

# 火碎流の発生条件に対する 火口形状の影響

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# Column collapse condition

**Positively or negatively buoyant**



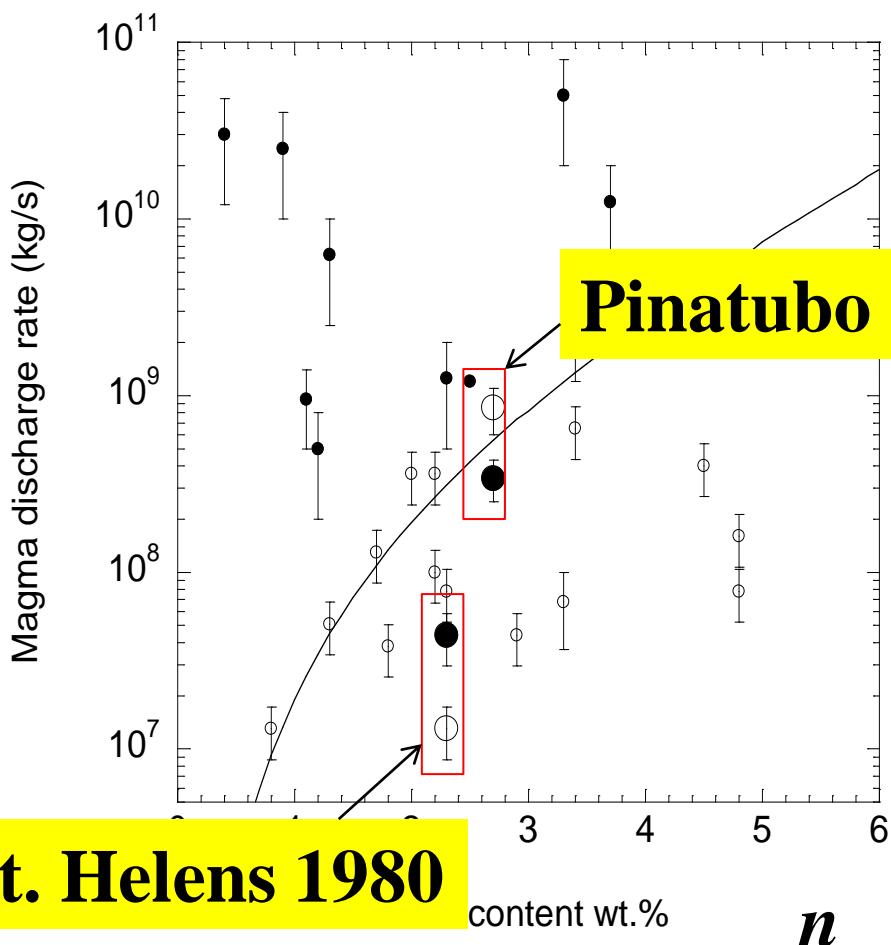
**Eruption column**

Example from the 1980 eruption  
at Mt. St. Helens (USGS)



**Pyroclastic flow**

# Previous model prediction based on 1-D steady eruption column model



Column collapse condition predicted by Carazzo et al. (2008)

with the assumptions

(1) A model for entrainment by Kaminski et al. (2005)

$$(2) v_a \approx 138\sqrt{n}$$

$v_a$ : velocity at the atmospheric  $P$  just above the vent

## Observations

- Pyroclastic flow
- Buoyant plume

# Problems to solve

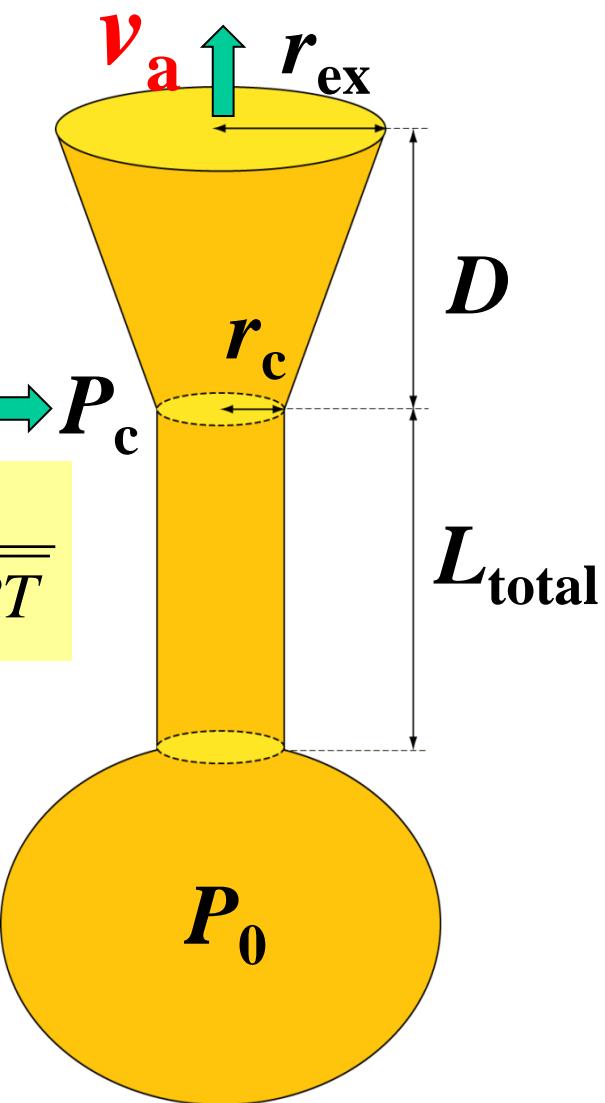
Crater

choking  $\rightarrow P_c$

$$q = \frac{P_c}{\sqrt{n_f RT}}$$

Conduit

Chamber



## Column dynamics

Bursik and Woods (1991)  
Kaminski and Jaupart (2001)  
Carazzo et al. (2008)

## Flow inside crater

Woods and Bower (1995)  
Ogden et al. (2008) etc.

## Flow in conduit

Wilson et al. (1980)  
CONFLOW (2000)  
Koyaguchi (2005) etc.

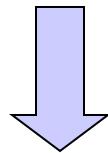
# Entrainment hypothesis

[Morton et al., 1956]

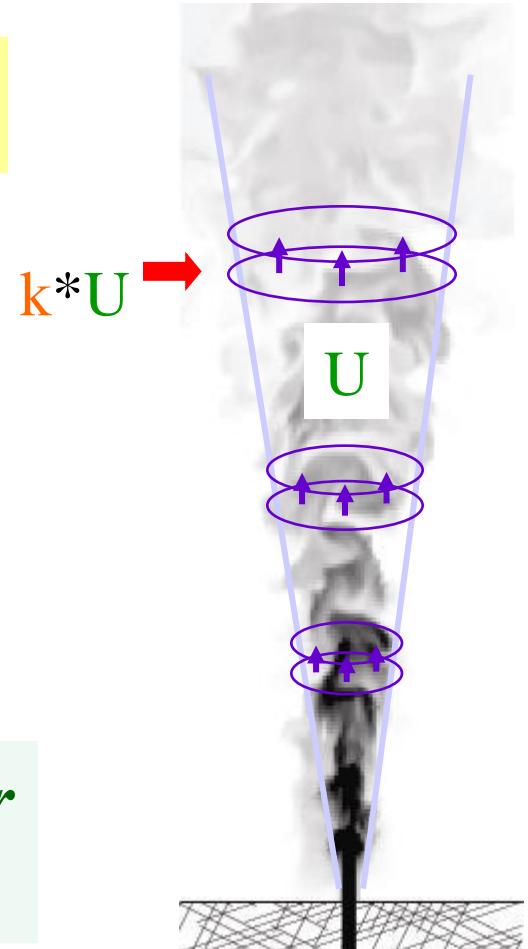
Inflow velocity =  $k$  \* mean velocity

