Problems in Solid Mechanics
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INVERSE ANALYSIS METHODS OF IDENTIFYING CRUSTAL CHARACTERISTICS USING GPS ARRYA DATA

INVERSE ANALYSIS METHODS OF IDENTIFYING CRUSTAL CHARACTERISTICS USING GPS ARRYA DATA

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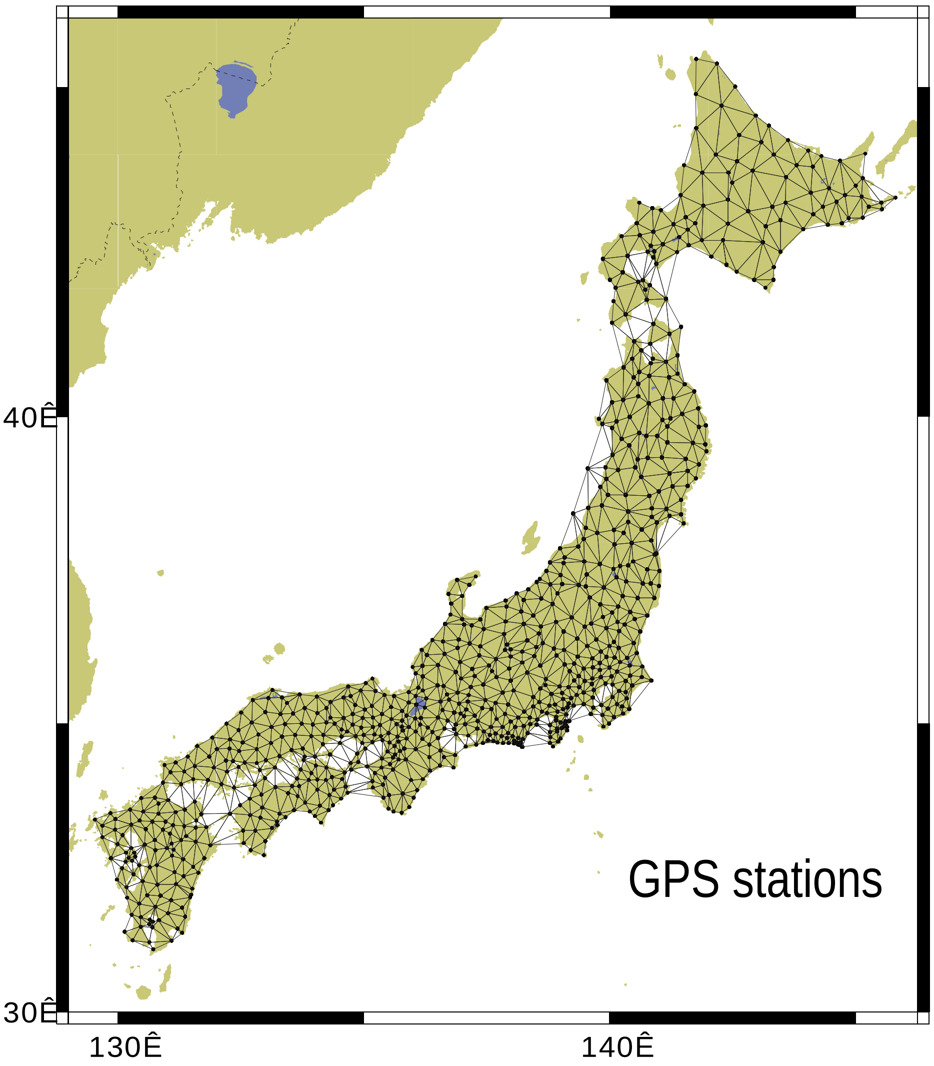
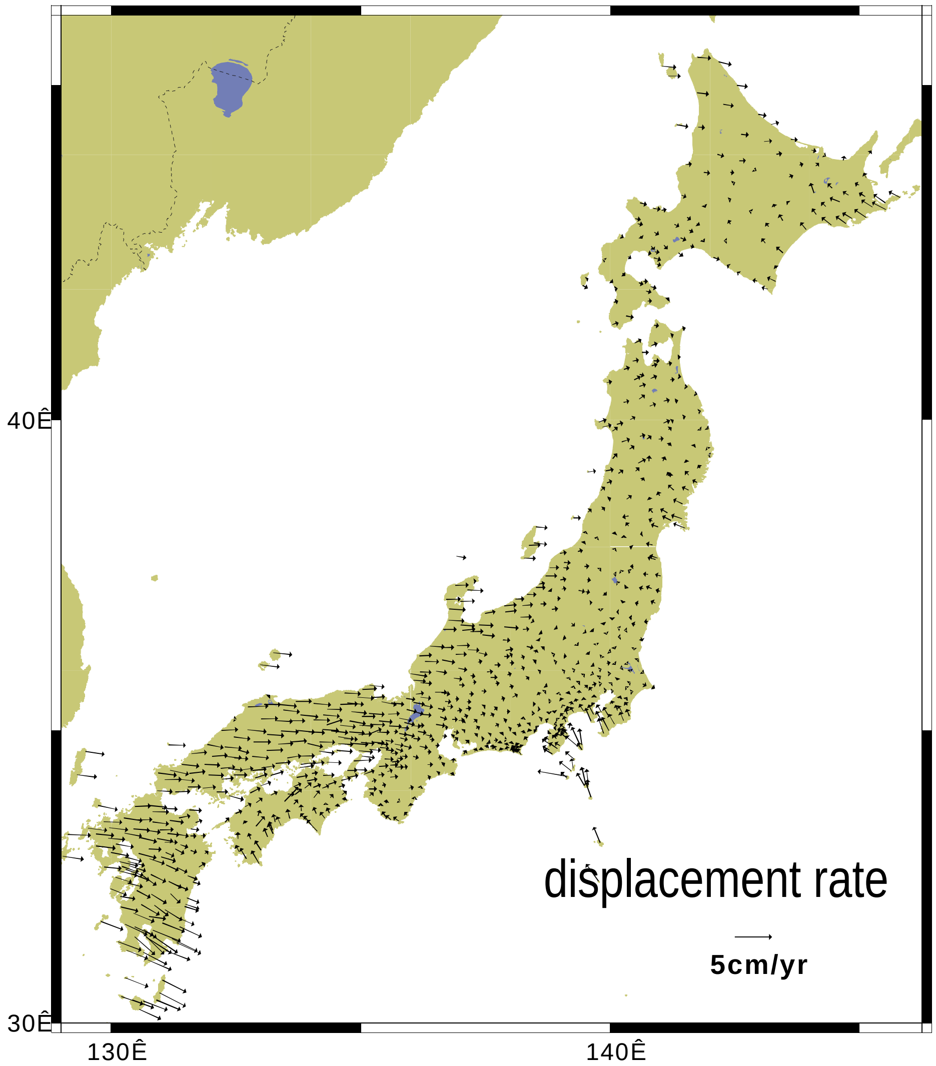
Contents

1. Stress inversion method: find equilibrating stress using measured strain and partial information on stress-strain relation
2. Elasticity inversion method: find local elasticity using densely measured displacement

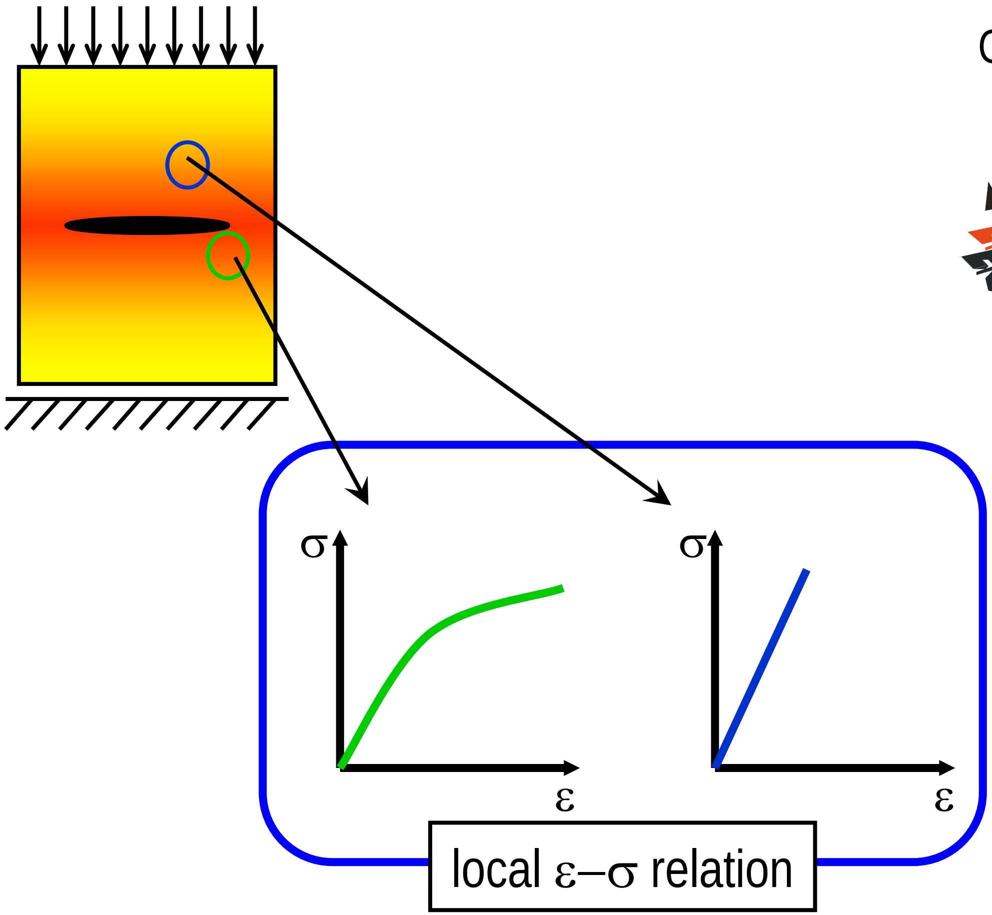
GPS NETWORK AND ITS DATA

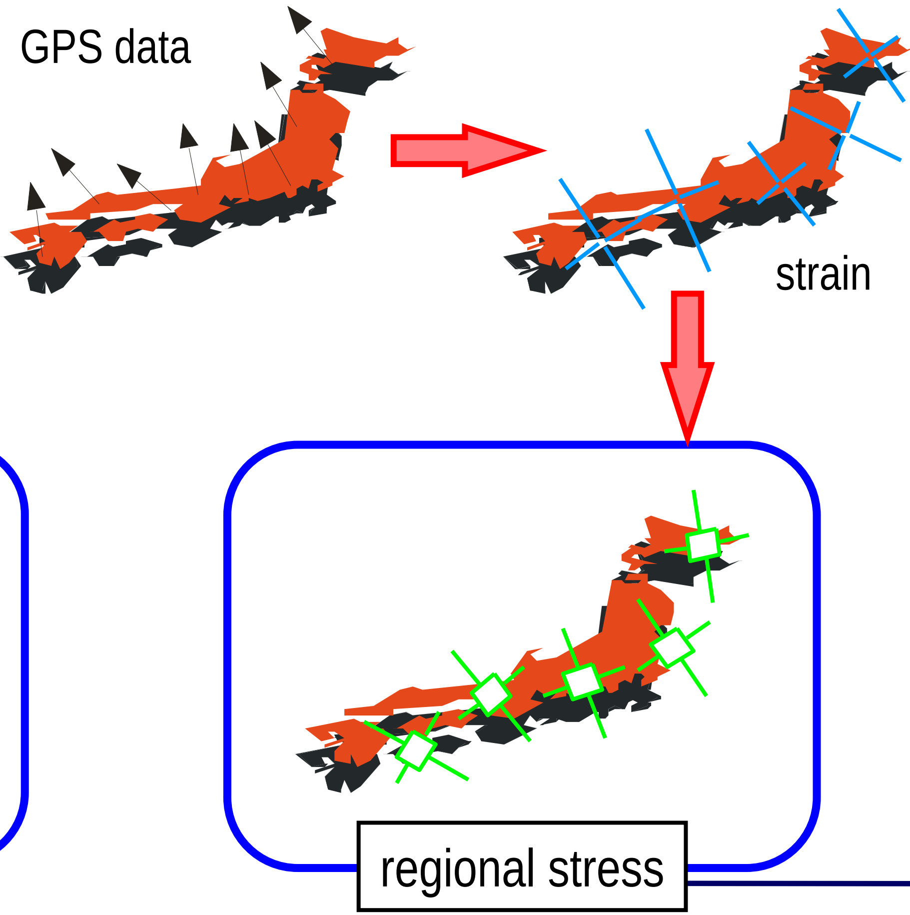




NEED FOR LOCAL STRESS PREDICTION

- Material Test of Next Generation
- Earthquake Prediction





IS STRESS INVERSION POSSIBLE?

- 3D State — Most Difficult
- 2D State — Possible?

3 UNKNOWNS

σ_{11}

σ_{22}

σ_{12}

2 EQUATIONS

equilibrium in x_1-direction

equilibrium in x_2-direction

1 CONDITION

STRAIN

STRESS INVERSION

STRESS INVERSION

Airy

$$\begin{cases} \sigma_{11} = a_{,22} \\ \sigma_{22} = a_{,11} \\ \sigma_{12} = -a_{,12} \end{cases}$$

partial information: $\sigma_{11} + \sigma_{22} = f(\varepsilon_{ij})$

$$f(\varepsilon_{ij}) = \kappa(\varepsilon_{11} + \varepsilon_{22})$$

$$\Downarrow$$

$$\sigma_{11} + \sigma_{22} = \kappa(\varepsilon_{11} + \varepsilon_{22})$$

2D bulk modulus?

self-equilibrating stress

B.V.P.

Poisson equation

measured strain

G.E. $\quad a_{,11} + a_{,22} = \kappa(\varepsilon_{11} + \varepsilon_{22})$

B.C. $\quad n_1 a_{,1} + n_2 a_{,2} = -n_1 r_2 + n_2 r_1$

resultant forces

EXTENSION TO OTHER DEFORMATION STATE

EXTENSION TO OTHER DEFORMATION STATE

- Dynamic State

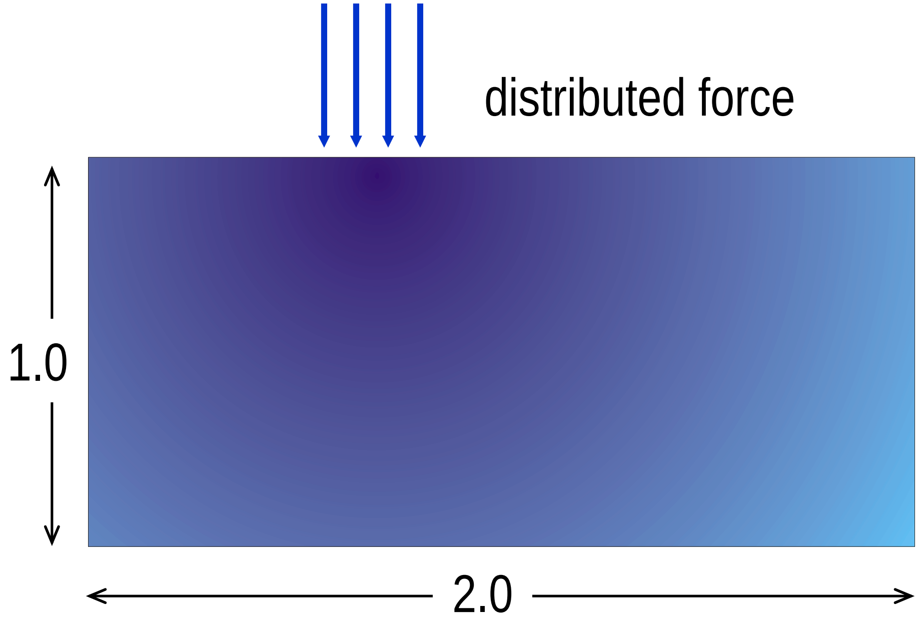
$$\begin{cases} \sigma_{11,1} + \sigma_{12,2} = \rho \ddot{u}_1 \\ \sigma_{12,1} + \sigma_{22,2} = \rho \ddot{u}_2 \\ \sigma_{11} + \sigma_{22} = f(\varepsilon) \end{cases}$$

- Finite Deformation State:

$$\begin{cases} \sum_{j,k} (\partial X_k / \partial x_j) \sigma_{1j,k} = 0 \\ \sum_{j,k} (\partial X_k / \partial x_j) \sigma_{2j,k} = 0 \\ \sigma_{11} + \sigma_{22} = f(\varepsilon) \end{cases} \qquad \left(\sigma_{ij} = \sigma_{ij}(X),\ x_i = x_i(X) \right)$$

NUMERICAL SIMULATION

- Conditions
  - elasto-plastic material with unknown yield function
  - prediction of stress and stress-strain relation

