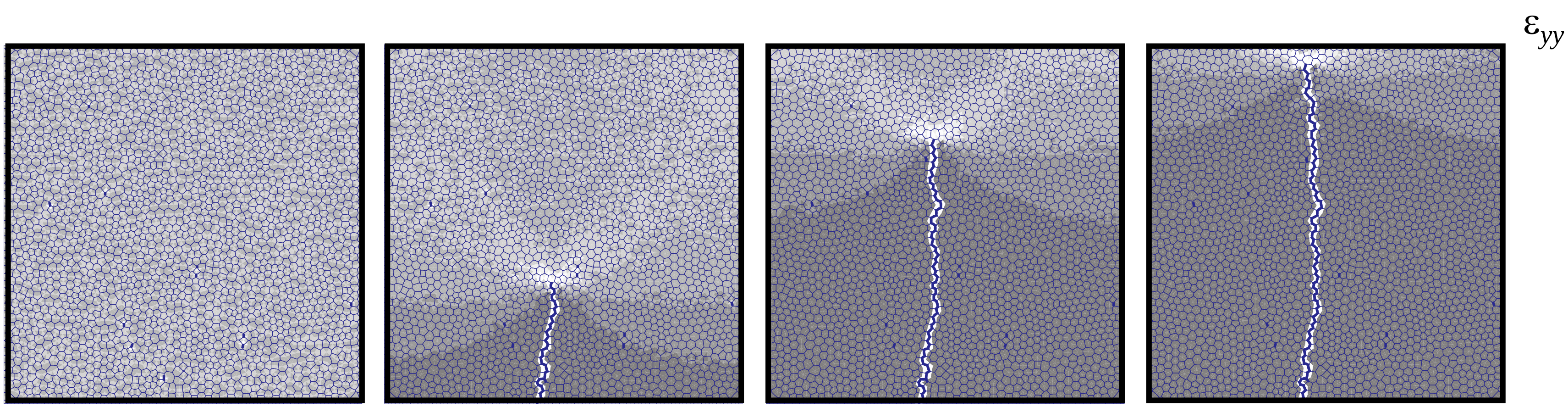
displacement norm w.r.t. NDF

displacement norm: $$\eta_d = \frac{\sqrt{\int_B \| u - \hat{u} \|^2 \, dv}}{\sqrt{\int_B \| \hat{u} \|^2 \, dv}}$$