$k=0.09$  (e.g., Woods, 1988)

based on self-similarity of ideal steady jets or plumes

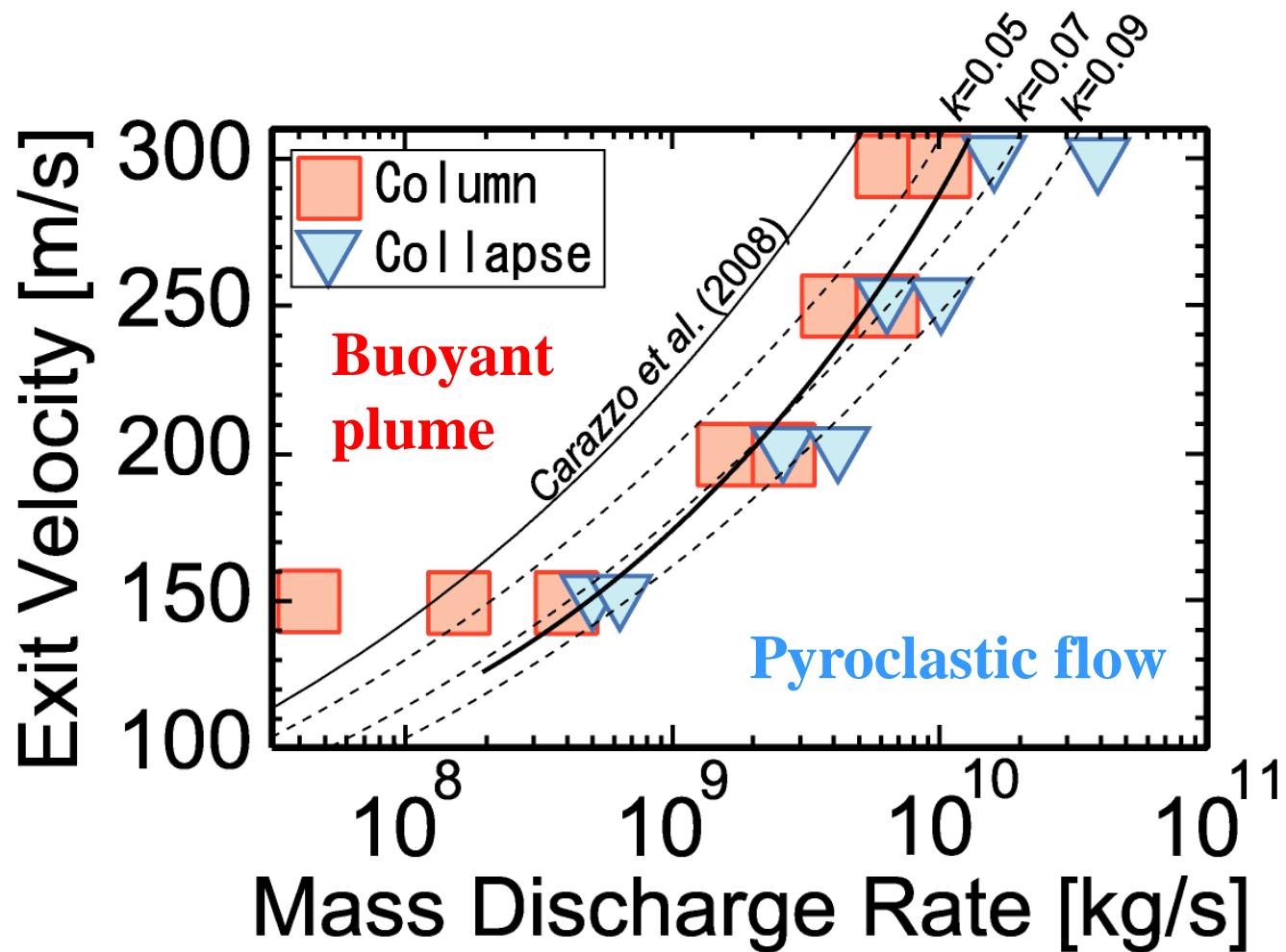


*Is this applicable to non-self-similar eruption clouds?*



# Comparison between the 1-D and 3D models

[Suzuki et al., 2005]



# Problems to solve

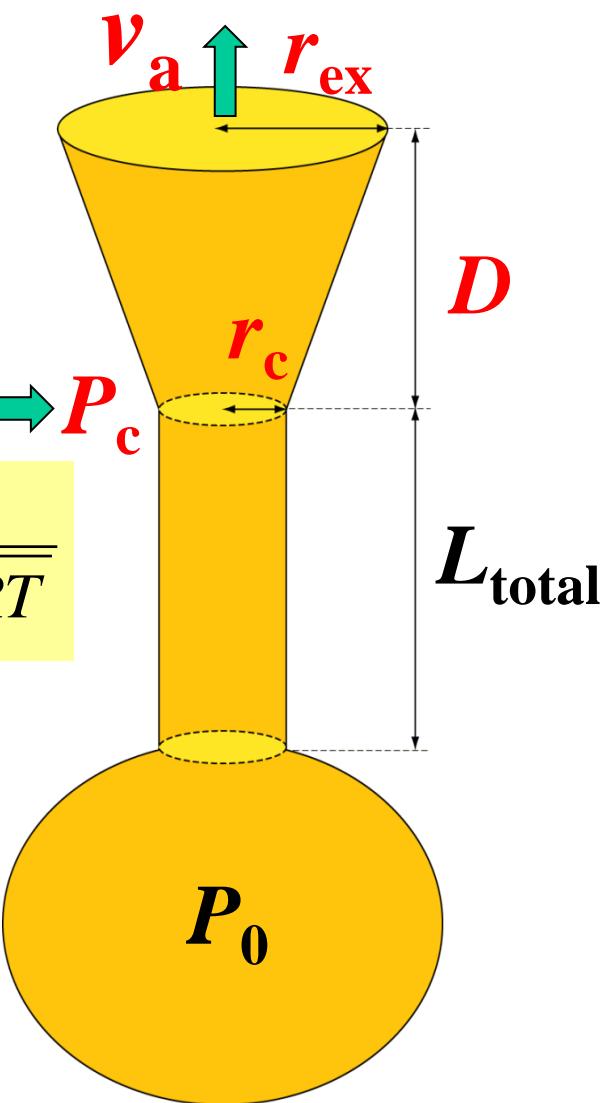
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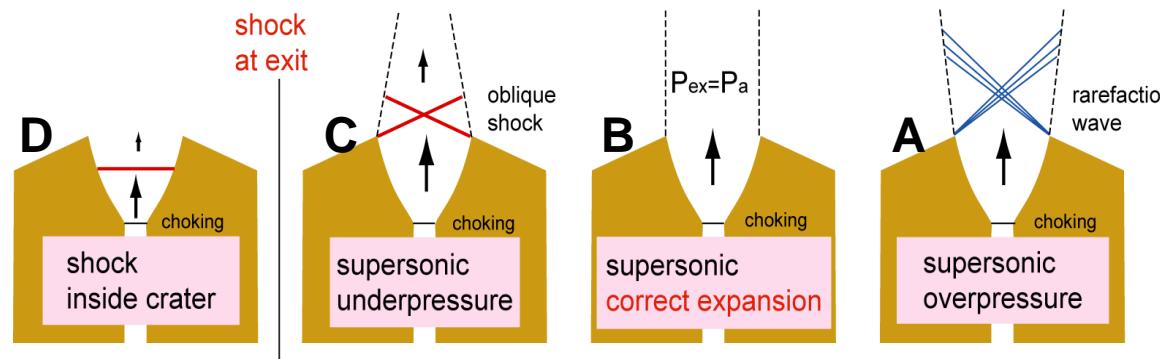
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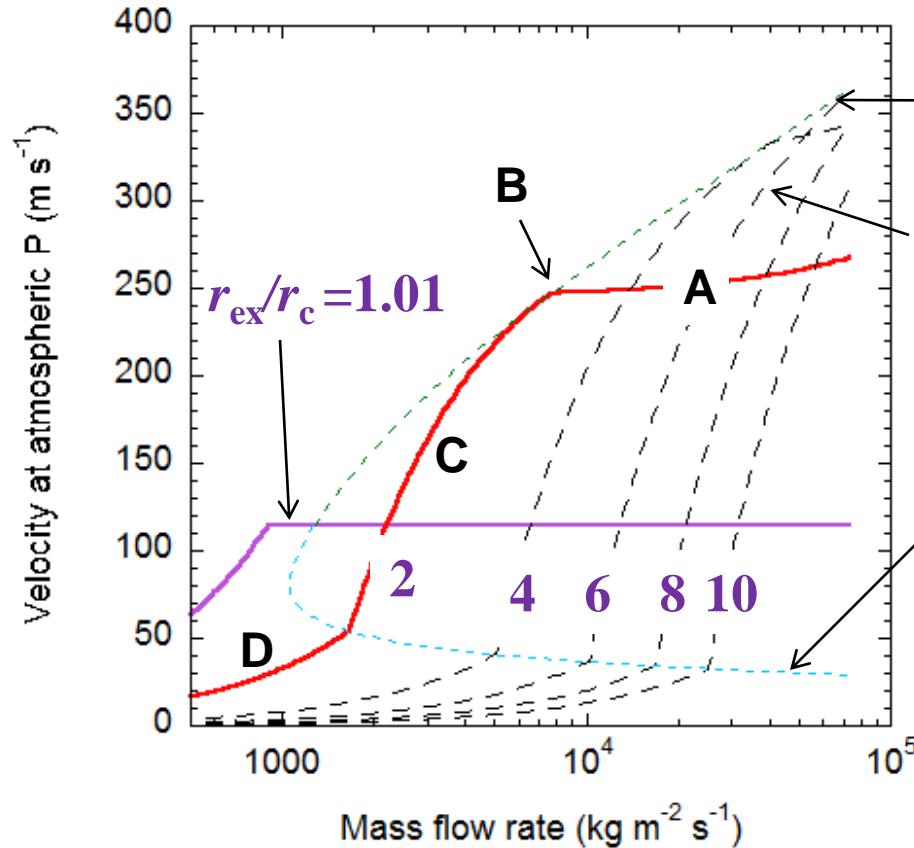
# Flow inside crater