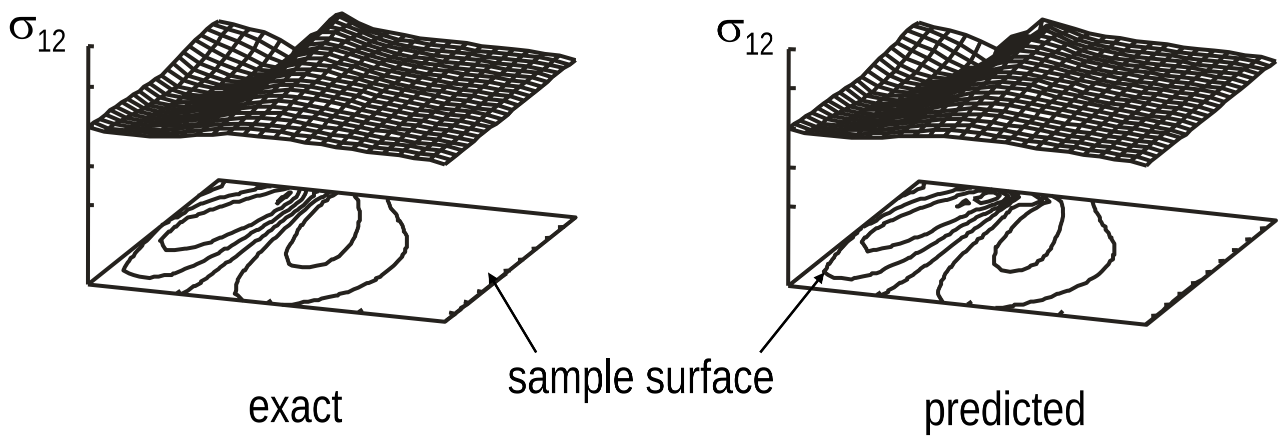


displacement field is used as input data

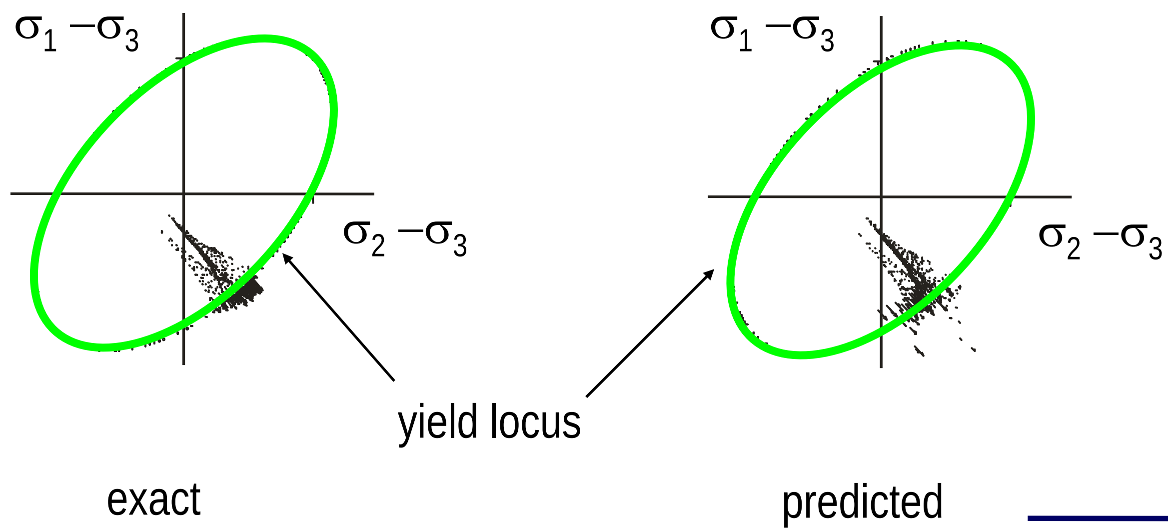
FEM computation with 20x48 elements

RESULTS OF INVERSION

distribution of σ_{12}

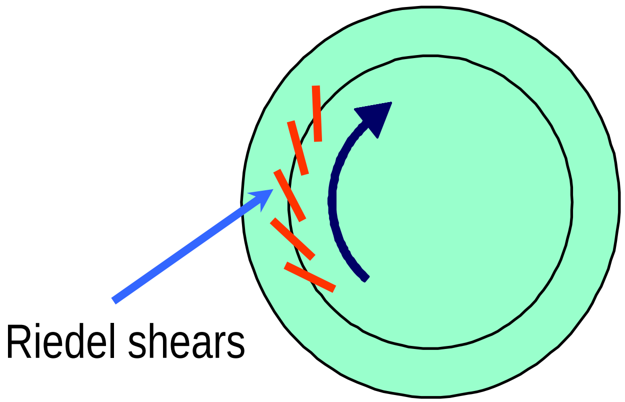


principle stress-
plastic strain rate

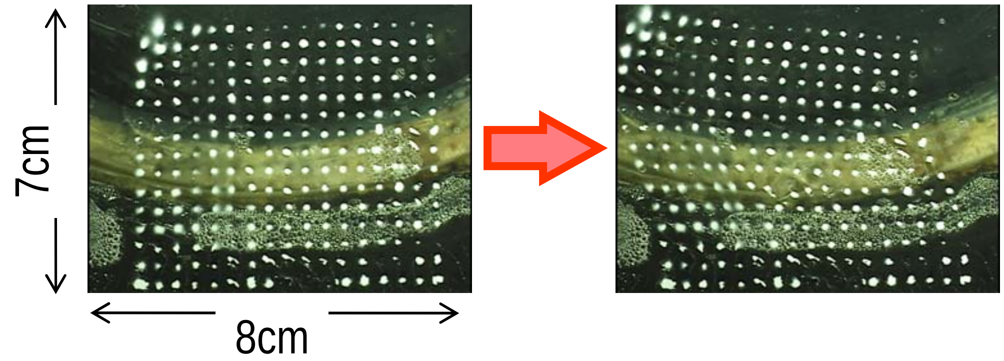


good agreement

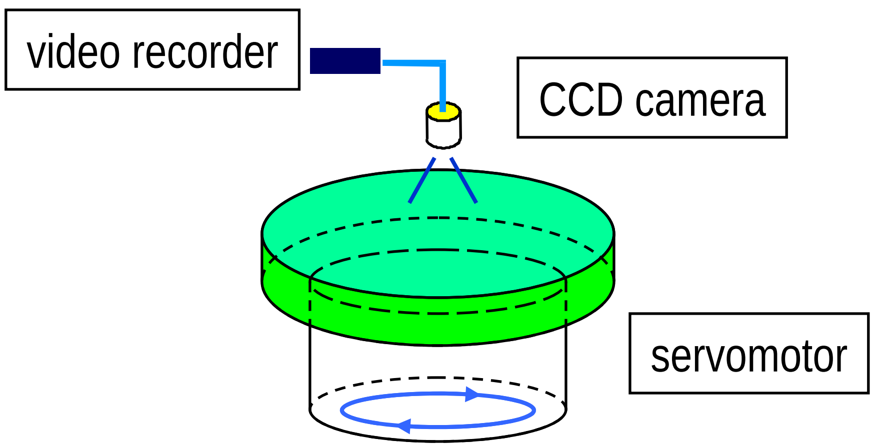
MODEL EXPERIMENT



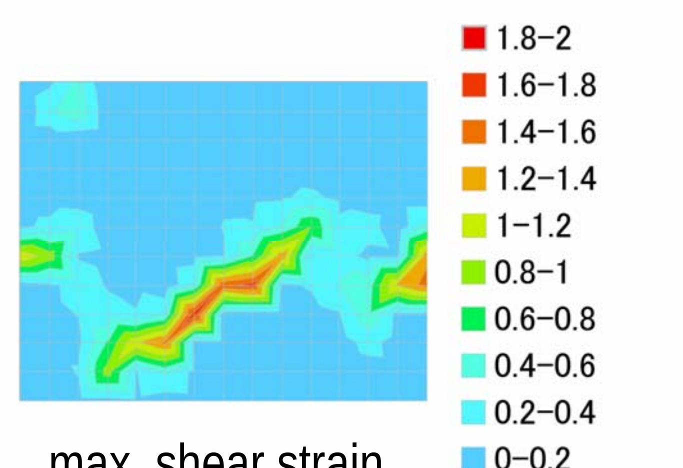
Torsional Shearing



examples of image

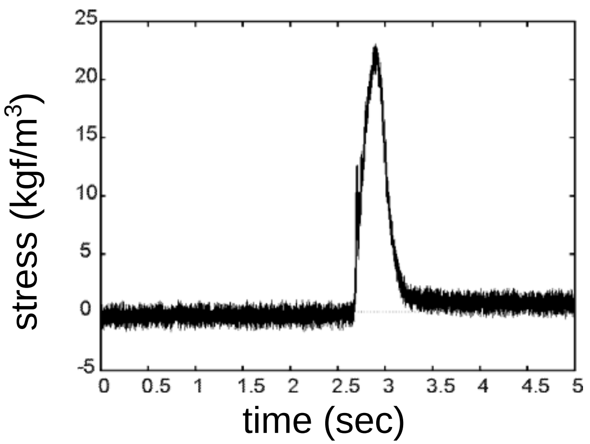


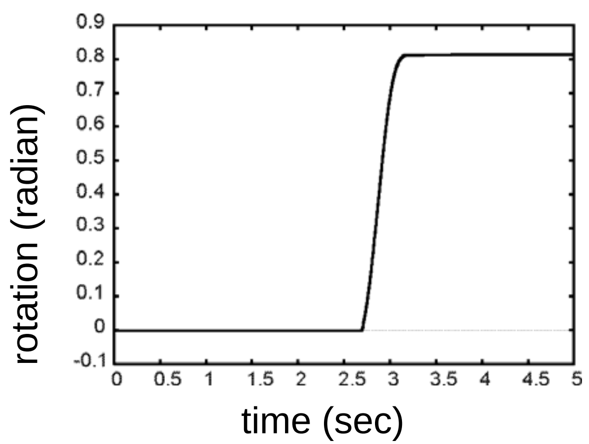
Experiment Apparatus

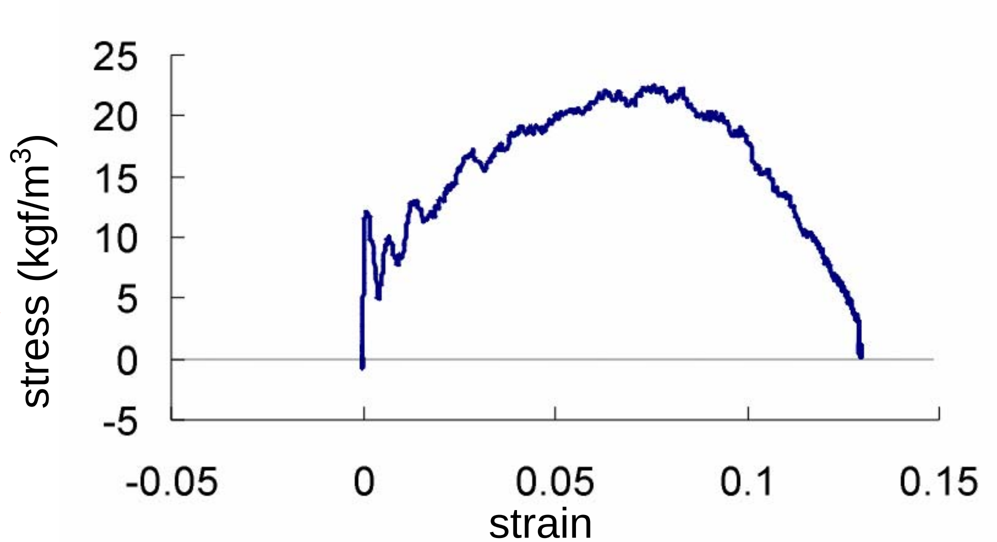


max. shear strain

OVERALL STRESS-STRAIN RELATION

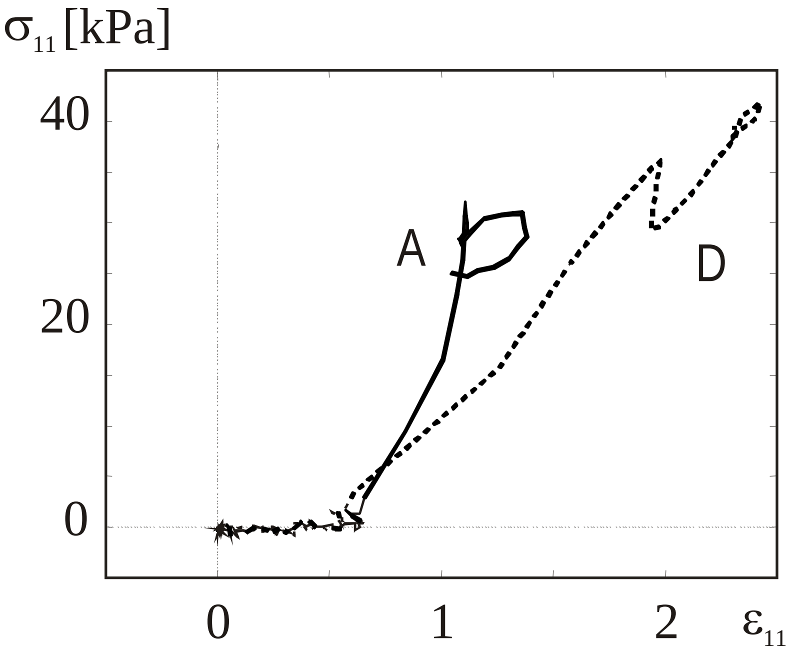




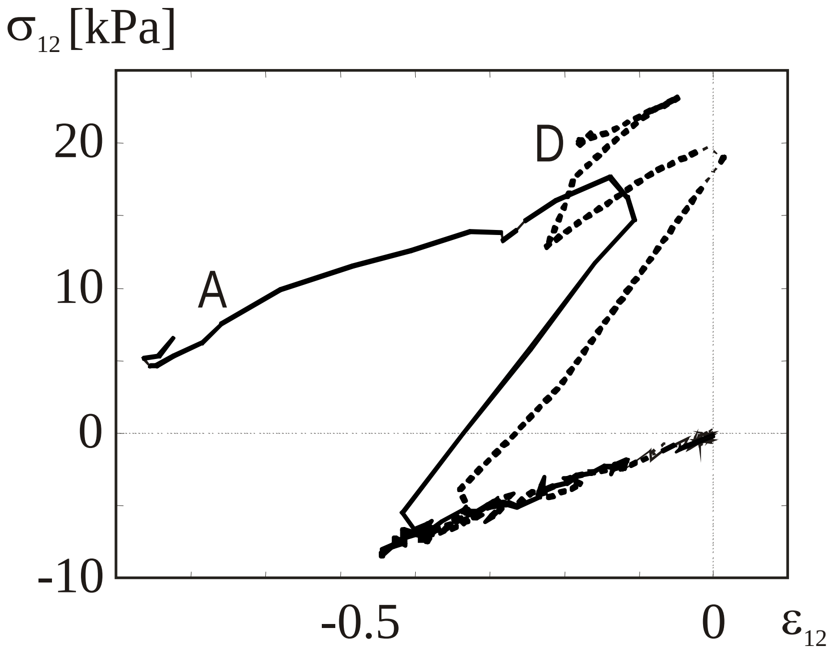


will be different from local relations?

LOCAL STRESS-STRAIN RELATIONS



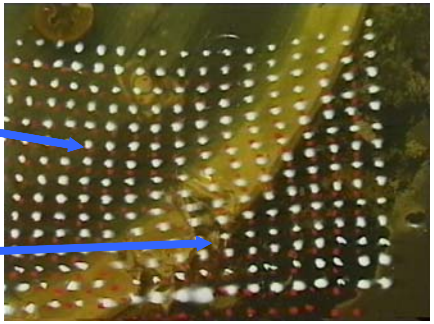
normal stress-strain



shear stress-strain

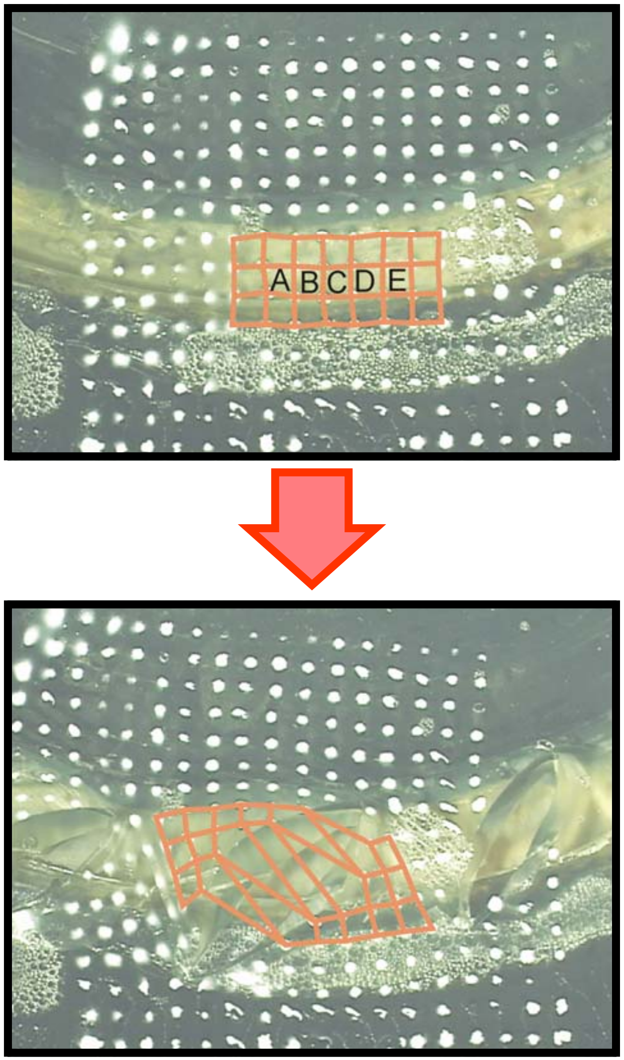
A: far from crack

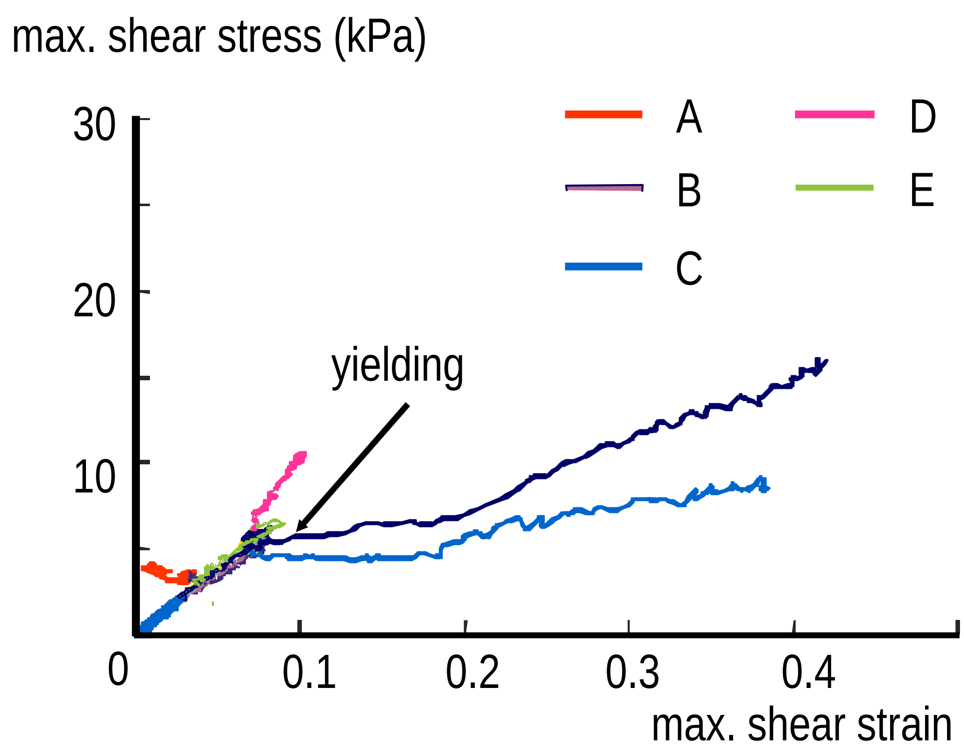
D: near crack



no common relations?