CHECK OF CRACK GROWTH



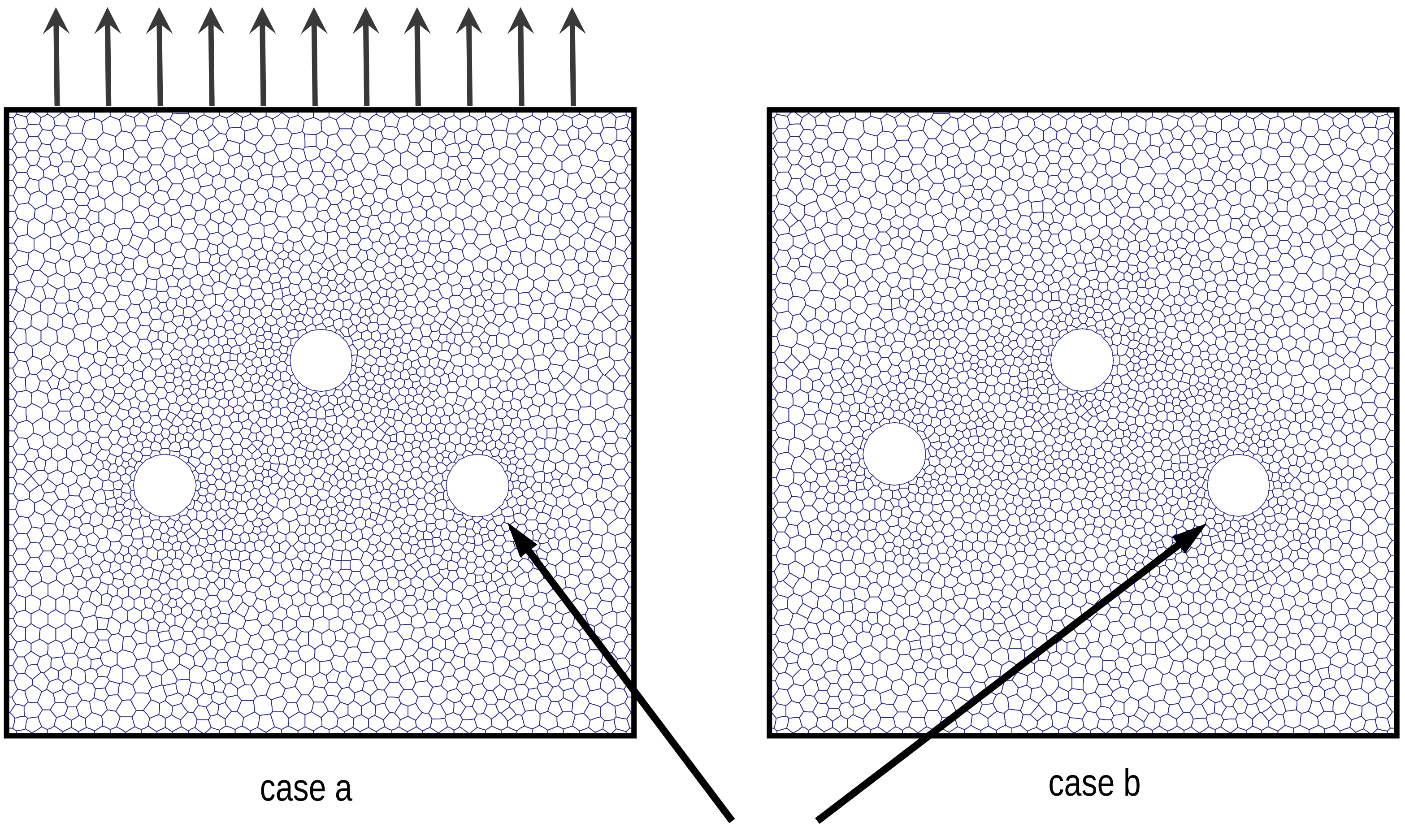
pattern of crack growth



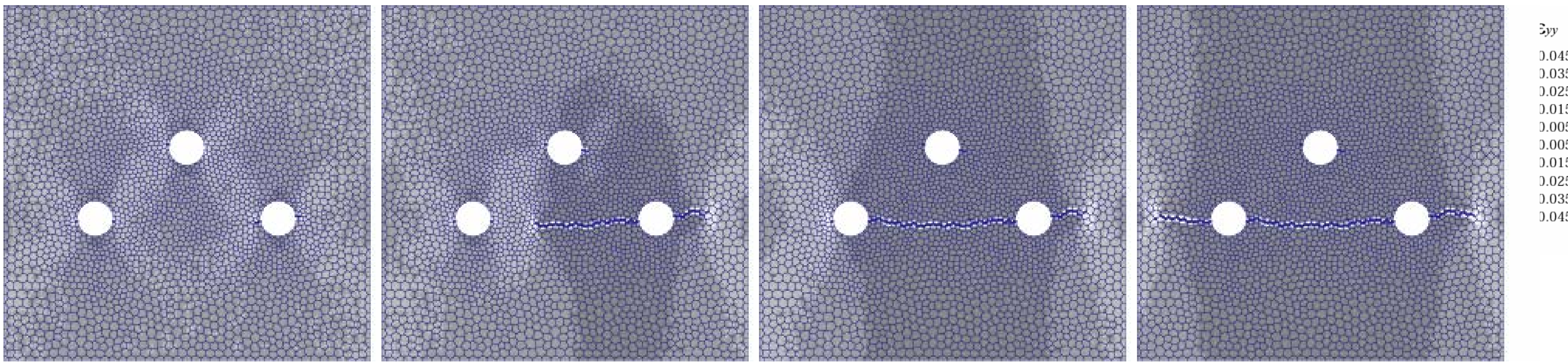