large  
 $r_{\text{ex}}/r_c$   
small  
 $MFR$



small  
 $r_{\text{ex}}/r_c$   
large  
 $MFR$

“ $v_a$ ”



Correct expansion

Velocity change for given  
 $r_{\text{ex}}/r_c (=2)$

Shock at exit

Mass flow rate ( $MFR$ )

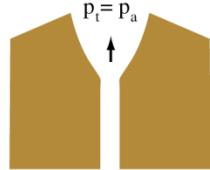
$$q \equiv \rho_c v_c = \frac{P_c}{\sqrt{n_f RT}}$$

# Boundaries of different flow regimes

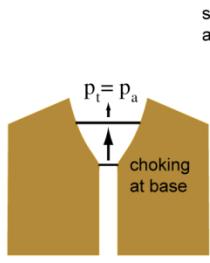
$$\text{Eq. (2)} \quad \frac{q}{q^*} = \left( \frac{r_{ex}}{r_c} \right)^2$$

*Subsonic eruption*

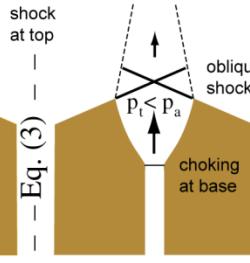
(a) subsonic throughout conduit and crater



(c) shock inside crater

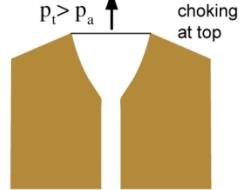


(d) overexpanded

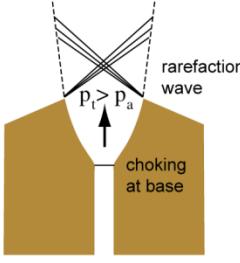


*Sonic or supersonic eruption*

(b) free decompression



(f) underexpanded



Eq. (2)

Eq. (1)

Eq. (4)

$$\text{Eq. (1)}$$

$$\frac{D}{l_{sc}} = \ln \left( \frac{r_{ex}}{r_c} \right)^2 + \frac{2V_L}{V_{Ga}} \frac{q}{q^*} \left\{ 1 - \left( \frac{r_c}{r_{ex}} \right)^2 \right\}$$

$$+ \frac{1}{2} \left( \frac{V_L}{V_{Ga}} \frac{q}{q^*} \right)^2 \left\{ 1 - \left( \frac{r_c}{r_{ex}} \right)^4 \right\}$$

Eq. (3)

$$\frac{1}{2} \left( \frac{q}{q^*} \right)^2 \left[ \left( \frac{r_{ex}}{r_c} \right)^{-4} \left\{ \frac{V_L}{V_{Ga}} + \left( \frac{q}{q^*} \right)^2 \left( \frac{r_{ex}}{r_c} \right)^4 \right\}^2 - \left( \frac{V_L}{V_{Ga}} + \frac{q}{q^*} \right)^2 \right]$$

$$+ \frac{V_L}{V_{Ga}} \frac{q}{q^*} \left\{ \left( \frac{r_{ex}}{r_c} \right)^{-4} \frac{q}{q^*} - 1 \right\} + \ln \left\{ \left( \frac{r_{ex}}{r_c} \right)^{-4} \frac{q}{q^*} \right\} + \frac{D}{l_{sc}} = 0$$

Eq. (4)

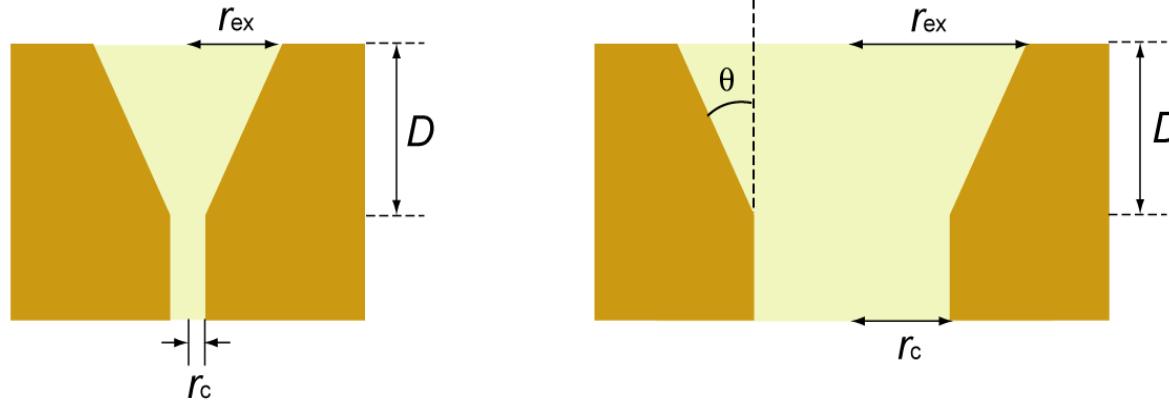
$$\frac{1}{2} \left( \frac{q}{q^*} \right)^2 \left\{ \left( \frac{r_{ex}}{r_c} \right)^{-4} \left( \frac{V_L}{V_{Ga}} + 1 \right)^2 - \left( \frac{V_L}{V_{Ga}} + \frac{q}{q^*} \right)^2 \right\}$$

$$+ \frac{V_L}{V_{Ga}} \left( 1 - \frac{q}{q^*} \right) - \ln \left( \frac{q}{q^*} \right) + \frac{D}{l_{sc}} = 0$$

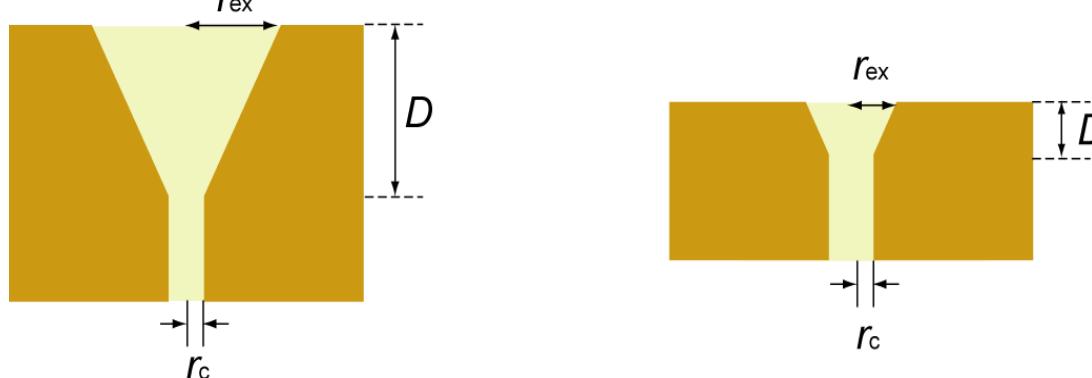
# Effect of the crater shape on $r_{\text{ex}}/r_c$ ratio