RESULTS OF INVERSION





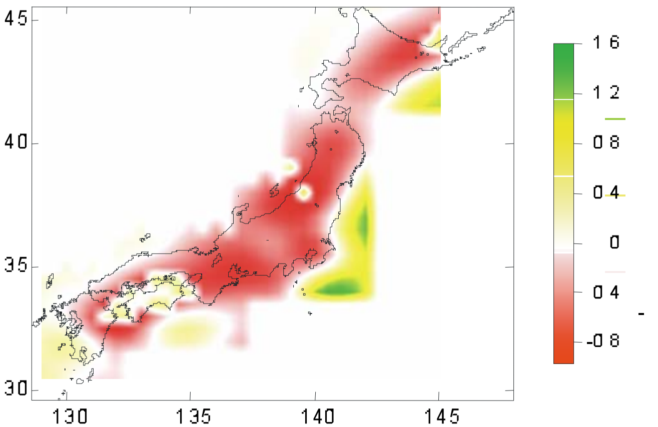
common elasto-plastic relations?

APPLICATION TO GPS ARRYA DATA

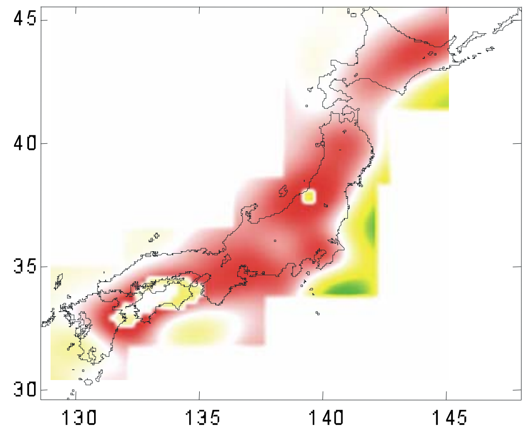
APPLICATION TO GPS ARRYA DATA

- verification of numerical analysis method
  - check numerical stability of solving boundary value problem
  - check dependency of parameters
- application of stress inversion method
  - geophysical interpretation of analysis results
  - critical examination of assumption of plane state
- development of crust deformation monitor
  - automatic processing of GPS array data

CONVERGENCE

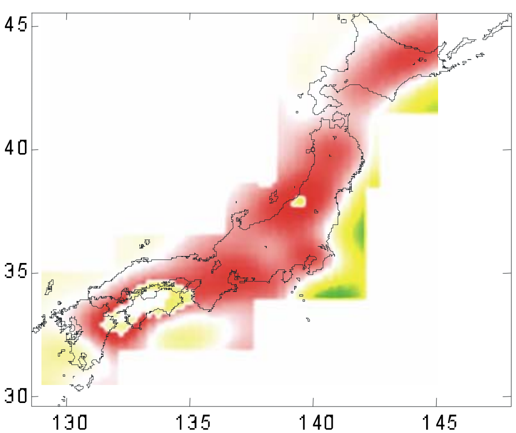


a) Δ=0.5



b) Δ=0.25

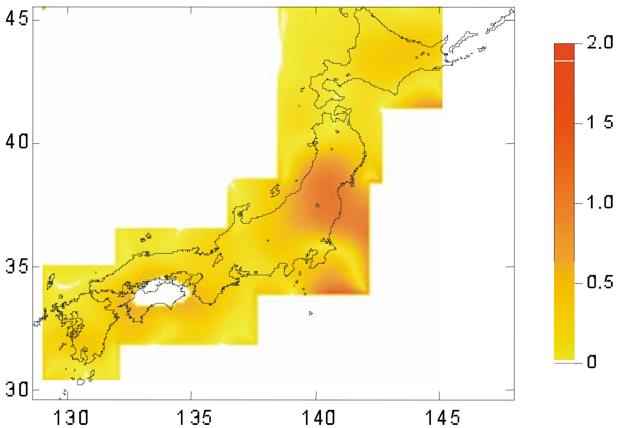
σ
(hydrostatic stress)



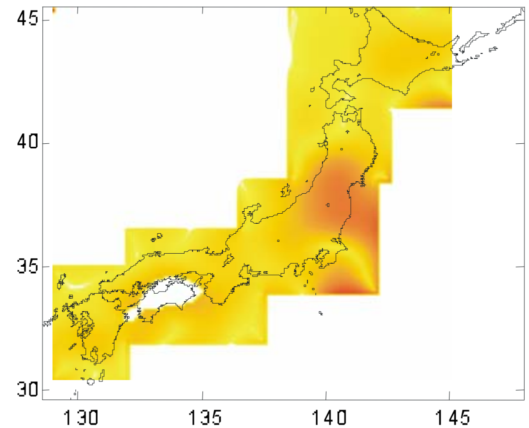
c) Δ=0.125

Δ: resolution of strain distribution (degree)

EFFECT OF REFERENCE

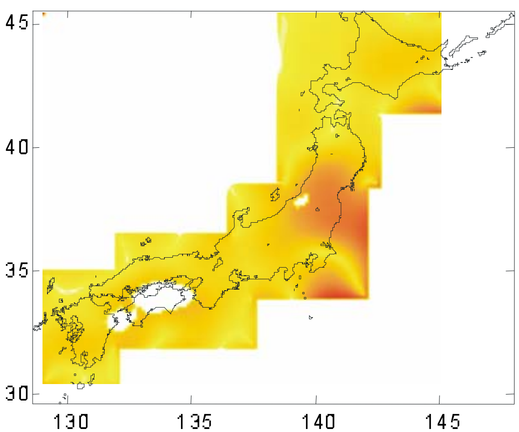


a) ν=0.1 (τ)

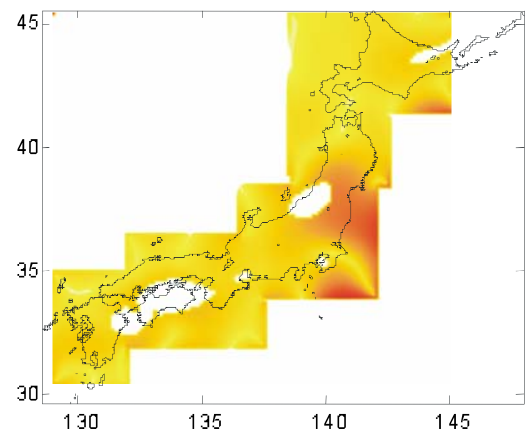


b) ν=0.2 (τ)

τ
(max. shear stress)

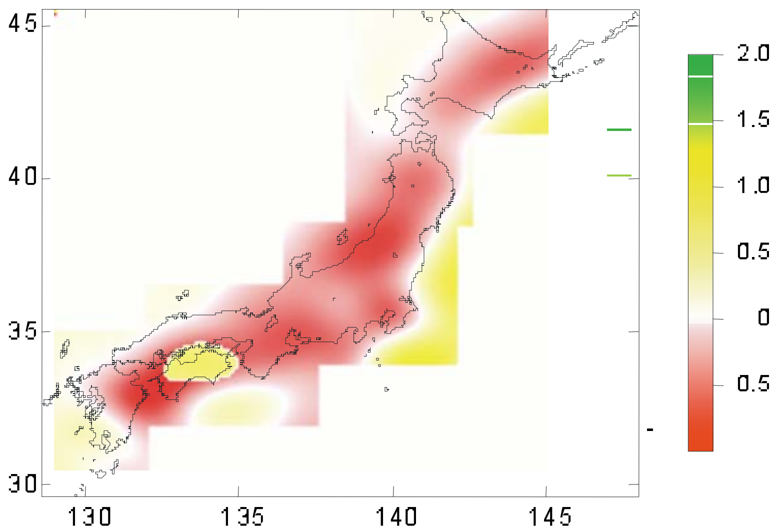


c) ν=0.3 (τ)

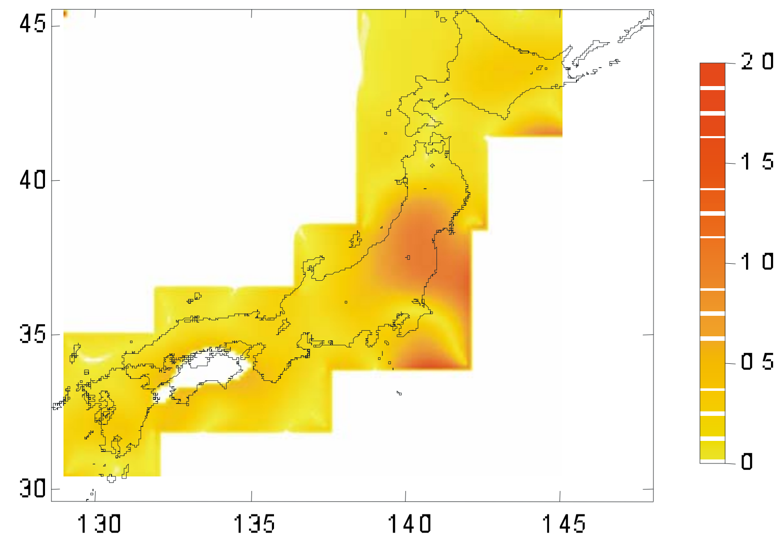


d) ν=0.4 (τ)

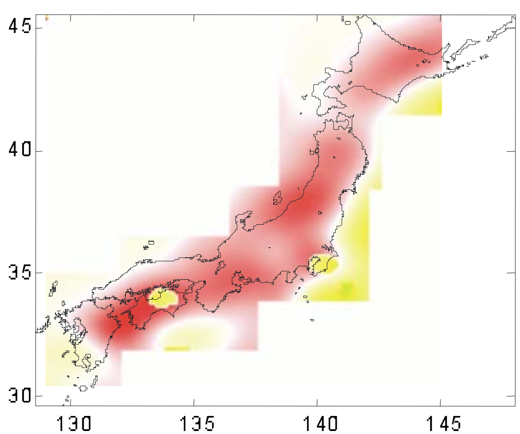
COMPARISON OF STRESS WITH STRAIN



σ (hydrostatic stress)

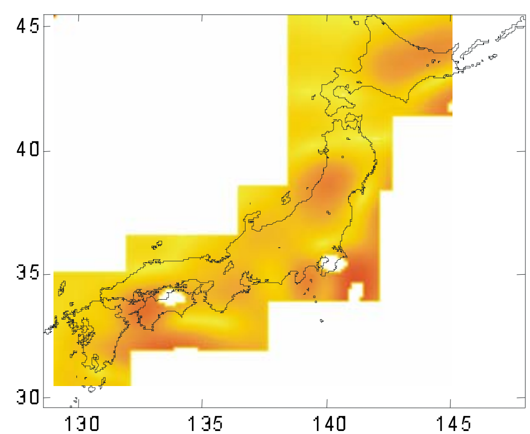


τ (max. shear stress)



ε

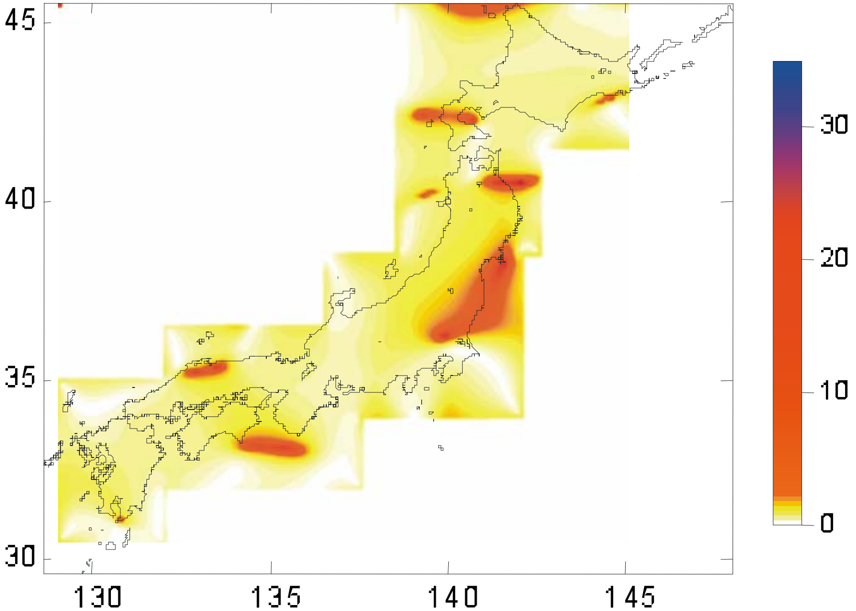
(vol. strain)



γ

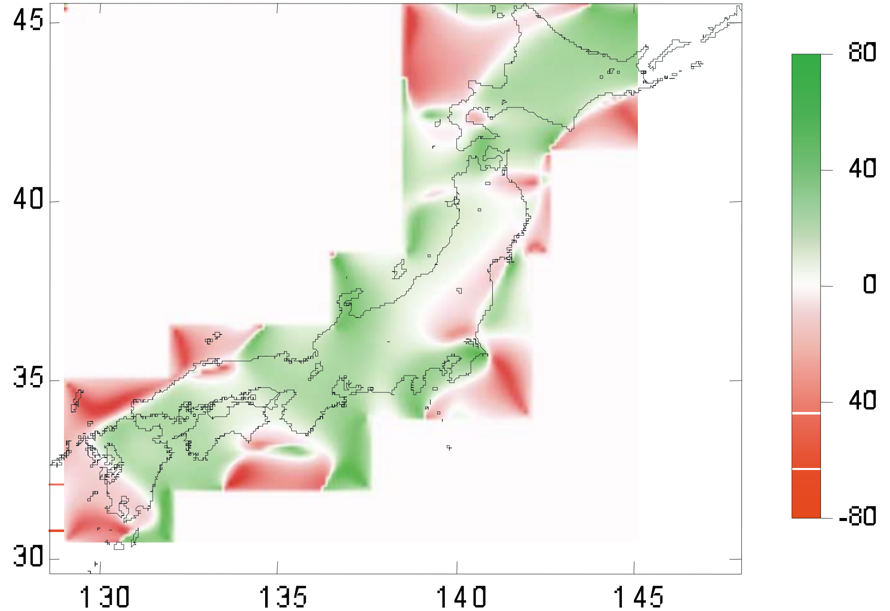
(max. shear strain)

REGIONAL CONSTITUTIVE RELATIONS



τ/γ

regional stiffness
(τ, γ: max. shear stress and strain)



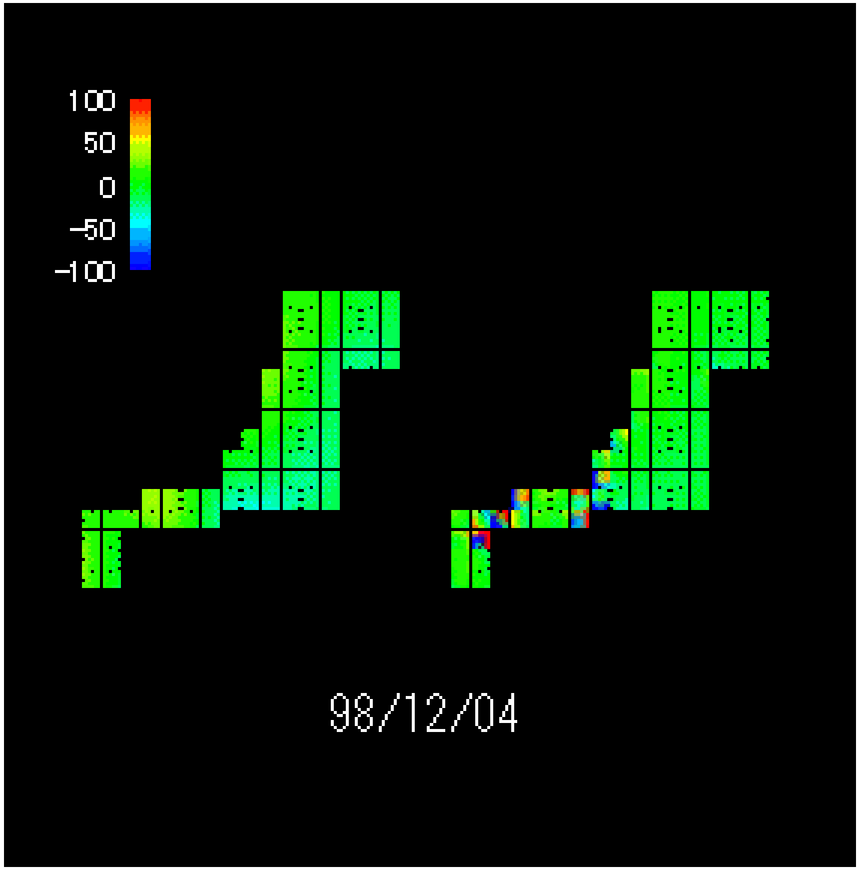
$\phi-\theta$

regional anisotropy
(ϕ, γ: principle stress and strain)