evolution of normal strain distribution

EXAMPLE: PLATE WITH 3 HOLES

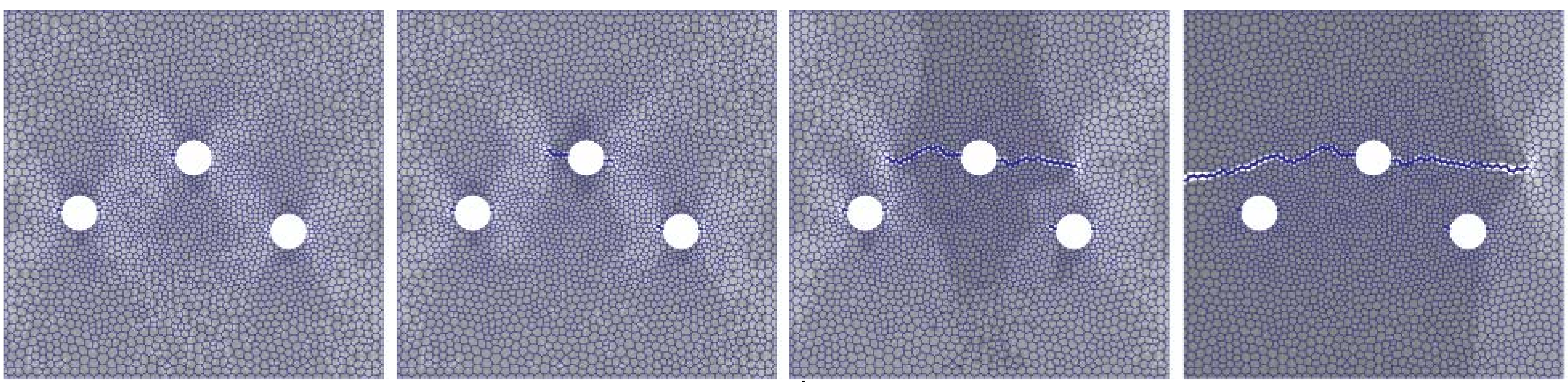
simulation of crack growth: crack stems from holes