$$\frac{r_{\text{ex}}}{r_c} \approx 1 + \frac{D \tan \theta}{r_c}$$

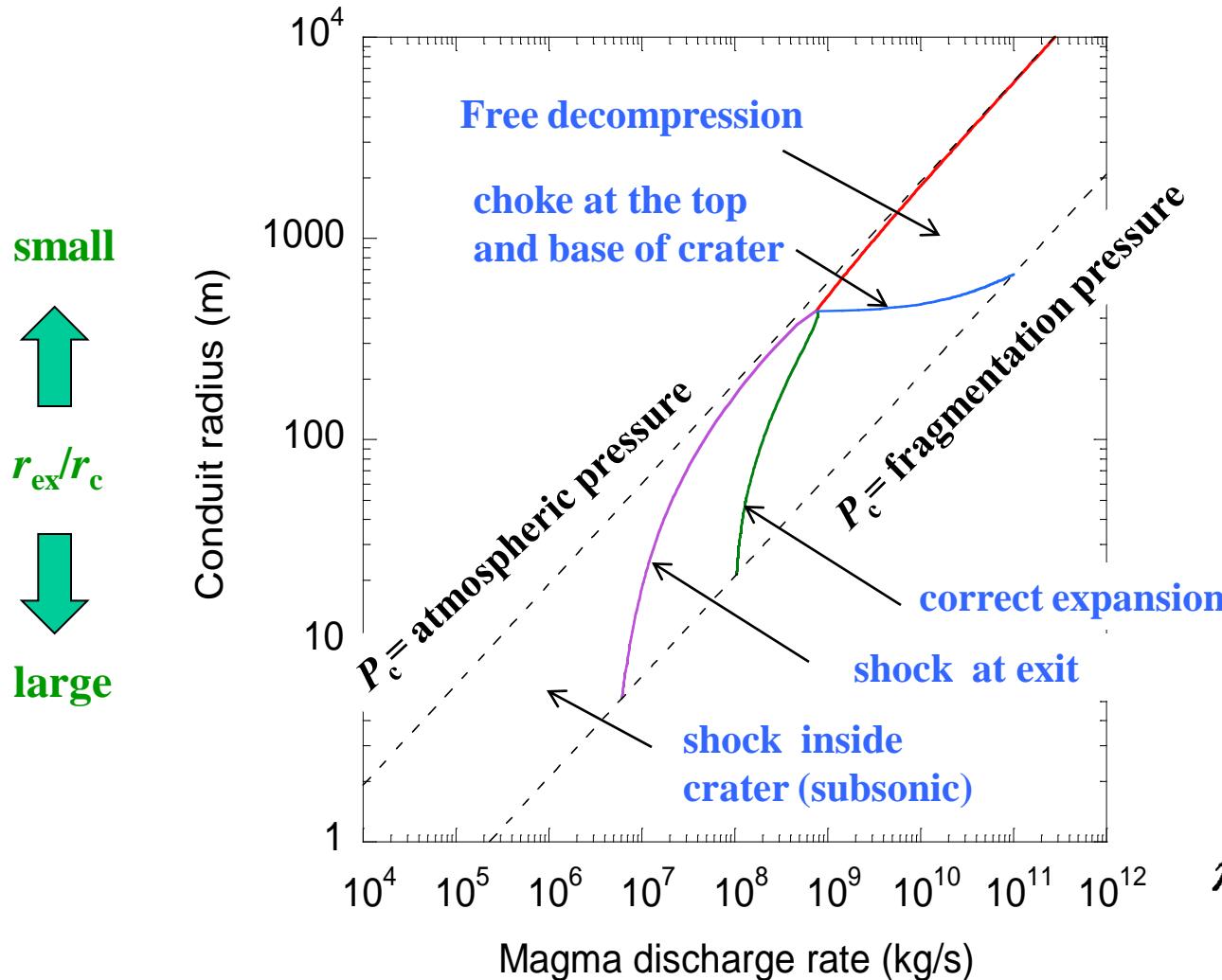
$r_{\text{ex}}/r_c$  decreases as  $r_c$  increases for given  $\theta$ .



$r_{\text{ex}}/r_c$  decreases as  $D$  decreases for given  $\theta$ .



# Flow regimes on “magma discharge rate vs $r_c$ diagram”



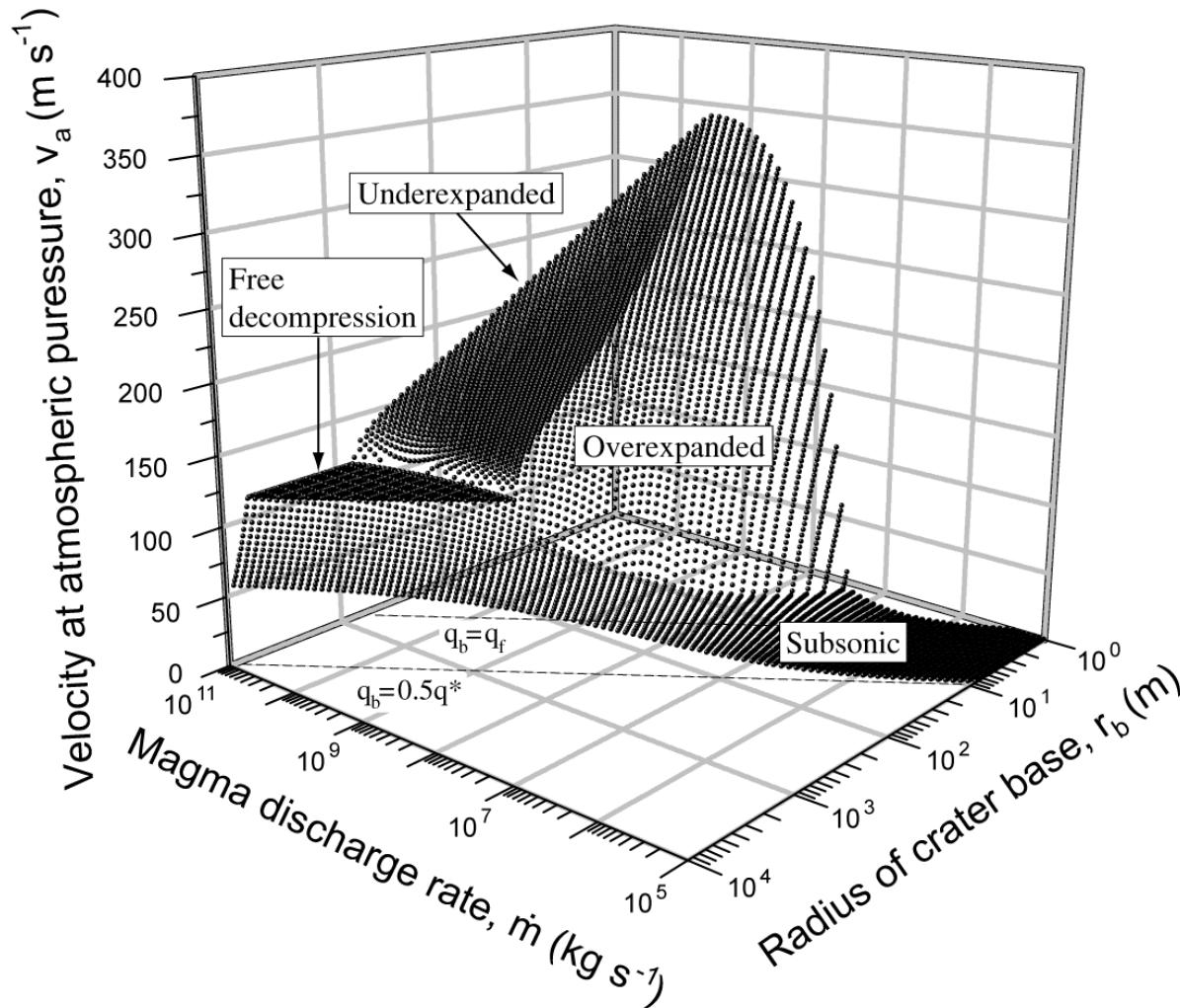
**Result for**  
 $n_0=0.04$   
 $D=500 \text{ m}$   
 $\theta=10^\circ$