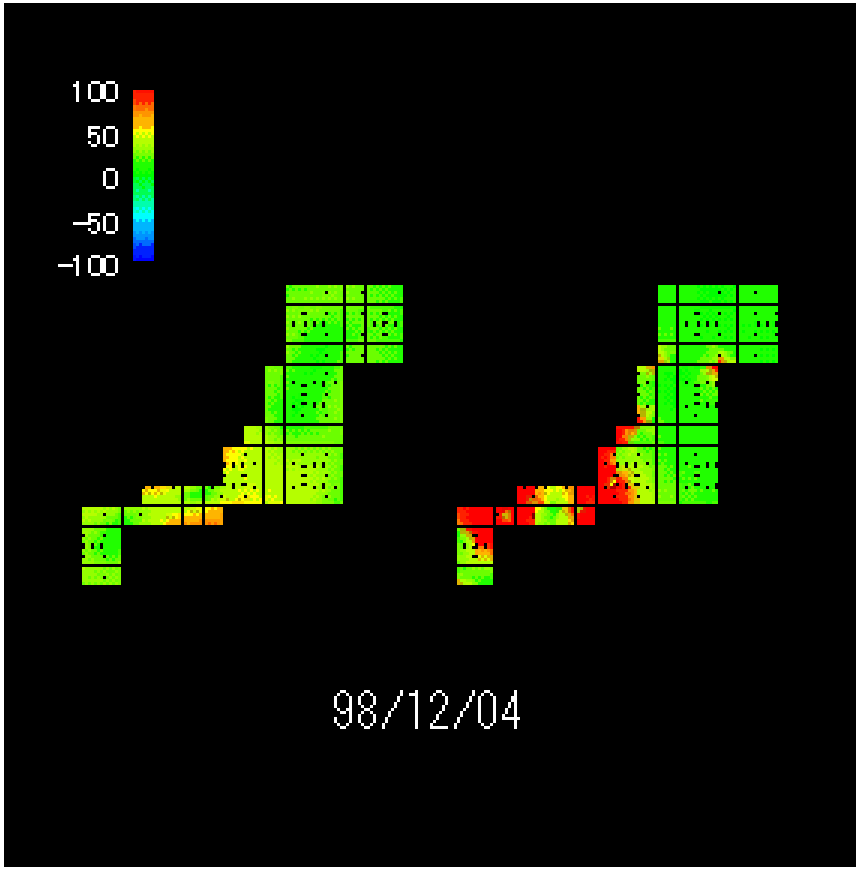
regional heterogeneity and anisotropy

CHANGE IN INVARIANT

1st invariant



2nd invariant

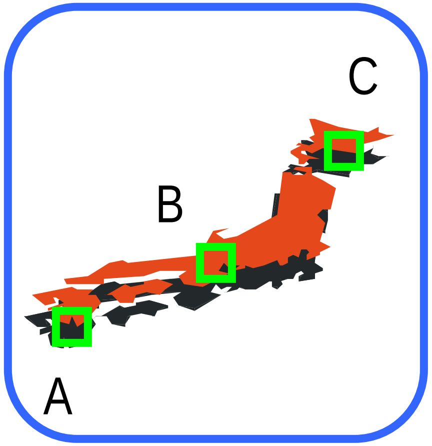


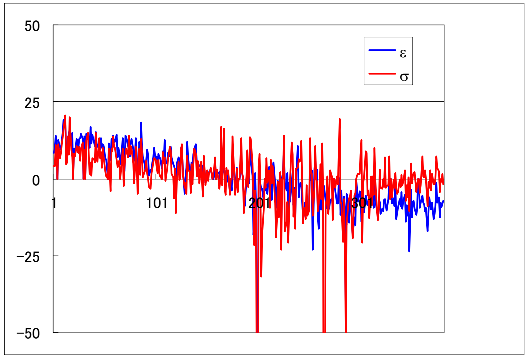
CHANGE IN REGIONAL STATE

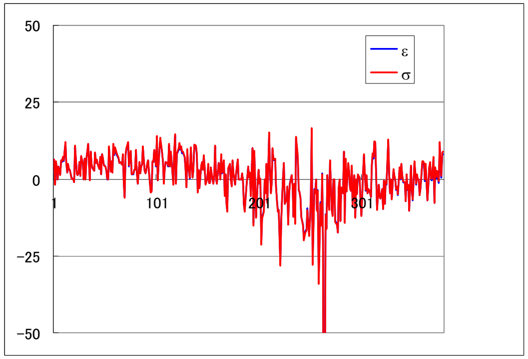
1st invariant

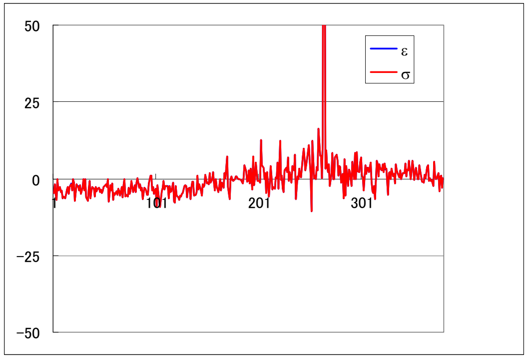
ε

σ







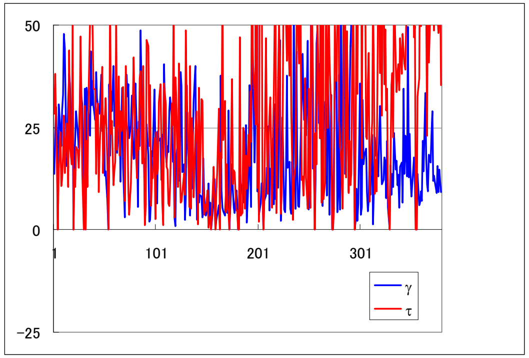


2nd invariant

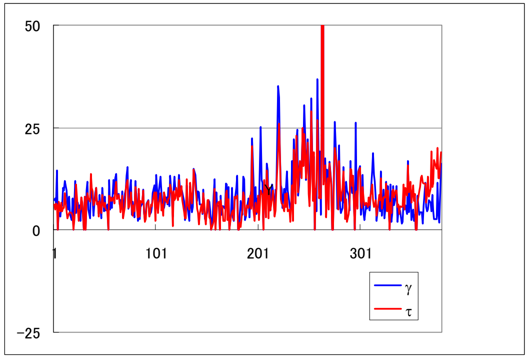
γ

τ

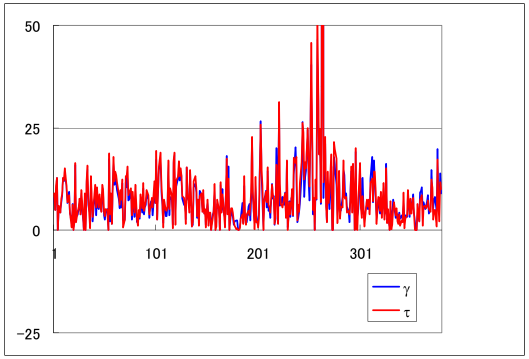
A



B



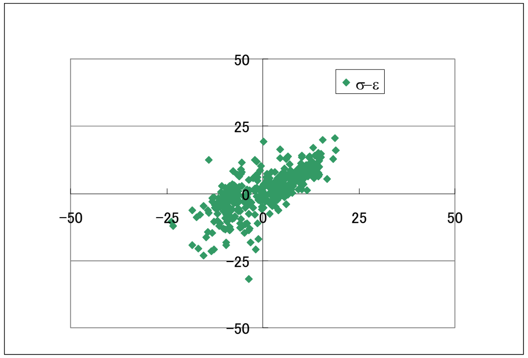
C

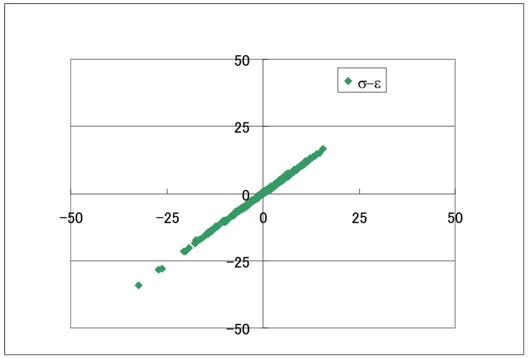


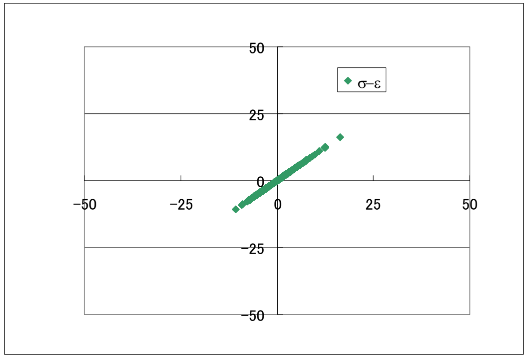
REGIONAL STRESS AND STRAIN

1st invariant

ε−γ



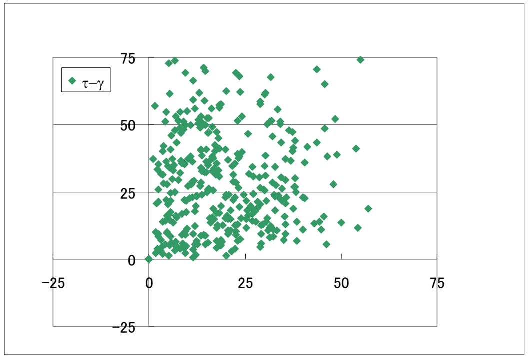




2nd invariant

γ−τ

A



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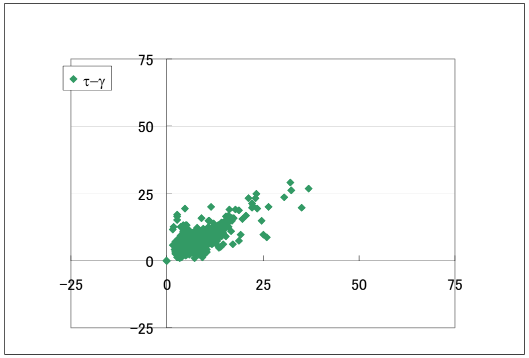
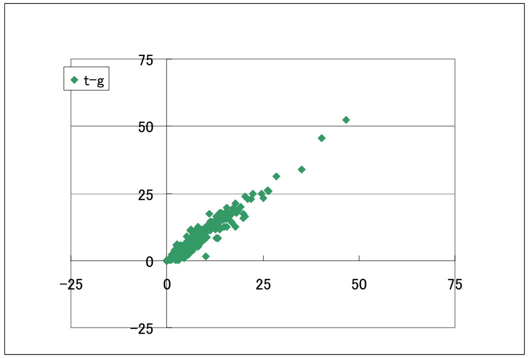
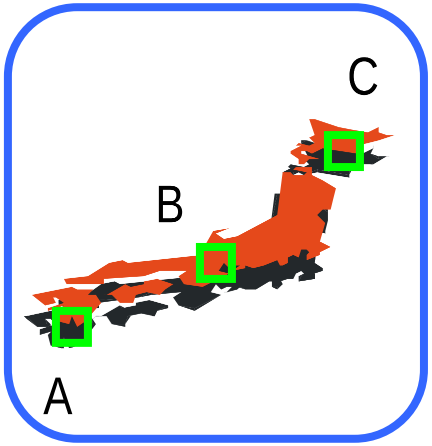


C





GPS DATA DURING 1998-1999

GPS DATA DURING 1998-1999

- GPS Data
  - no spatial filtering to get rid of measurement noise
  - linear interpolation between two GPS station

- More Sophisticated Treatment of BVP
  - FEM with triangle element
  - weak form
  - regionally averaged field quantities

APPLICATION TO GPS NETWORK DATA

APPLICATION TO GPS NETWORK DATA

BVP in Rate Form and Weak Form

strain rate

G.E. $\dot{a}_{,11}+\dot{a}_{,22}=\kappa(\dot{\varepsilon}_{11}+\dot{\varepsilon}_{22})$

B.C. $n_1\dot{a}_{,1}+n_2\dot{a}_{,2}=-n_1\dot{r}_2+n_2\dot{r}_1$

$$\boxed{\int \varphi_{,1}\dot{a}_{,1}+\varphi_{,2}\dot{a}_{,2}-\kappa(\varphi_{,1}\dot{u}_1+\varphi_{,2}\dot{u}_2)dS=0}$$

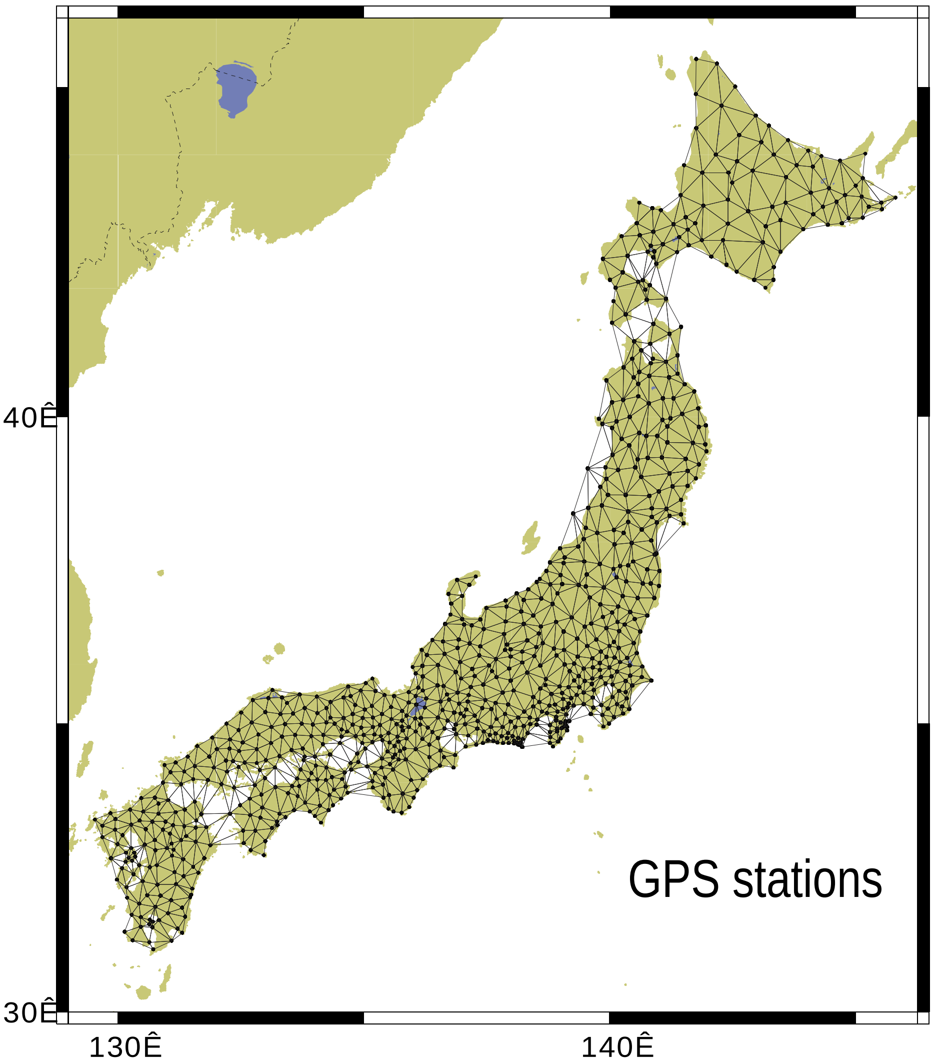
displacement rate

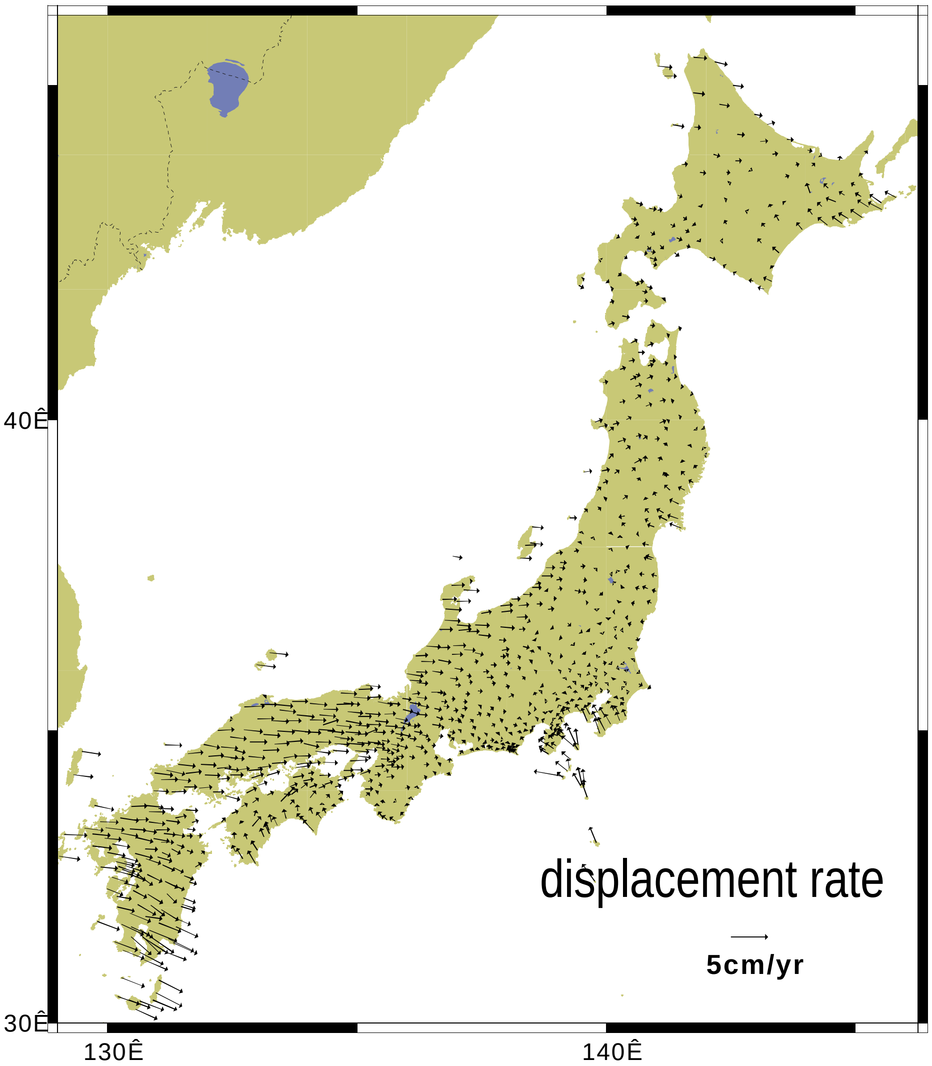
Computation of Average Quantities

$$\boxed{\begin{aligned}\langle\dot{\varepsilon}_{11}\rangle &= \frac{1}{\Omega}\int_{\partial\Omega} n_1\dot{u}_1 dL \\ \langle\dot{\sigma}_{11}\rangle &= \frac{1}{\Omega}\int_{\partial\Omega} n_2\dot{a}_{,2} dL\end{aligned}}$$