DIFFERENCE IN CRACK GROWTH: DISTRIBUTION OF NORMAL STRAIN

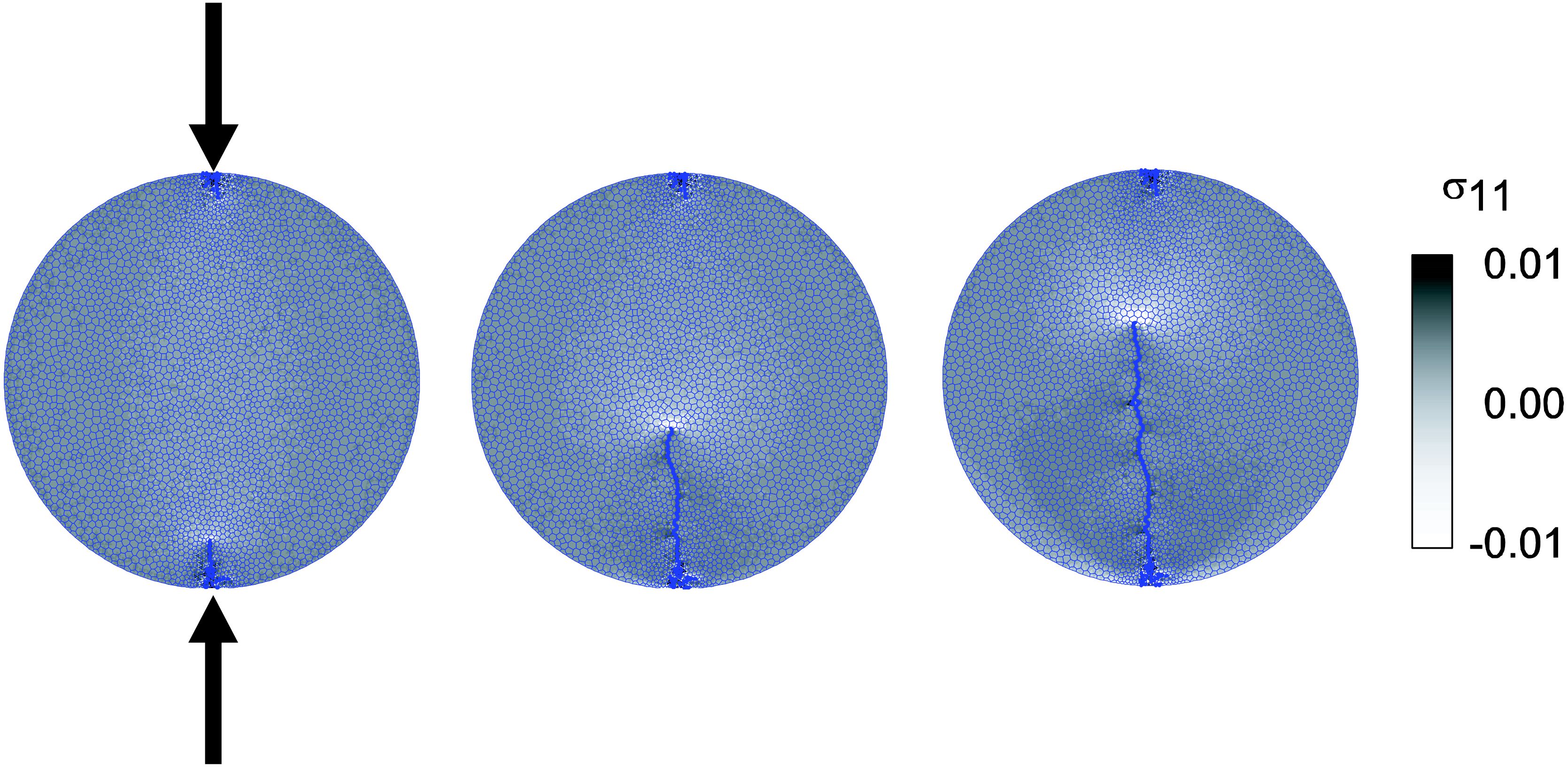


case a



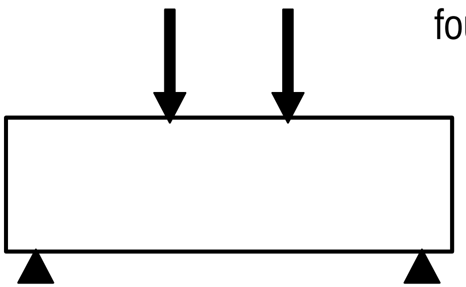
case b

SIMULATION OF BRAZILIAN TEST

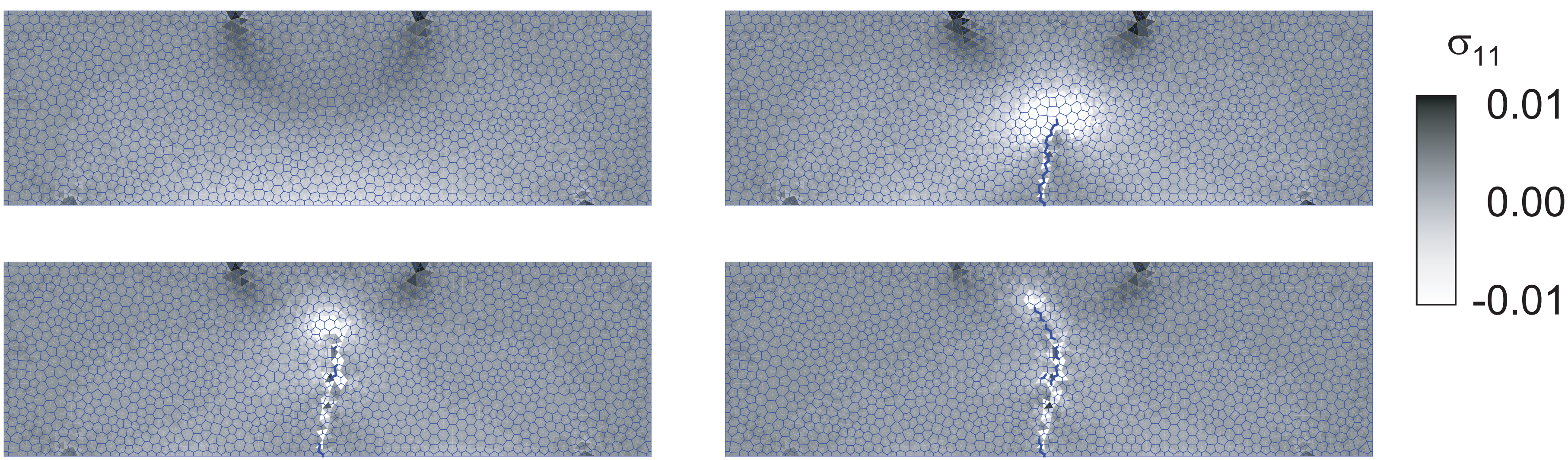


Brazilian test

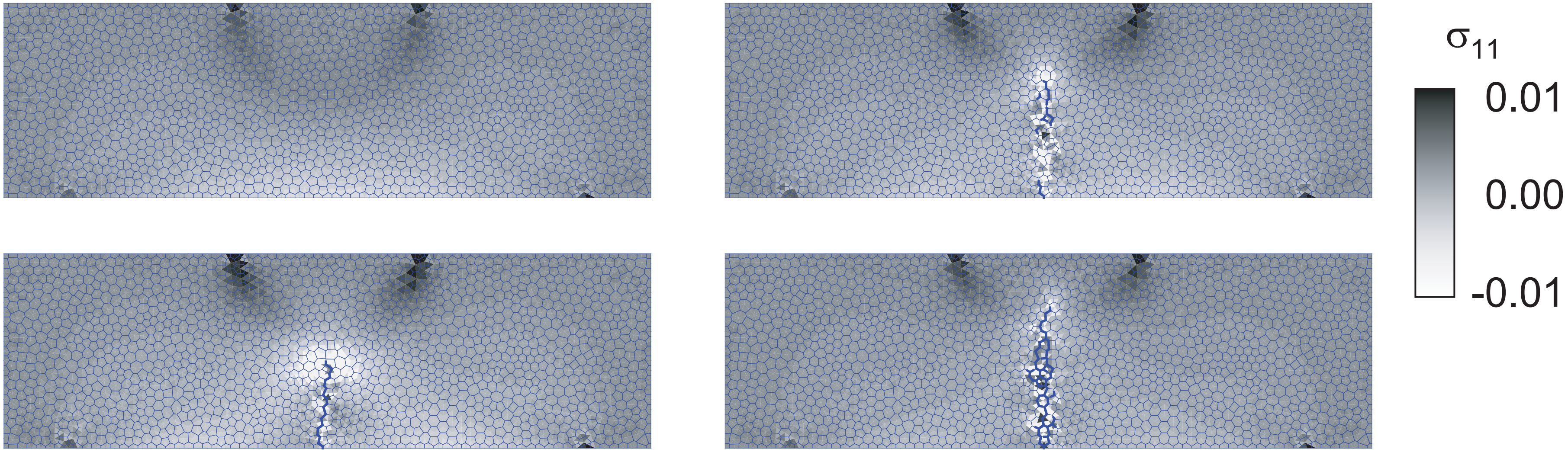
SIMULATION OF FOUR POINT BENDING TEST



four point bending test



FOUR POINT BENDING WITH IDEALLY HOMOGENEOUS MATERIALS



FEM-β puts two source of local heterogeneity, 1) mesh quality for particle discretization and 2) crack path along Voronoi boundary. An ideally homogeneous material which is modeled with best mesh quality sometimes fail to simulate crack propagation.

CONCLUDING REMARKS

Particle Discretization

- discretization scheme using set of non-overlapping characteristic functions

Continuum Mechanics Problem

- essentially same accuracy as FEM with uniform strain
- applicable to non-linear plasticity

Failure Analysis

- simple but robust treatment of failure
- Monte-Carlo simulation for studying local heterogeneity effects on failure
  - candidates of failure patterns are pre-determined by spatial discretization