$$\pi r_c^2 q = \frac{\pi r_c^2 P_c}{\sqrt{n_f RT}}$$

# Velocity at the atmospheric $P$ (" $v_a$ ") on "magma discharge rate vs $r_c$ diagram"

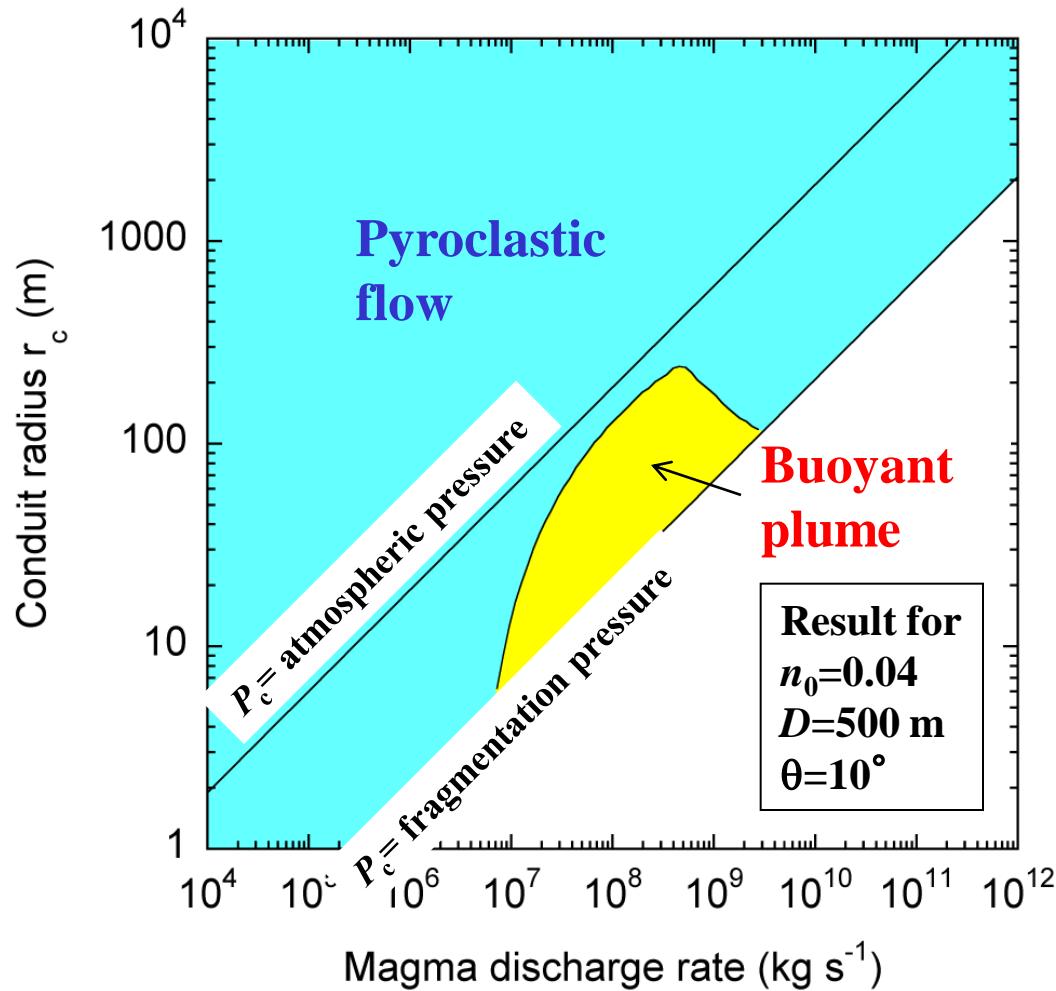
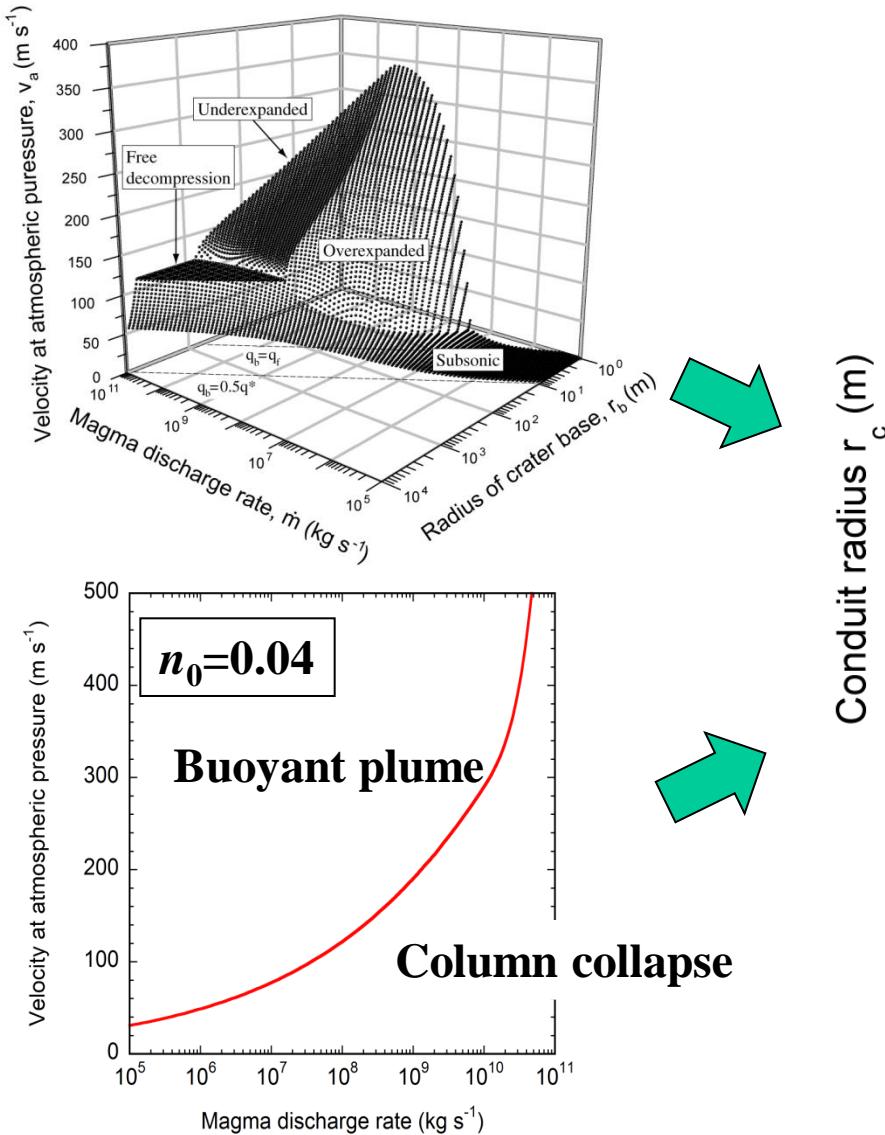
(based on Woods and Bower, 1995; Ogden et al., 2008)

" $v_a$ "