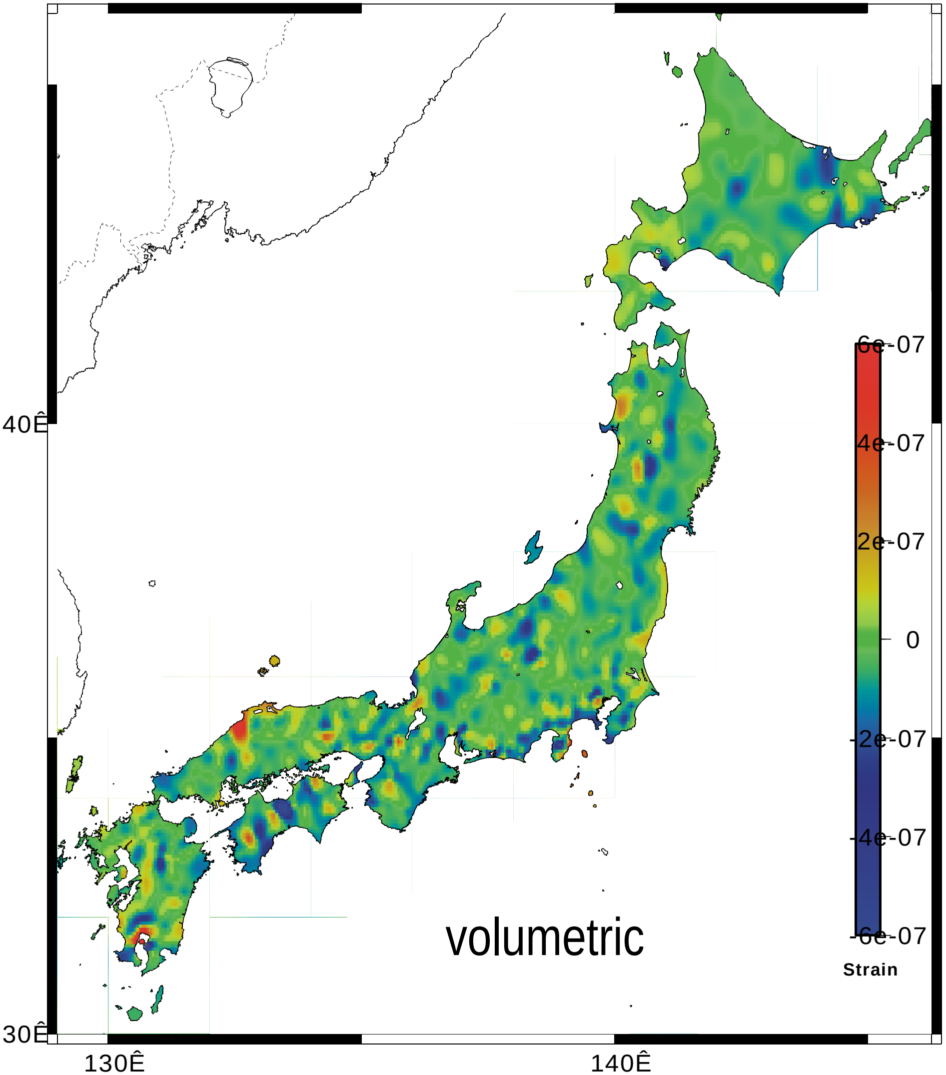
average stress rate computed by using 1st-order derivative

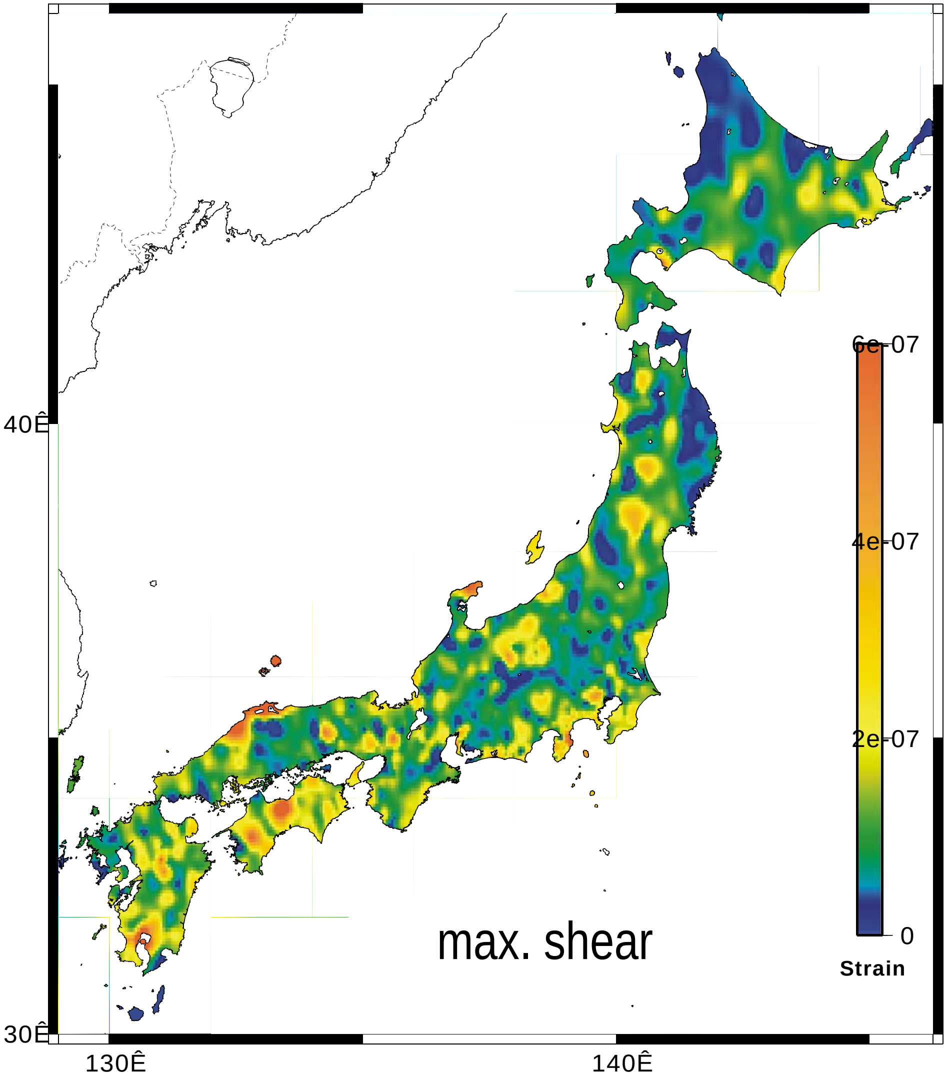
GPS NETWORK AND ITS DATA



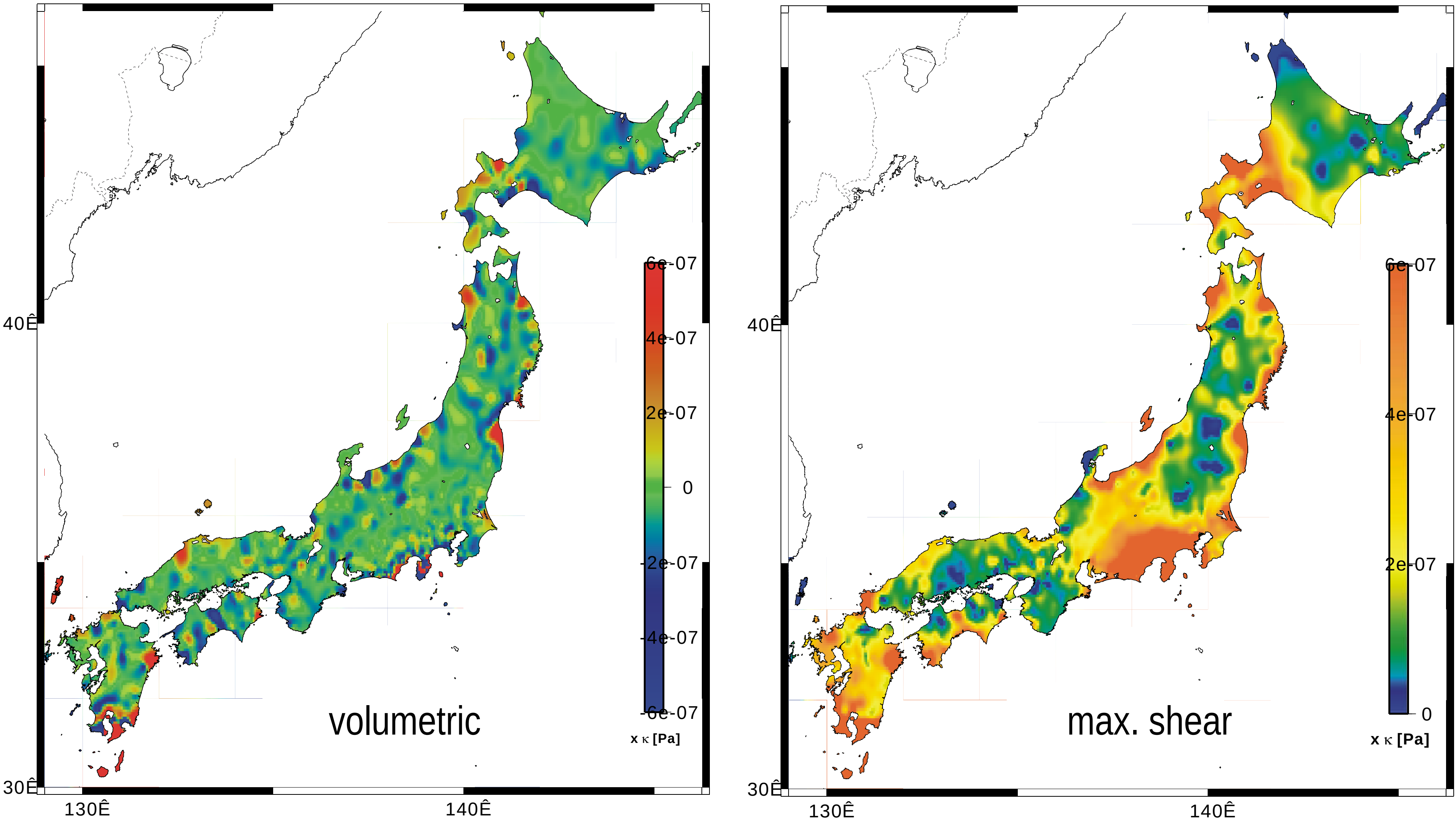


REGIONAL STRAIN RATE

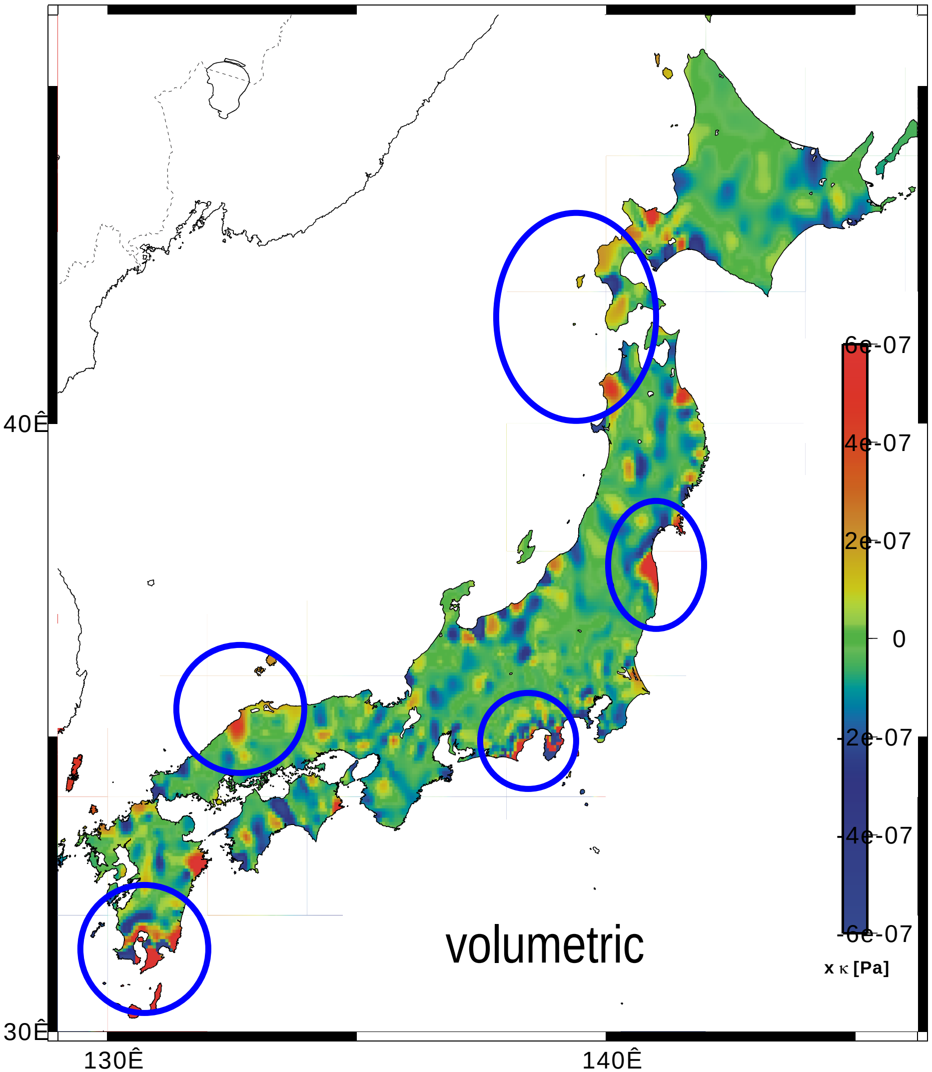


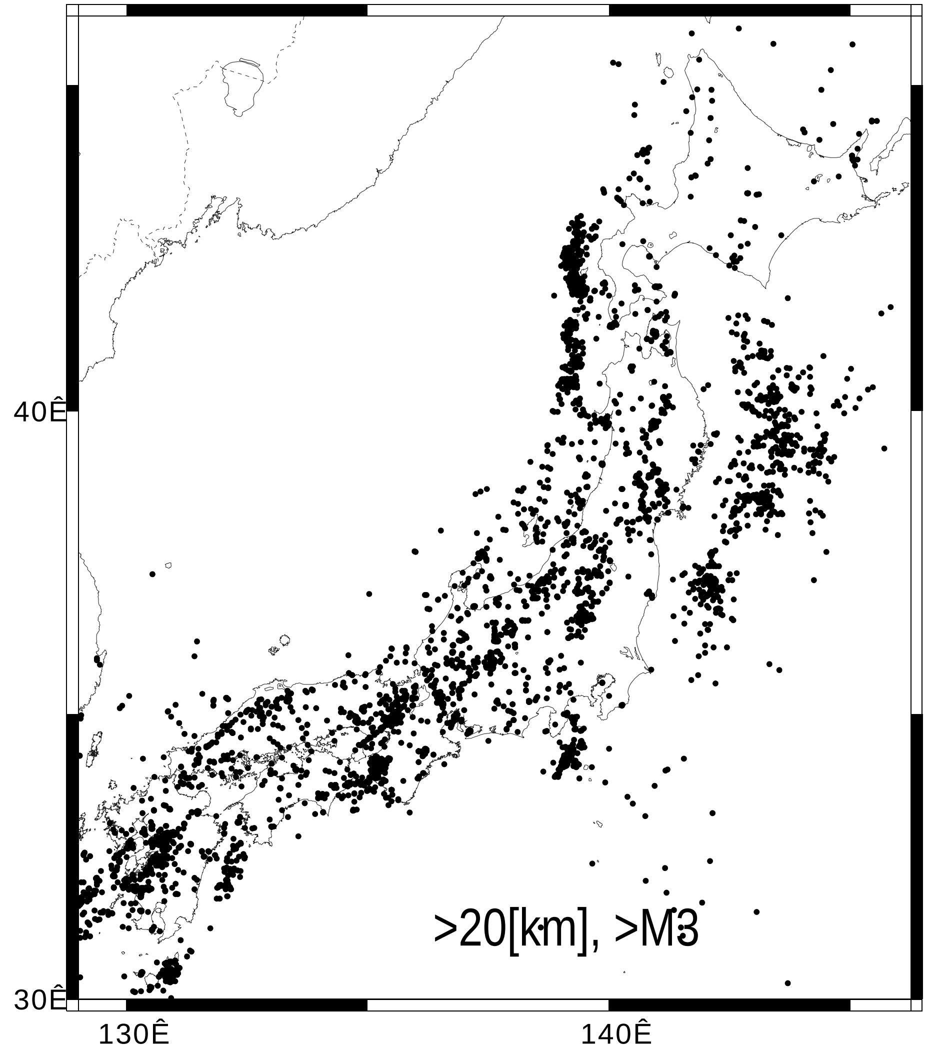


REGIONAL STRESS RATE



COMPARISON WITH SEISMIC EVENTS?





REGIONAL CONSTITUTIVE RELATIONS

find $\kappa(\mathbf{x})$ s.t.

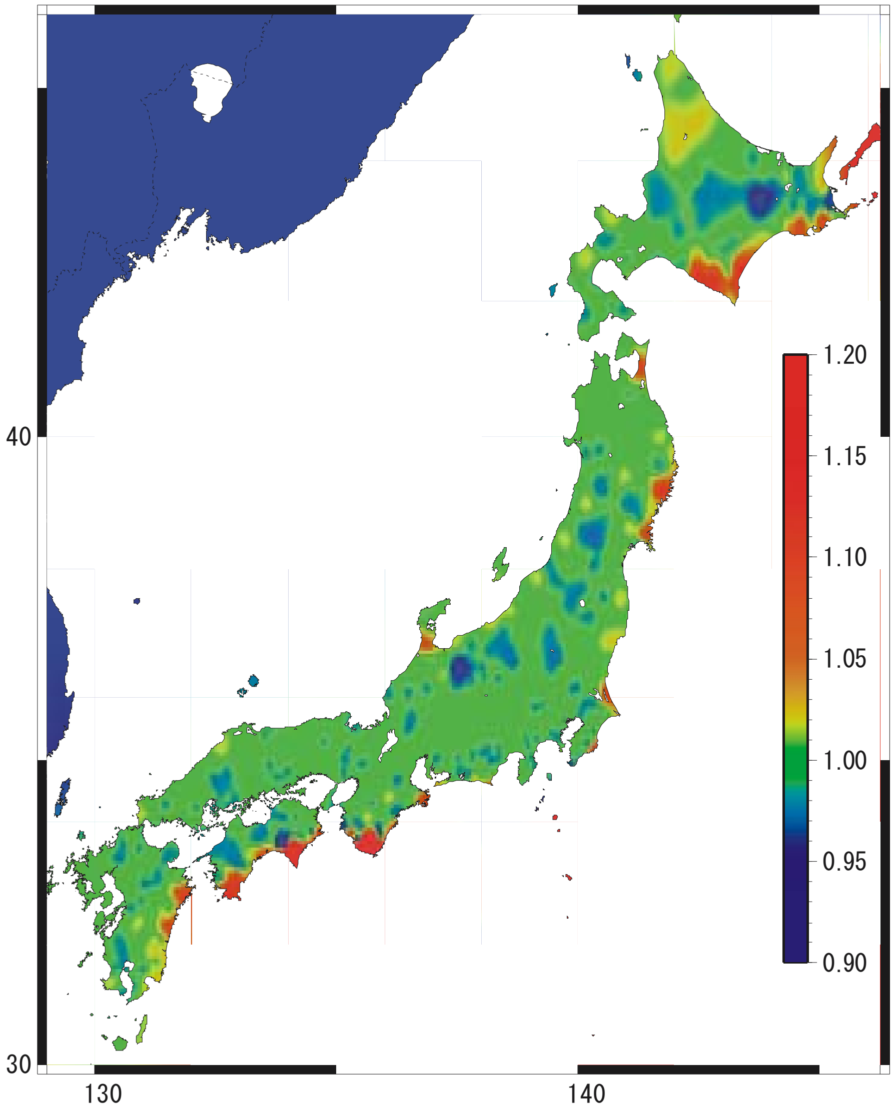
$$\dot{\tau}(\mathbf{x};\kappa) = \kappa(\mathbf{x})\dot{\gamma}(\mathbf{x})$$

$\dot{\tau}, \dot{\gamma}$ maximum shear

κ regional stiffness

κ is originally used to relate σ & ε through $\sigma = \kappa\ \varepsilon$.

not too far from known geological structure



DRAWBACKS OF STRESS INVERSION

- Need to Know One Constitutive Relation
  - bulk stress and bulk strain
  - isotropy assumption

- Need to Know Boundary Traction/Resultant Force
  - assumption of uniform stress
  - fast decrease of non-uniform boundary traction

- Difficulty in Understanding Plane-Stress-State Model

another analysis method needed?