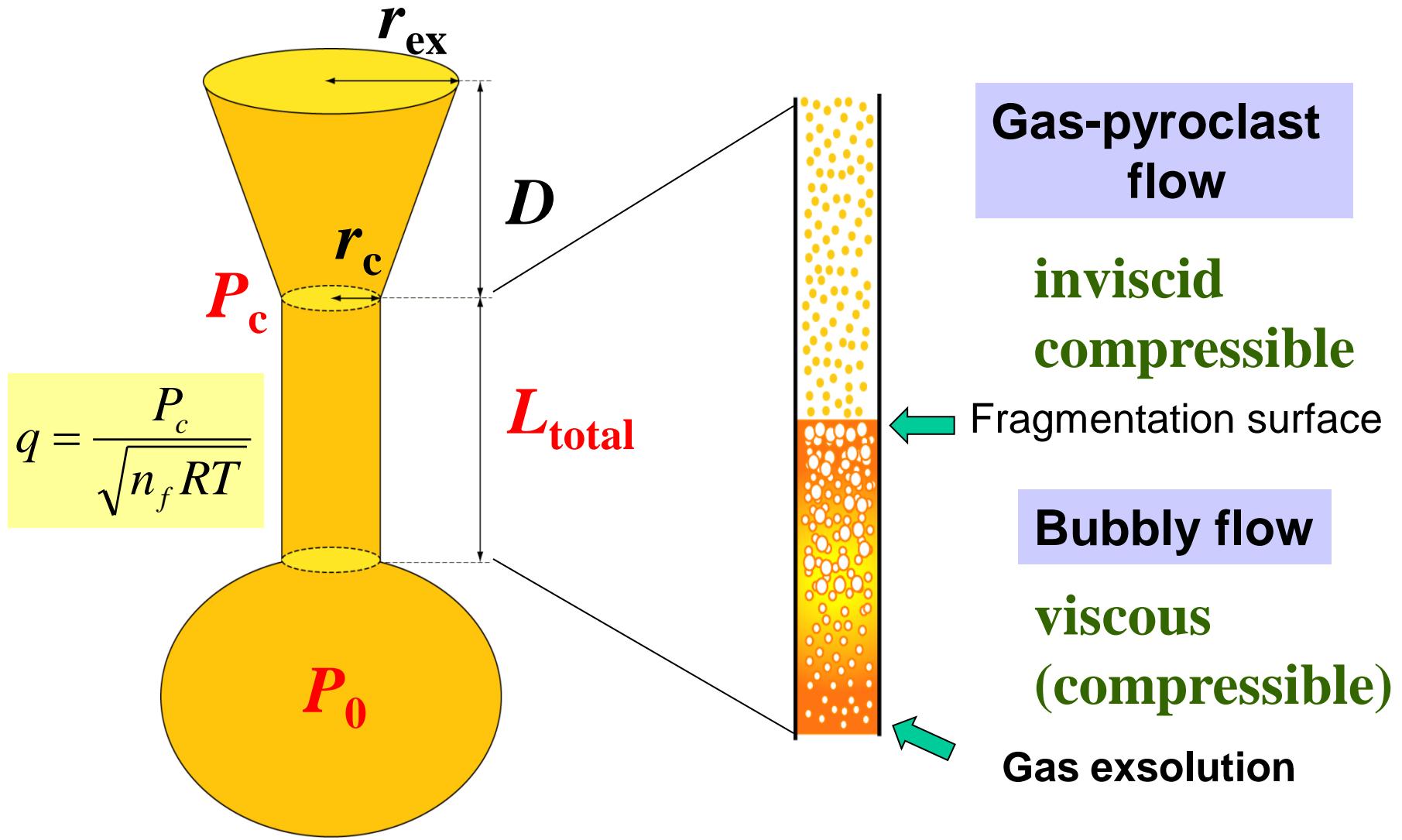
**Result for**  
 $n_0=0.04$   
 $D=500\text{ m}$   
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# Proclastic-flow and buoyant plume regions on “magma discharge rate vs $r_c$ diagram”



# Mass flow rate through a conduit

1-D conduit flow model (e.g., Wilson et al., 1980; Koyaguchi, 2005)



# Semi-analytical solution of 1-D steady conduit flow (Koyaguchi, 2005)

small

$$\gamma \equiv \frac{g \rho_l^2 r_c^2 \sqrt{n_f RT}}{8\eta P_f}$$

large

$$\gamma \ll 1$$

$$\gamma \gg 1$$

$$\frac{P_c}{P_f} \approx \exp\left(-\frac{gL_{total}}{n_f RT}\right)$$

$$\frac{P_c}{P_f} \approx \exp\left(-\frac{\Delta P}{\rho_l n_f RT}\right)$$

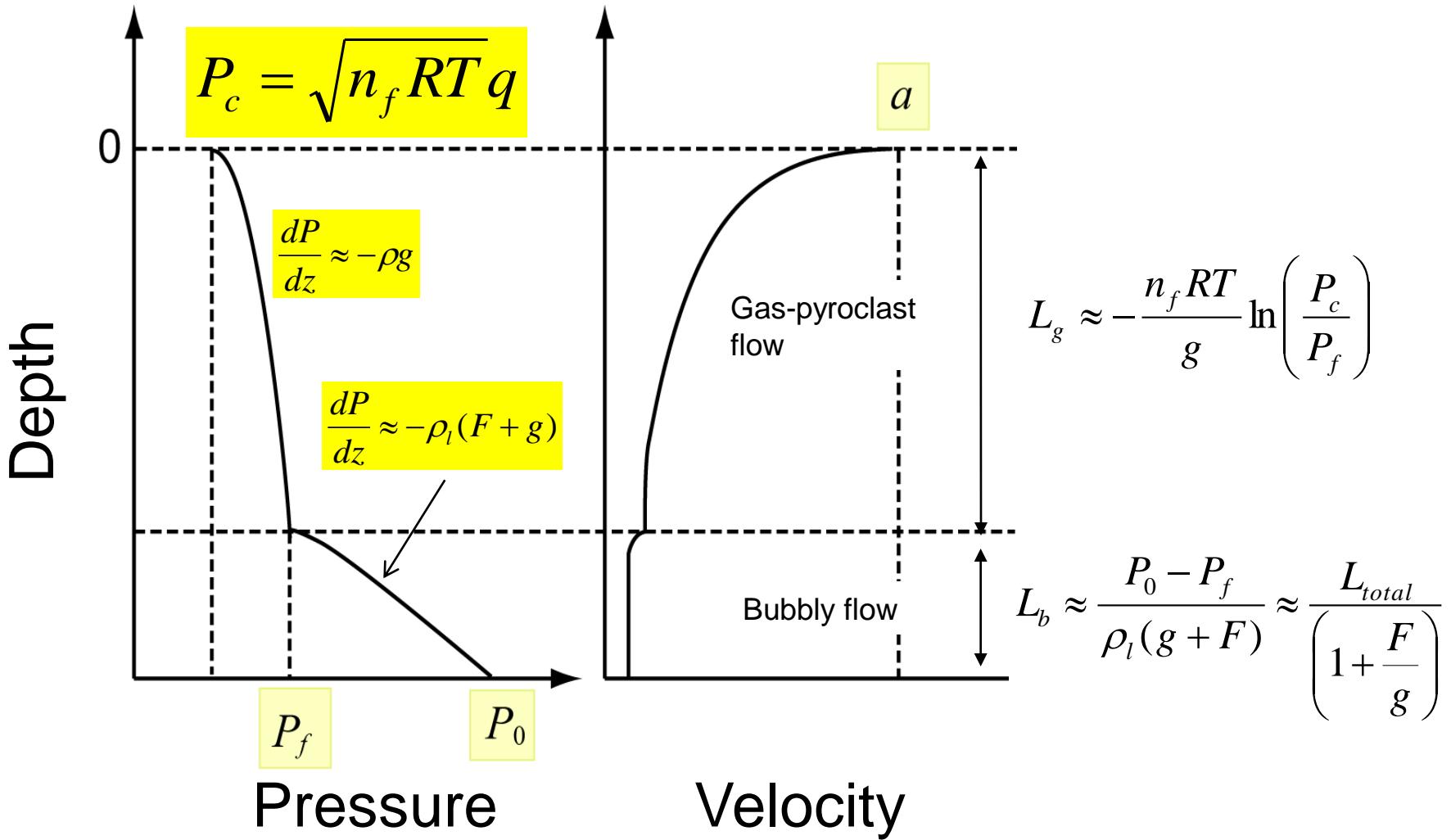
**Chamber depth control**

$L_{total}$ : chamber depth.