DRAWBACKS OF ELASTICITY INVERSION

DRAWBACKS OF ELASTICITY INVERSION

- Sensitive to Displacement Error
  - need to make fine discretization of target body
  - need to have some strong modes of deformation

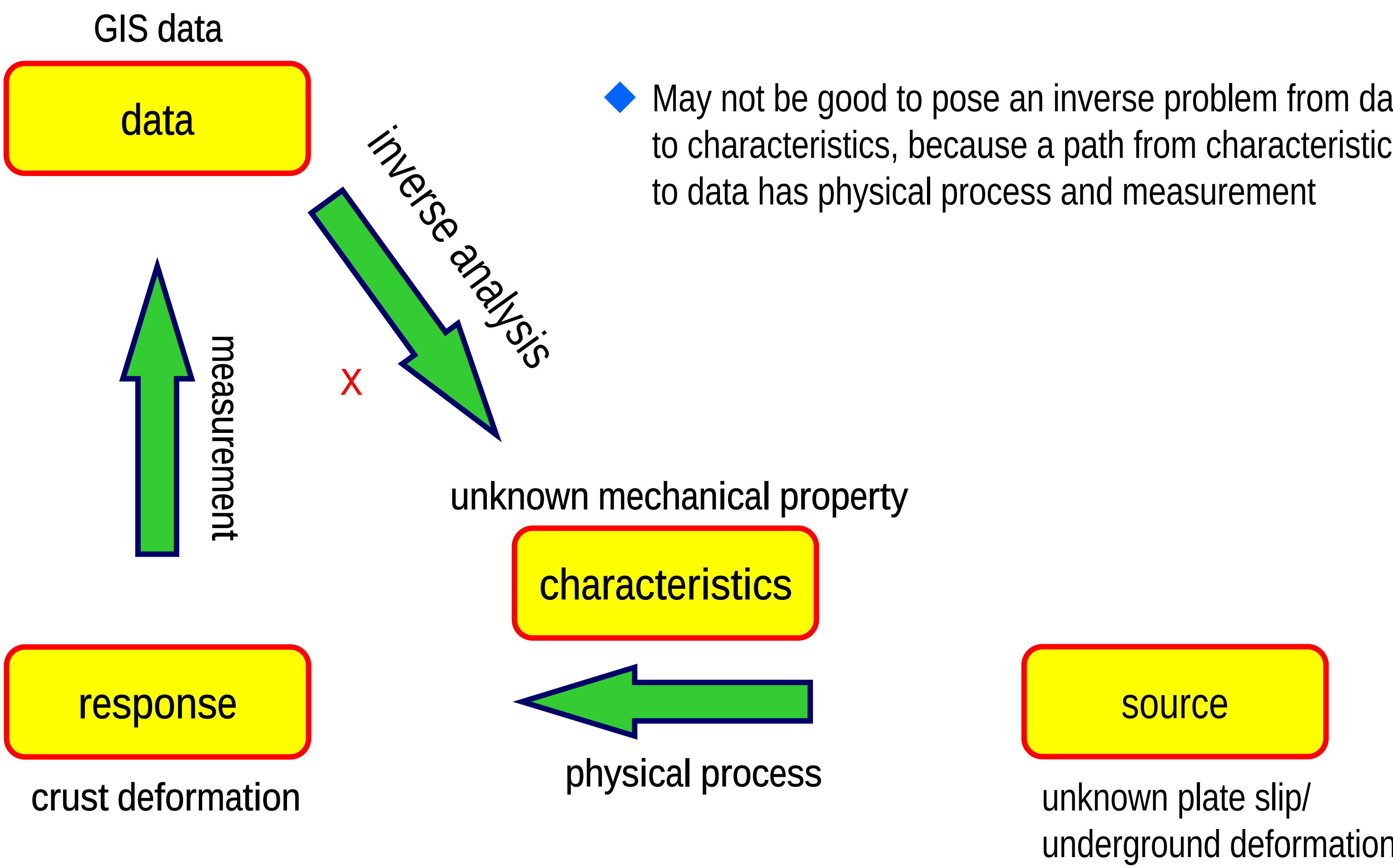
Why is it so?

- no mistakes in mathematics
- poor understanding of physics

PHYSICAL PROCESS AND MEASUREMENT

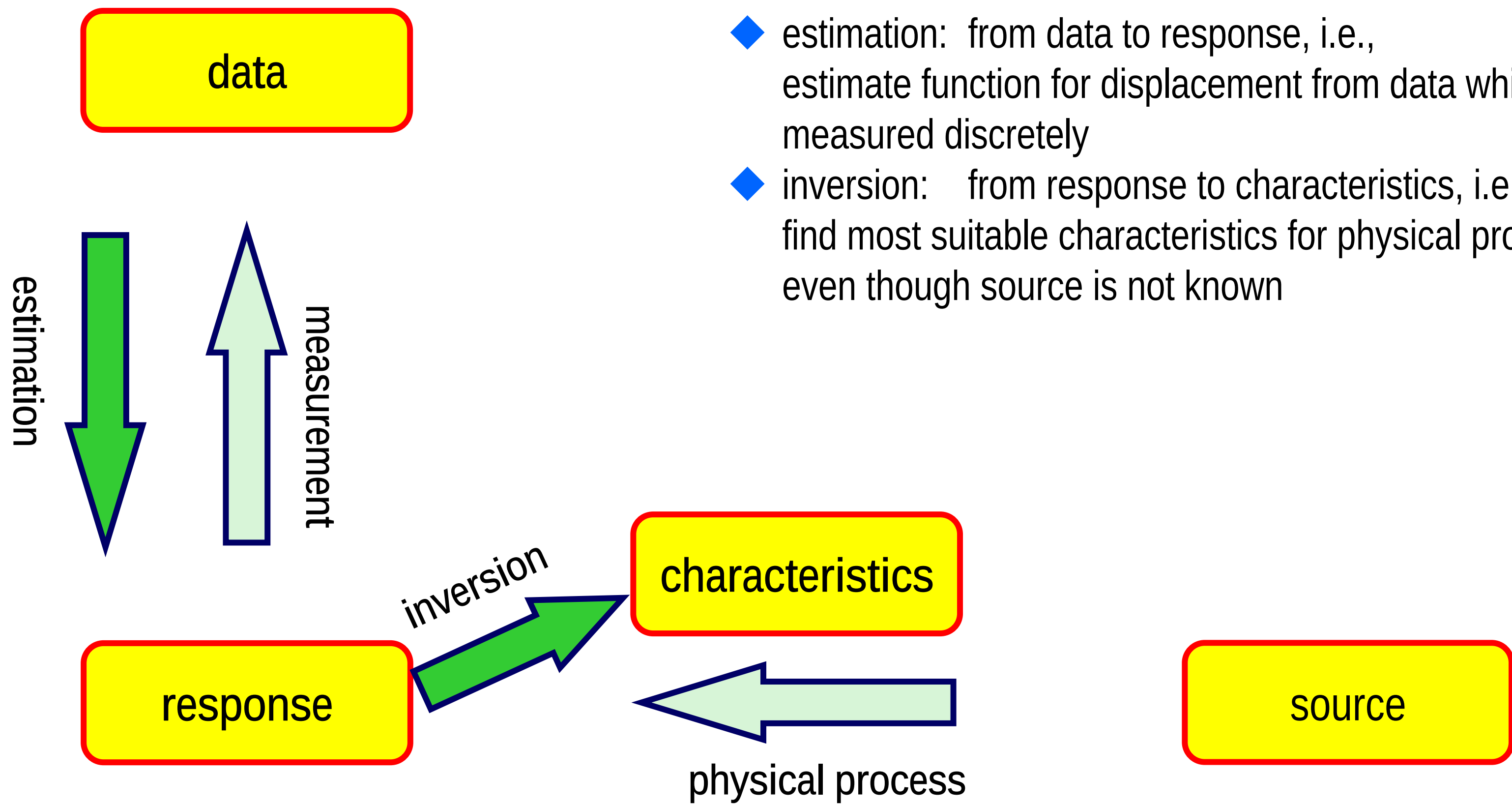
PHYSICAL PROCESS AND MEASUREMENT

- May not be good to pose an inverse problem from data to characteristics, because a path from characteristics to data has physical process and measurement

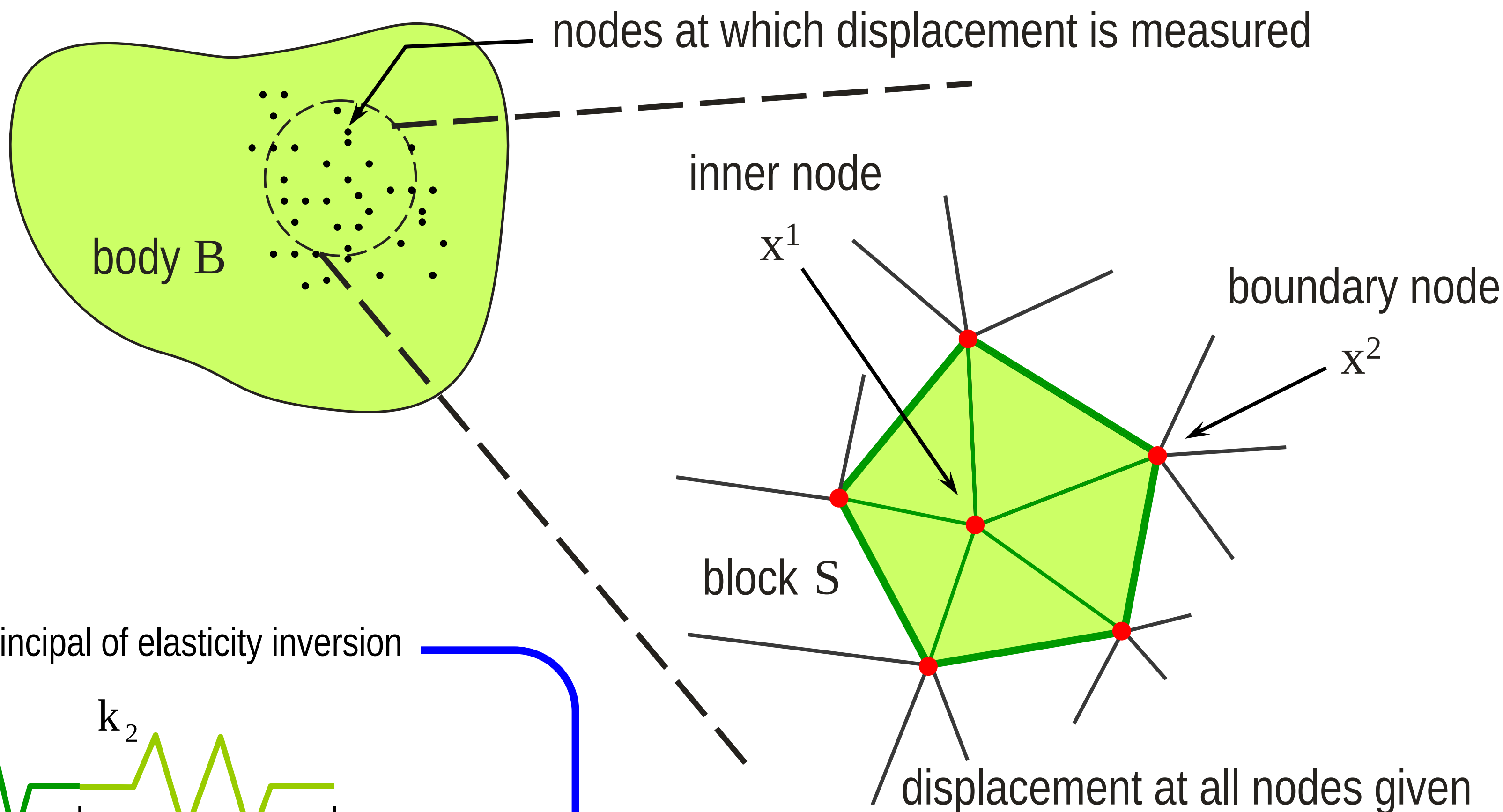


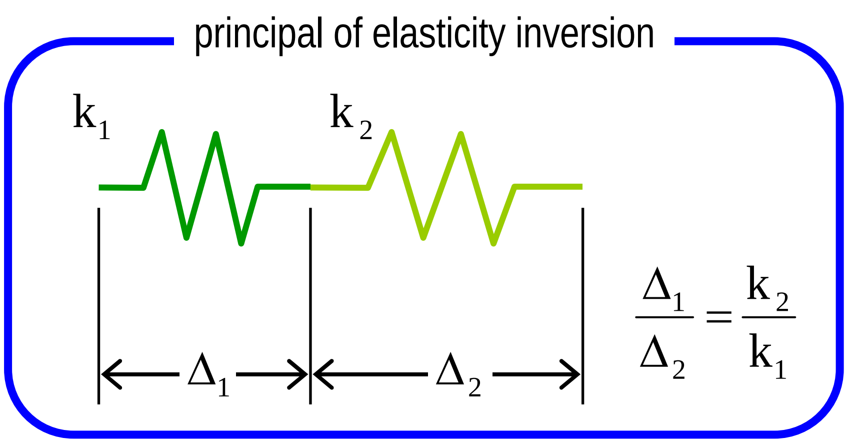
ESTIMATION AND INVERSION

- estimation: from data to response, i.e.,
  estimate function for displacement from data which are measured discretely
- inversion: from response to characteristics, i.e.,
  find most suitable characteristics for physical process even though source is not known



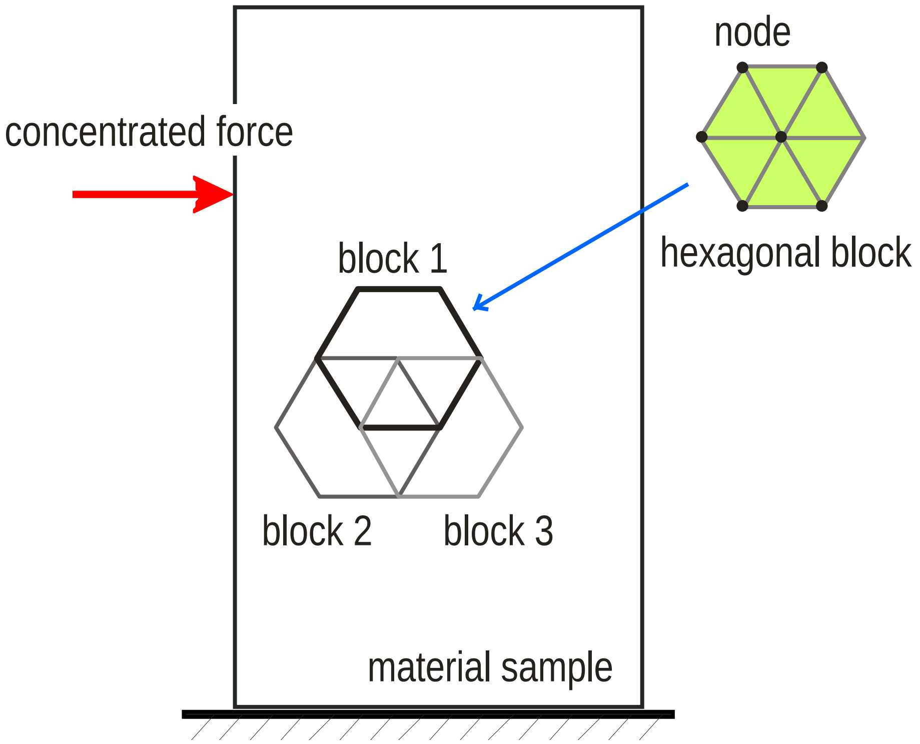
BLOCK IN CONTINUUM





no need to consider interaction of S with outside region

IDENTIFICATION OF DISPLACEMENT MODE



material sample test

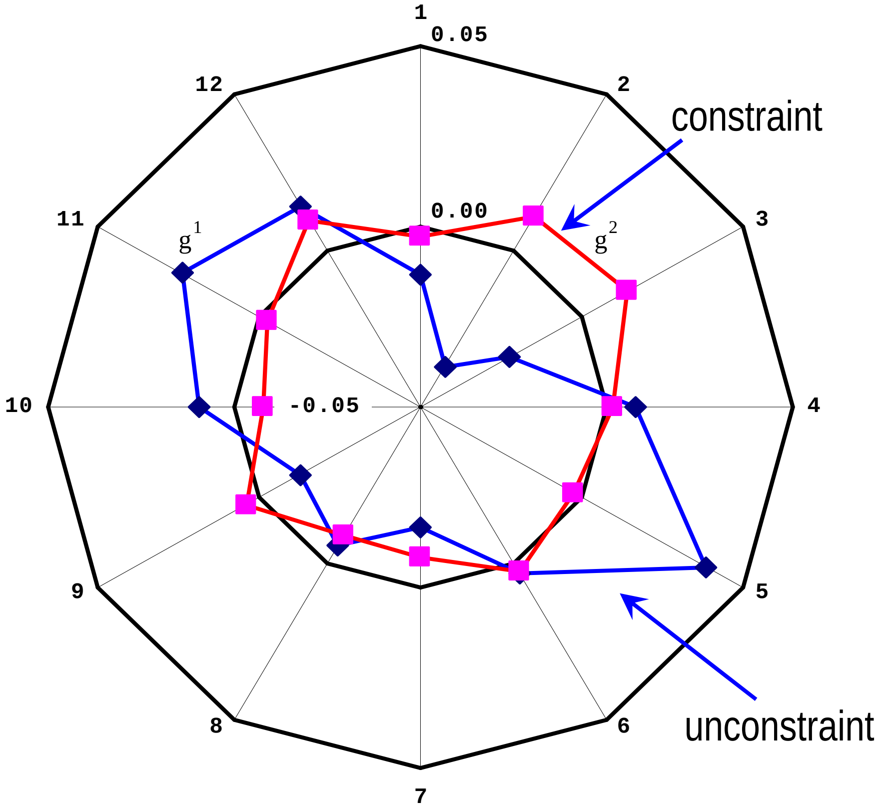
apply several BC's, and measure displacement at nodes of a hexagonal block.