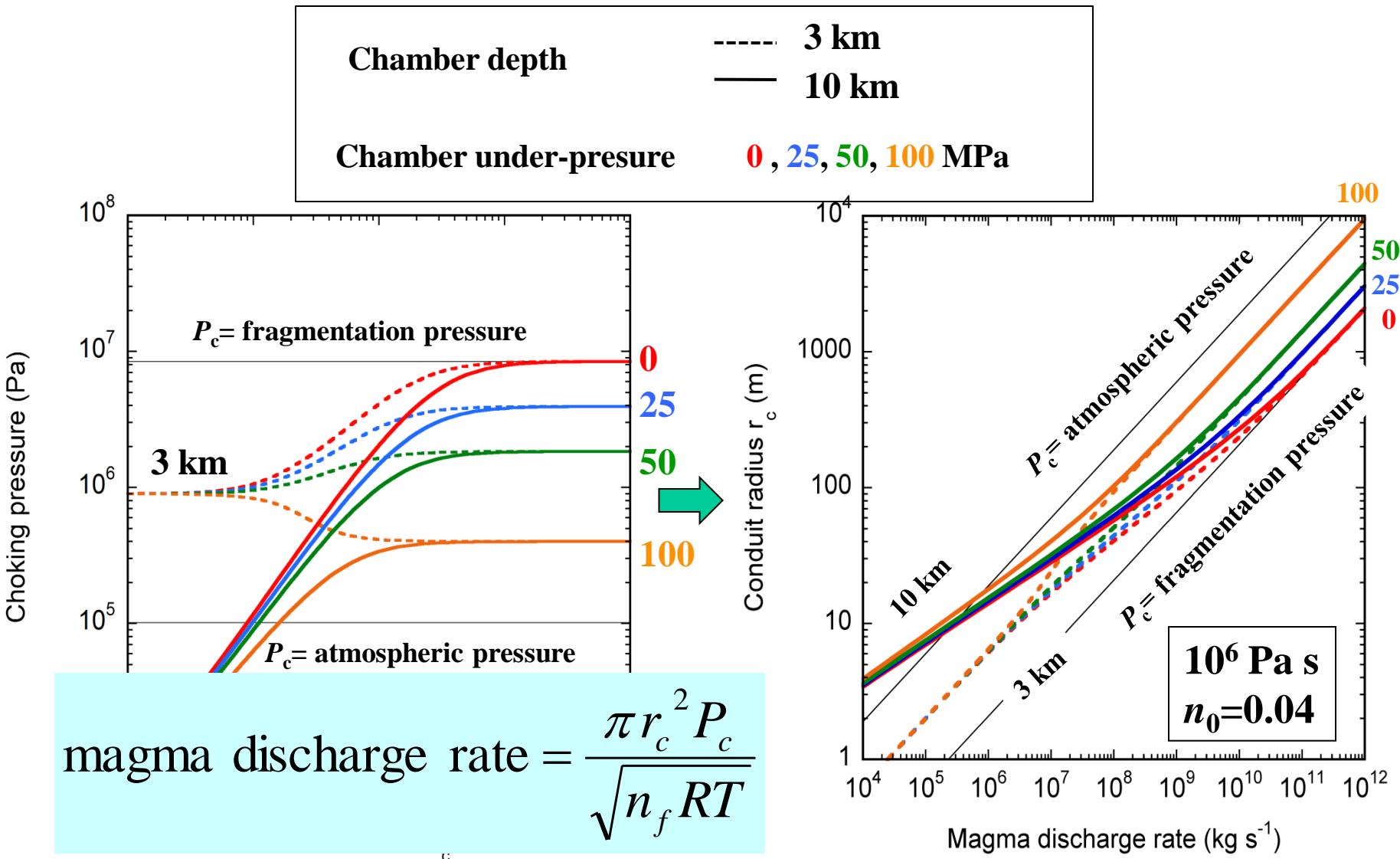
$\Delta P = P_{lith} - P_0$ : degree of chamber under-pressure.