1. identify displacement modes (a characteristic set of nodal displacement)
2. identify local elasticity

IDENTIFICATION OF DISPLACEMENT MODE

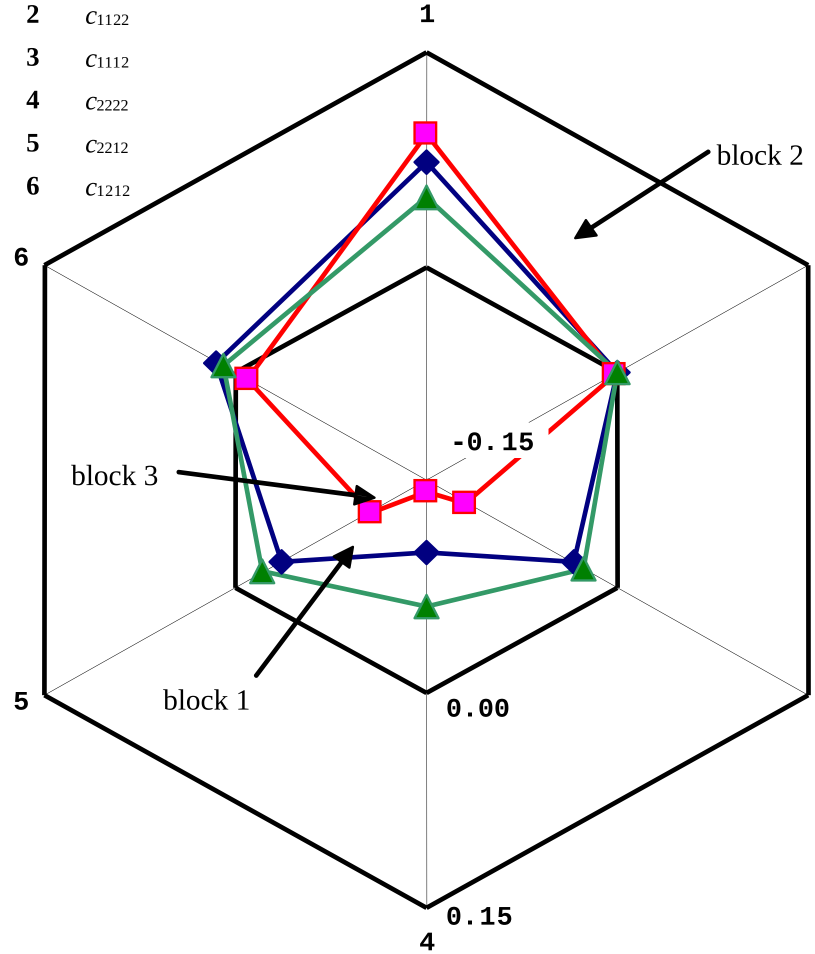
IDENTIFICATION OF DISPLACEMENT MODE

displacement mode



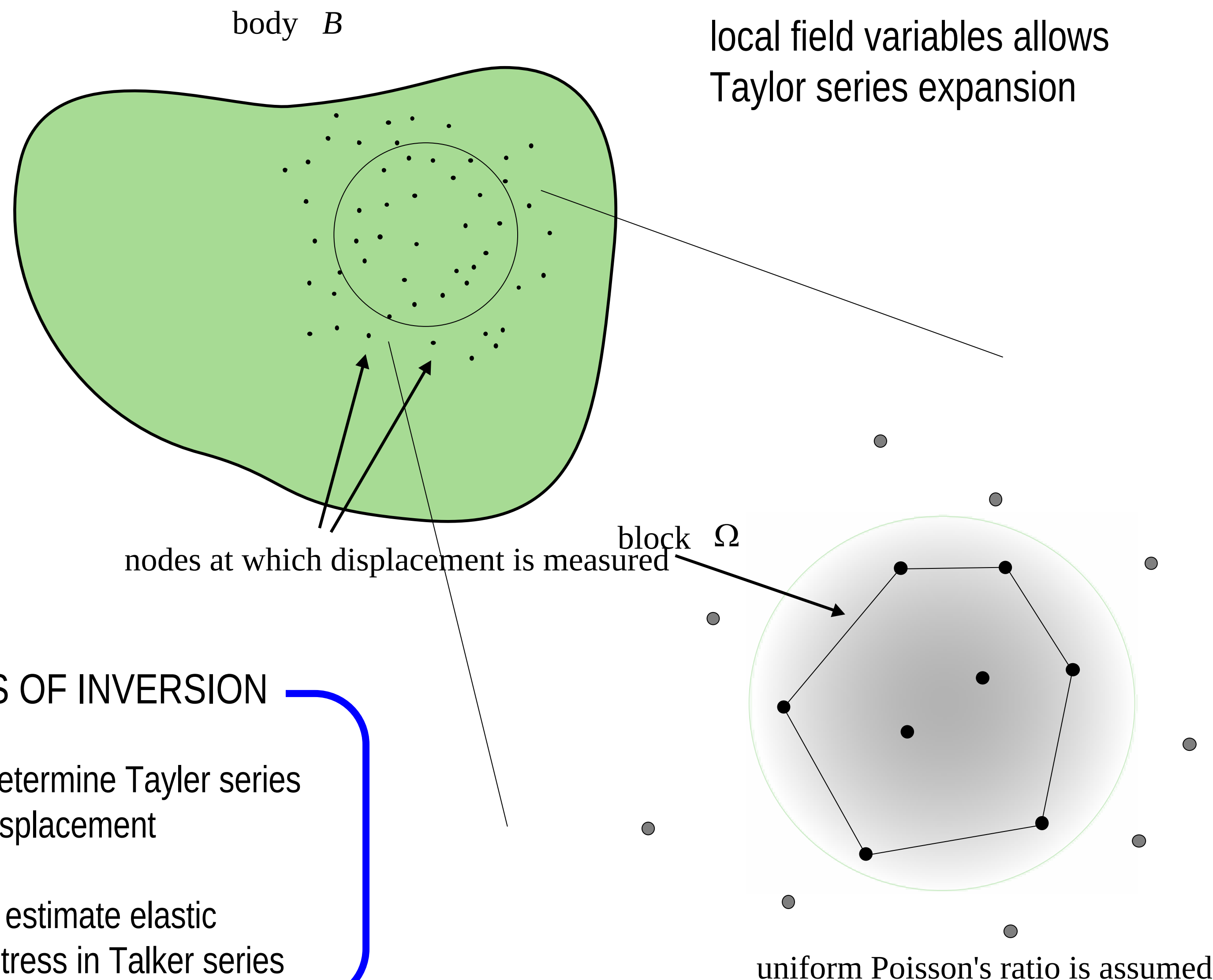
elastic parameters

1	c_{1111}
2	c_{1122}
3	c_{1112}
4	c_{2222}
5	c_{2212}
6	c_{1212}



ELASTICITY INVERSION METHOD

local field variables allows
Taylor series expansion



BASIC PROCEDURES OF INVERSION

1. use displacement data to determine Tayler series expansion coefficients of displacement

2. use equilibrium equation to estimate elastic parameters by expanding stress in Talker series

DETERMINATION OF DISPLACEMENT COEFFICIENT

1. Taylor Expansion: $\{a_{ip}\}$

$$u_i(\mathbf{x}) = \sum_{p=1}^{P} a_{ip} f_p(\mathbf{x}) \qquad \{a_{ip}\} = \{u_i, u_{i,1}, u_{i,2}, \tfrac{1}{2}u_{i,2}, u_{i,11}, \tfrac{1}{2}u_{i,12}, u_{i,22}, \cdots\}$$

$$\{f_p\} = \{1, x_1, x_2, x_1^2, x_1 x_2, x_2^2, \cdots\}$$

2. Displacement Data: $\{\overline{u}_i^n\}$

$$\overline{u}_i^n = \sum_{p=1}^{P} f_{pn} a_{ip}$$

3. Solution of Matrix Equation $f_{pn} = f_p(\mathbf{x}^n)$

$$a_{ip} = \boxed{\sum_{\alpha=1}^{A} \frac{1}{\lambda^\alpha} \left(\sum_n u_i^n \varphi_n^\alpha \right) \psi_p^\alpha} + \underline{\sum_{\beta=1}^{P-A} b_i^\beta \psi_p^\beta} \qquad \left\{\lambda^\alpha, \varphi_n^\alpha, \psi_p^\alpha\right\} : \text{SVD of } f_{pn}$$

fully determined undetermined

ESTIMATION OF POISSON RATIO ν

1. Elasticity Tensor Expressed in Terms of Poisson Ratio

$$c_{ijkl} = c^0_{ijkl} + \nu c^1_{ijkl}$$

2. Equation of Equilibrium and Its Taylor Expansion

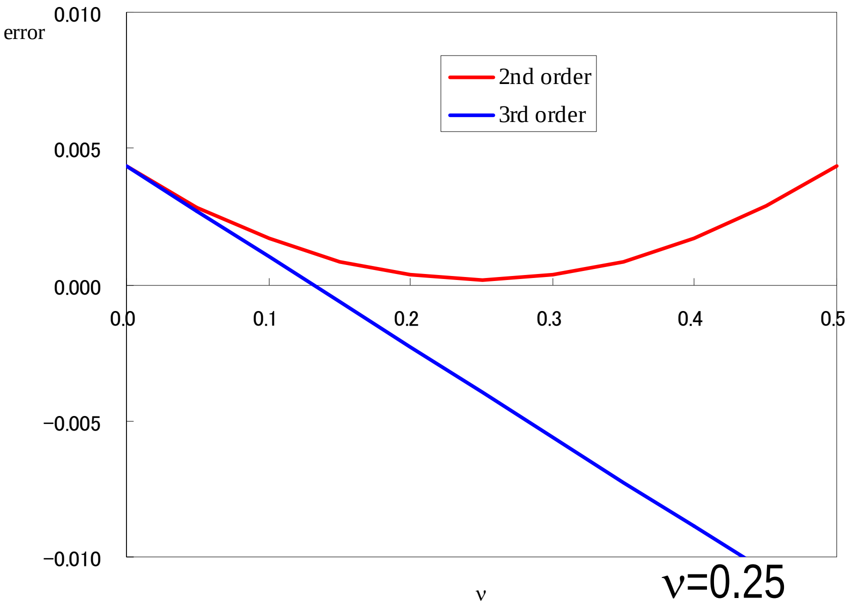
$$b_i(\mathbf{x}) = (c_{ijkl} u_{k,l}(\mathbf{x}))_{,j} = \sum_p b_{ip} f_p(\mathbf{x}) = 0$$

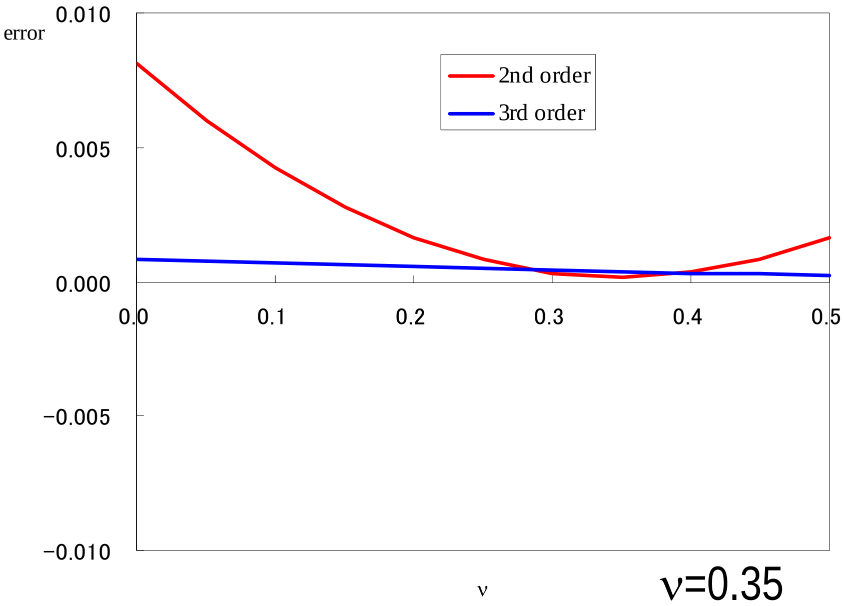
3. Coefficient of Expansion: $b_{ip}=0$ for 0^{th} Order (p=1)

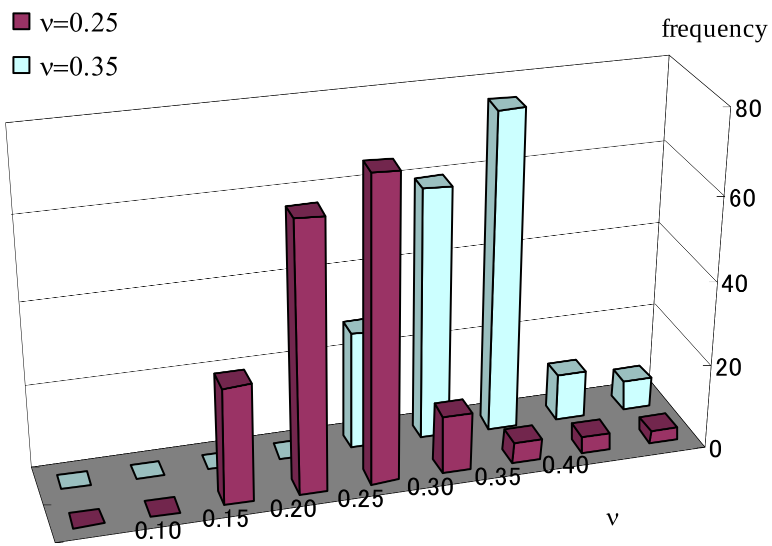
$$\begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \left(\begin{bmatrix} 2 & 0 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & 0 & 2 \end{bmatrix} + \nu \begin{bmatrix} 0 & 0 & 0 & \frac{1}{2} & -1 & 0 \\ 0 & -1 & \frac{1}{2} & 0 & 0 & 0 \end{bmatrix} \right) \begin{bmatrix} a_{14} \\ a_{24} \\ \vdots \\ a_{26} \end{bmatrix}$$

linear equation of ν is derived

NUMERICAL SIMULATION (1)







measurement	200
expansion of displacement	3rd order
expansion of equilibrium	0th or 1st order

NUMERICAL SIMULATION (2)

1. Measurement Error: $\{e_i^n\}$

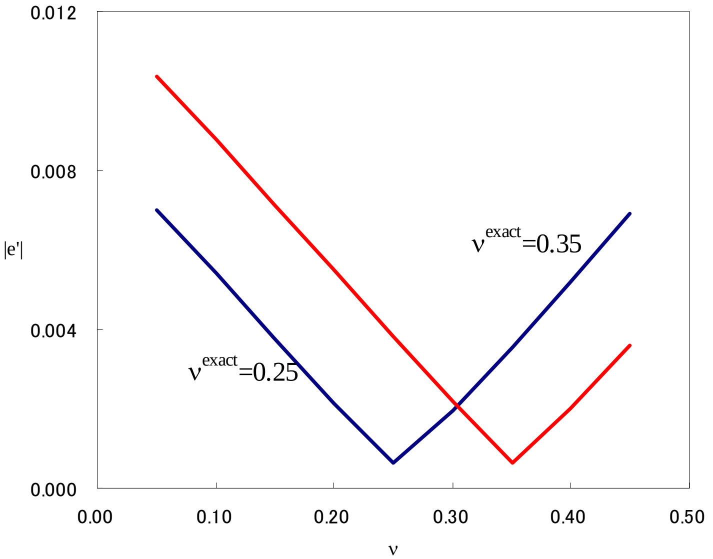
$$\bar{u}_i^n = \sum_{p=1}^{P} f_{pn} a_{ip} + e_i^n$$

2. Find ν such that

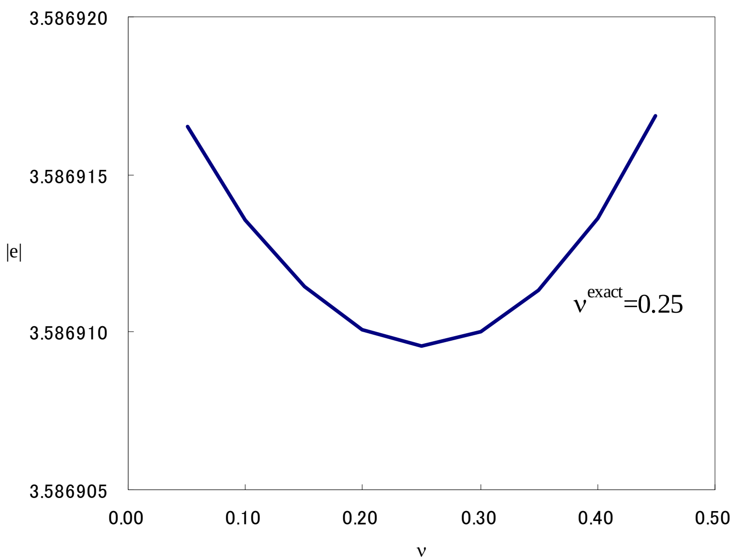
$$\begin{array}{ll} \text{minimize} & |e|^2 = \sum (e_i^n)^2 \\ \text{subjected to} & b_{ip}(\nu) = 0 \end{array}$$

measurement	200
expansion of displacement	3rd order
expansion of equilibrium	0th or 1st order

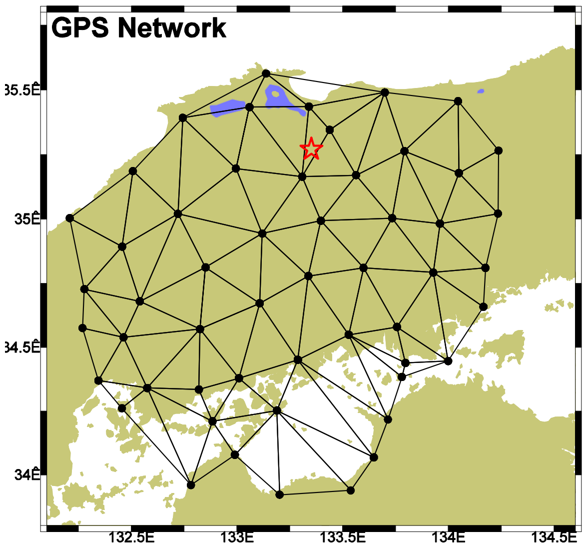
0th



1st

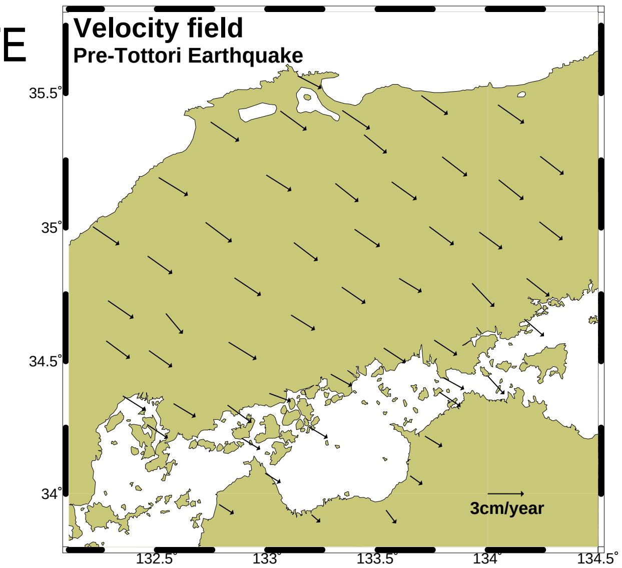


APPLICATION OF LOCAL GPS DATA

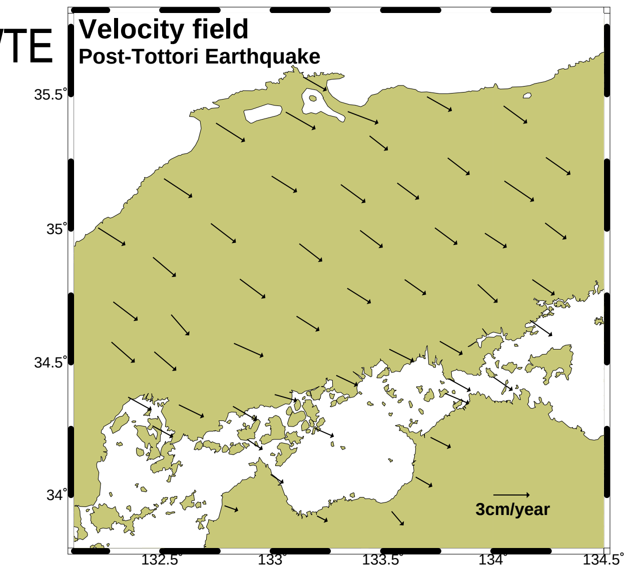


GPS array

pre-WTE



post-WTE



2000 Western Tottori Earthquake (M_{JMA}=7.3) examine change in deformation and elasticity before and after this earthquake

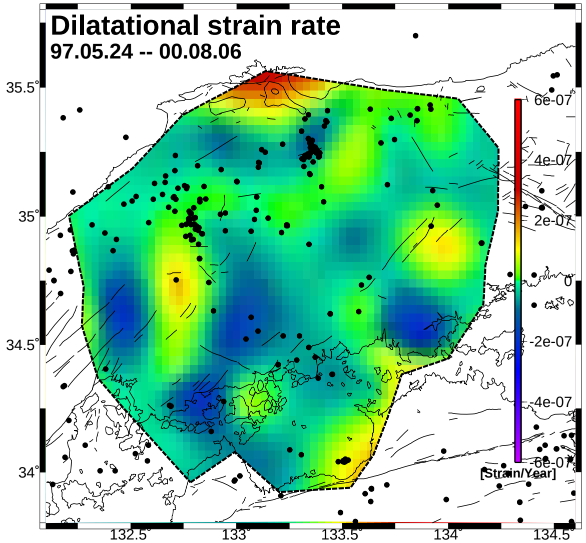
- 53 GPS observation points
- 82 triangular elements
- GPS data obtained from 1997 to 2002
- annual and biannual sinusoidal variations excluded

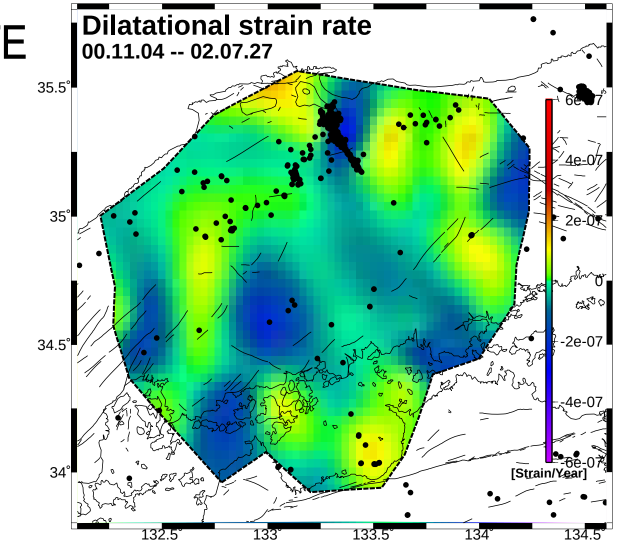
STRAIN RATE

pre-WTE

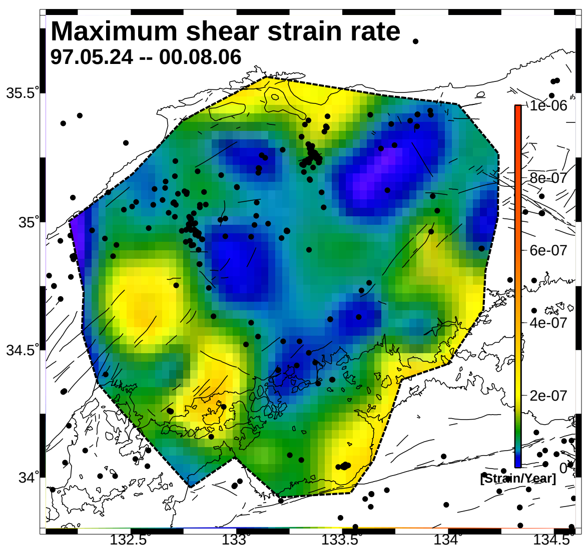
post-WTE

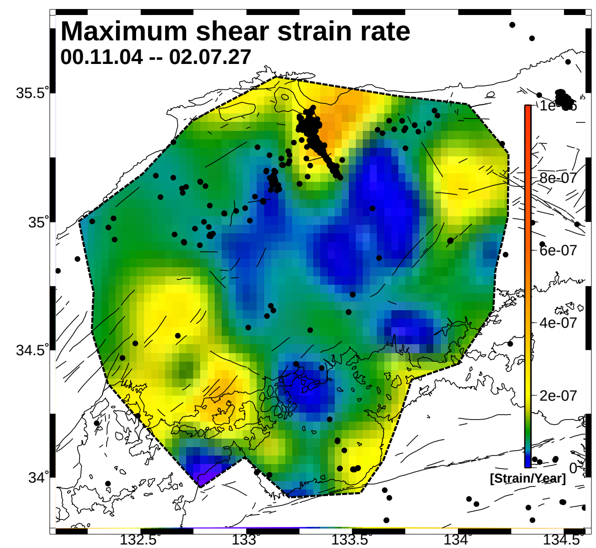
dilatational





shear



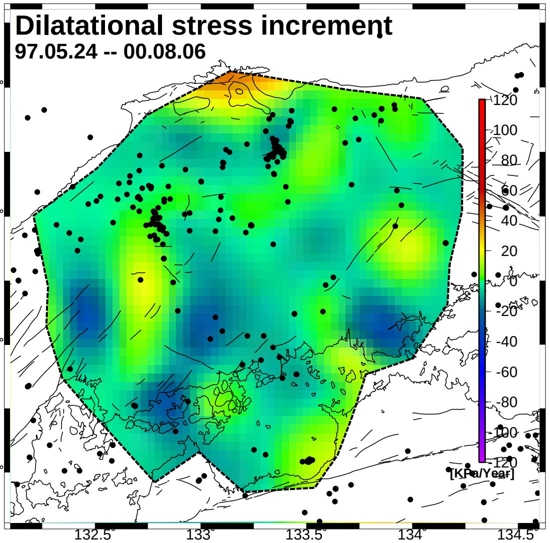


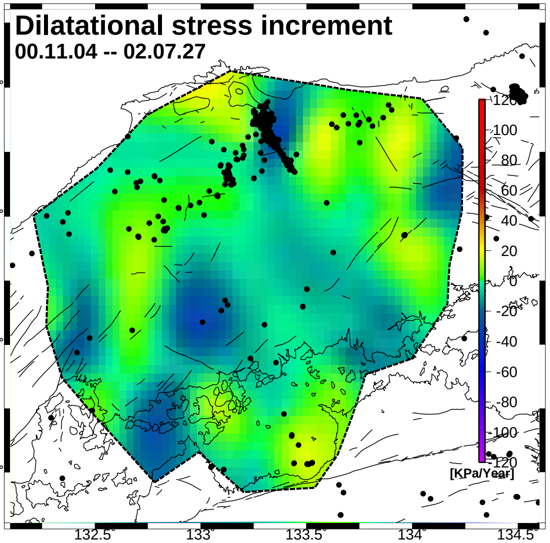
STRESS RATE

pre-WTE

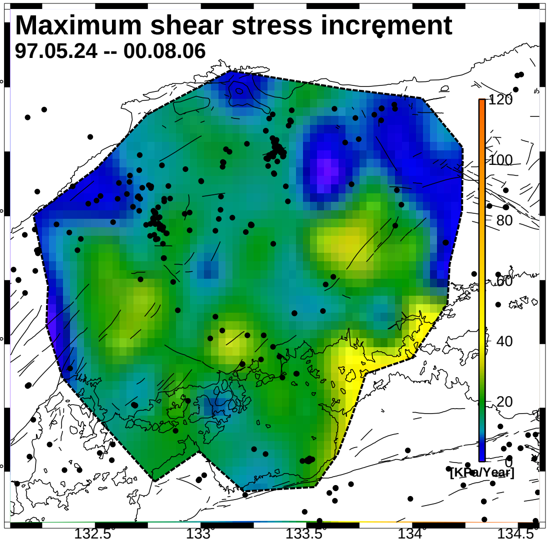
post-WTE

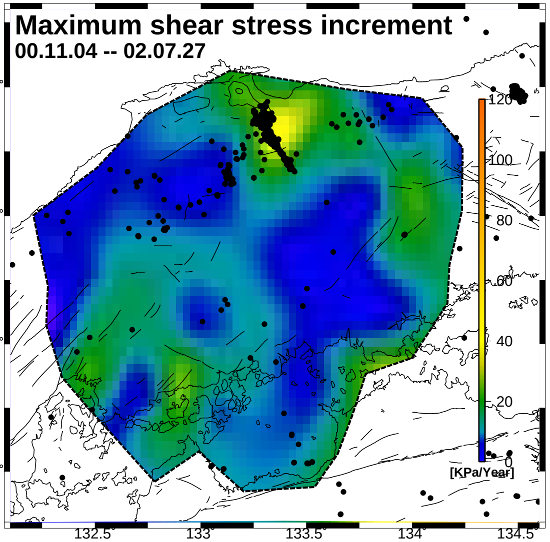
dilatational





shear





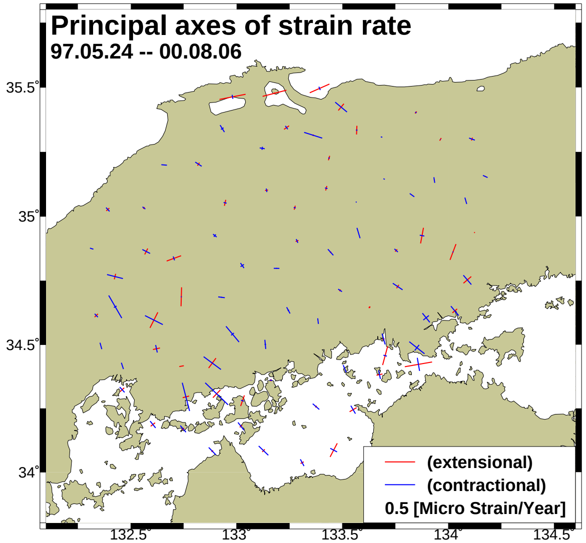
PRINICPLE AXIS

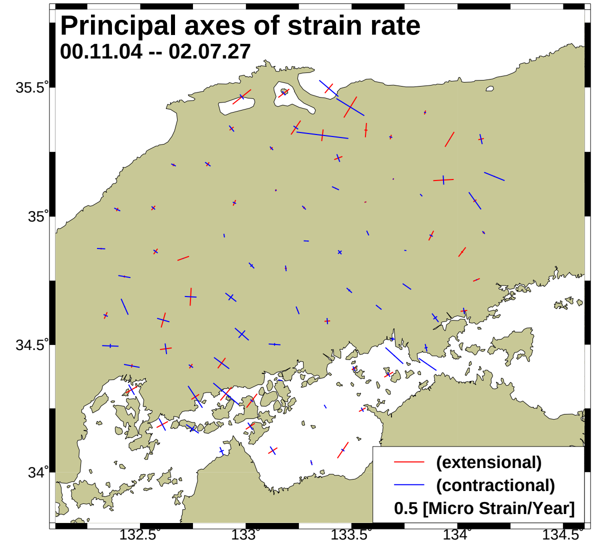
PRINICPLE AXIS

pre-WTE

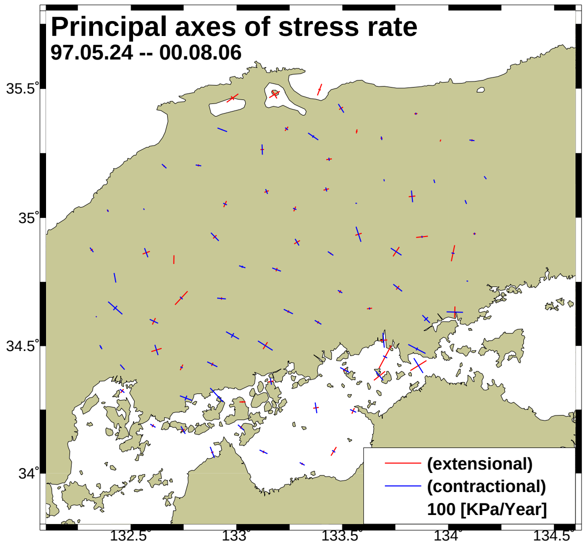
post-WTE

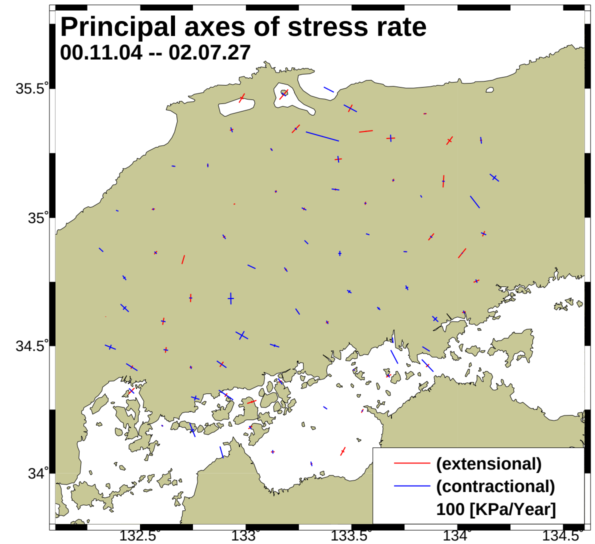
strain





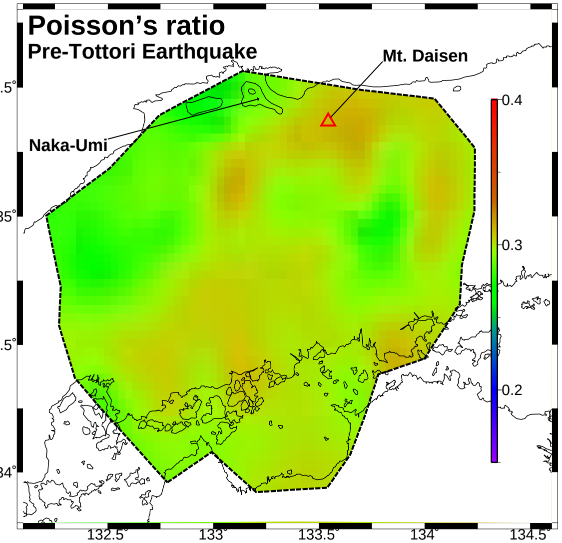
stress



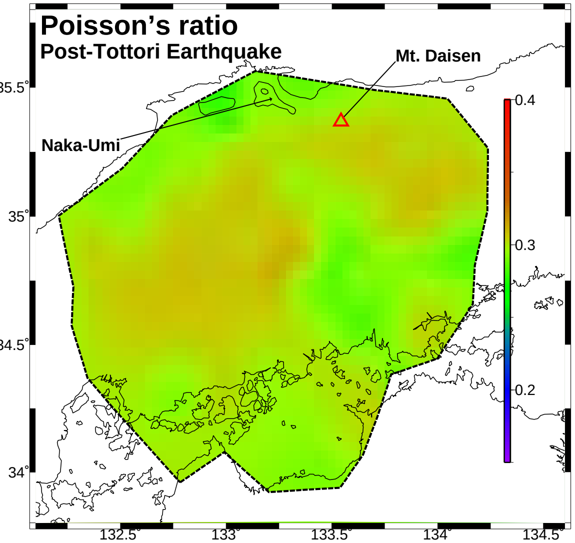


POISSON RATIO

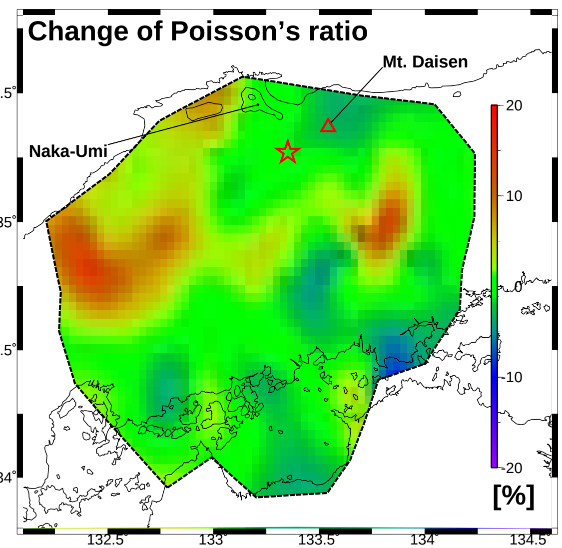
pre-WTE



post-WTE



difference



ν has been reduced near the source faults. The comparison with other analyses are being made.

CONCLUDING REMARKS

CONCLUDING REMARKS

- Two inverse analysis methods
  - stress inversion
    find Airy's stress function by solving Poisson's equation
  - elasticity inversion
    find elastic parameters by estimating displacement expansion coefficients

- Development of new inversion is needed for geophysics where experiments cannot be made.

- Application
  - small material samples used for bio-mechanics
  - geomaterials
  - new image analysis with higher spatial resolution