# Semi-analytical solution of 1-D steady conduit flow (Koyaguchi, 2005)

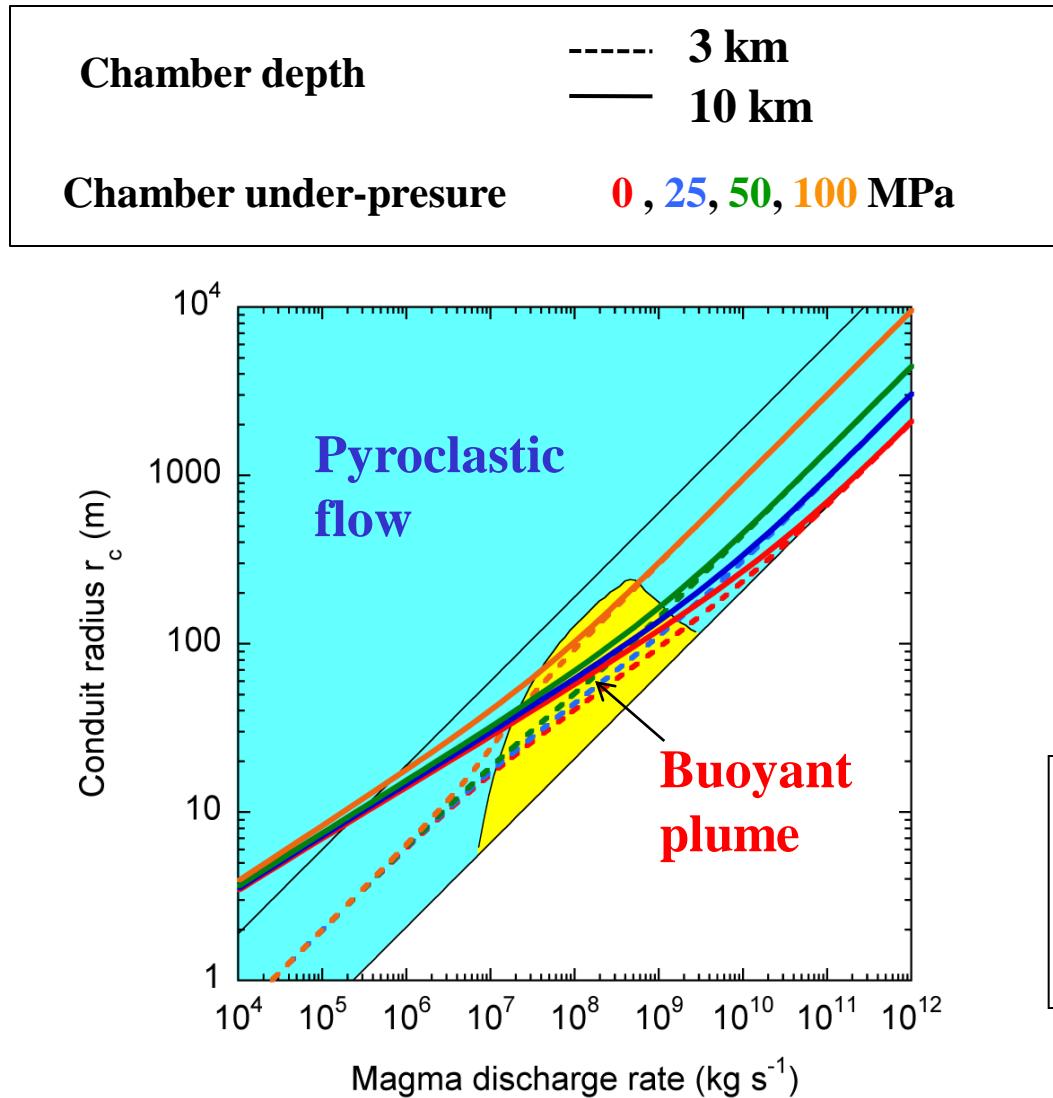
$q = \rho v$ ; mass flow rate



# “Magma discharge rate vs $r_c$ relationship” derived from 1-D conduit flow model

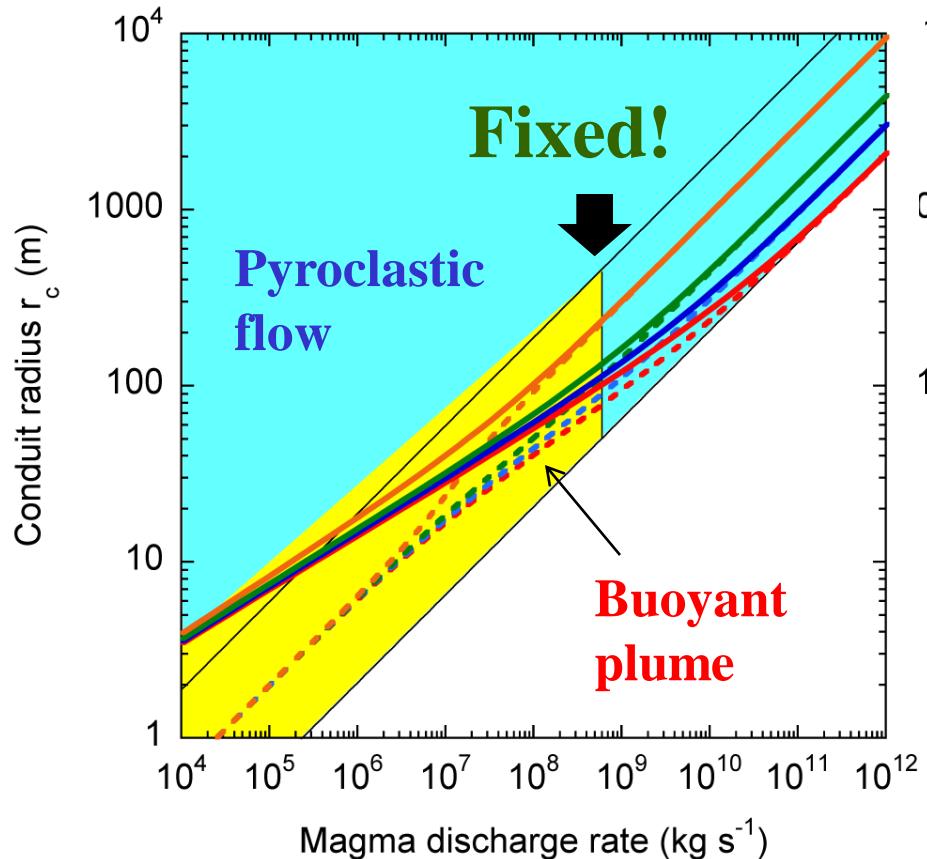


# Column collapse condition on “magma discharge rate vs $r_c$ diagram”

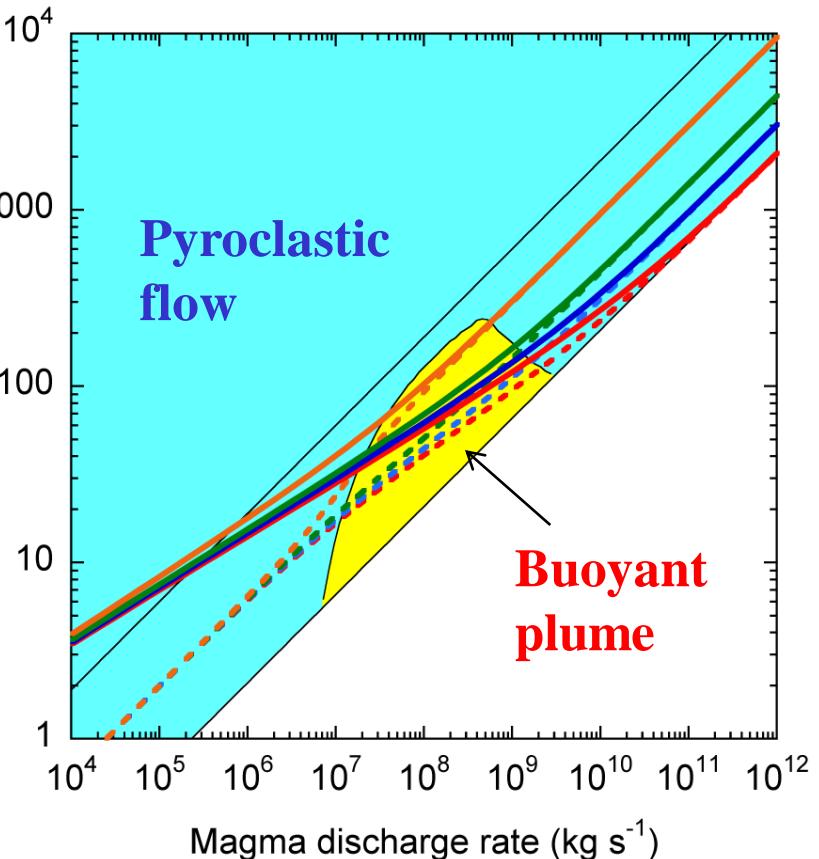


# Difference from the previous study

## Previous study



## This study



$$v_a \approx 138\sqrt{n}$$

(e.g. Carazzo et al. 2008)

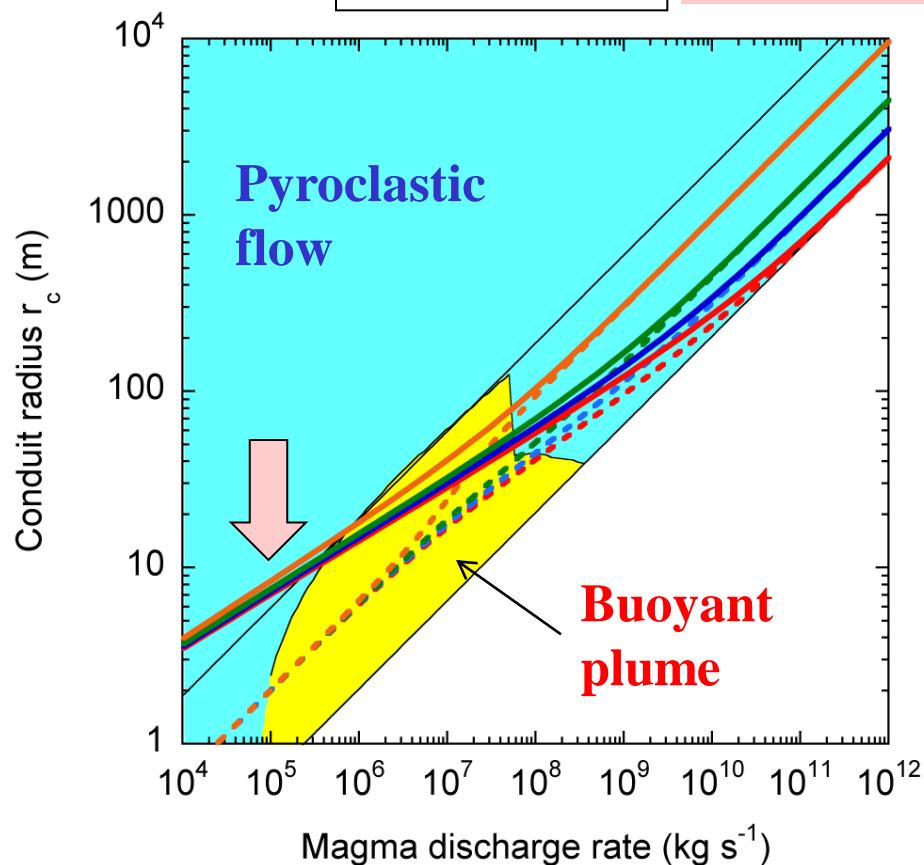
**variable**  $v_a$

# Dependence on crater shape

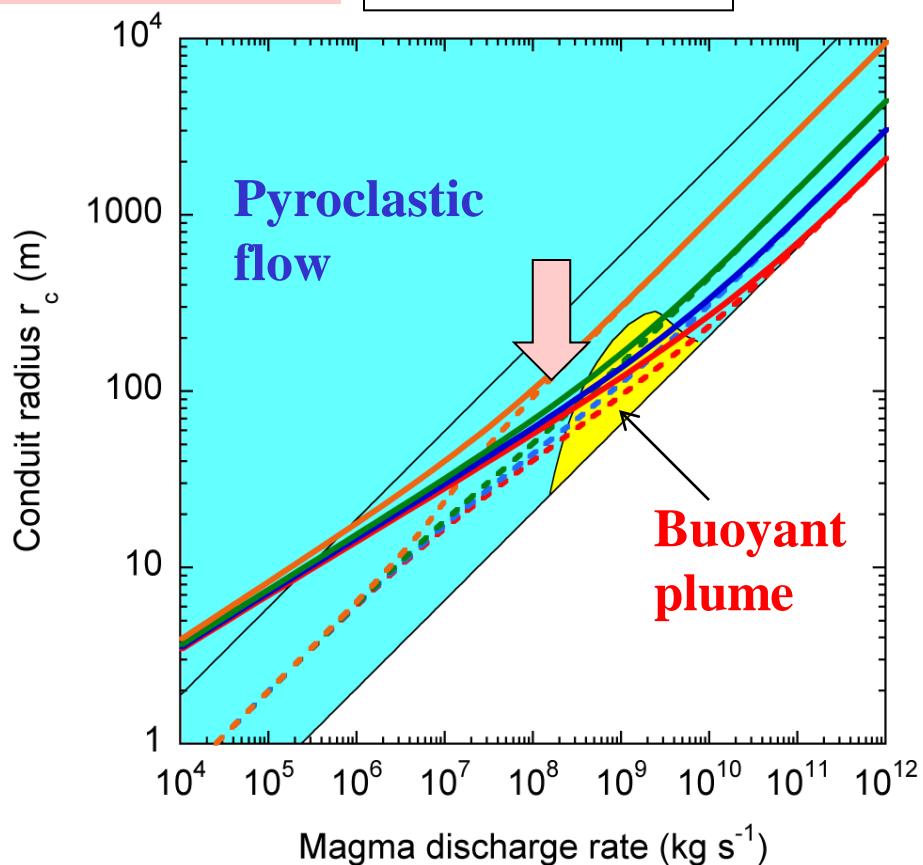
**D=500 m**  
**θ=1°**

$$\dot{m}_L \approx \frac{\pi P_a D^2 \tan^2 \theta}{\sqrt{n_f RT}}$$

**D=1000 m**  
**θ=20°**

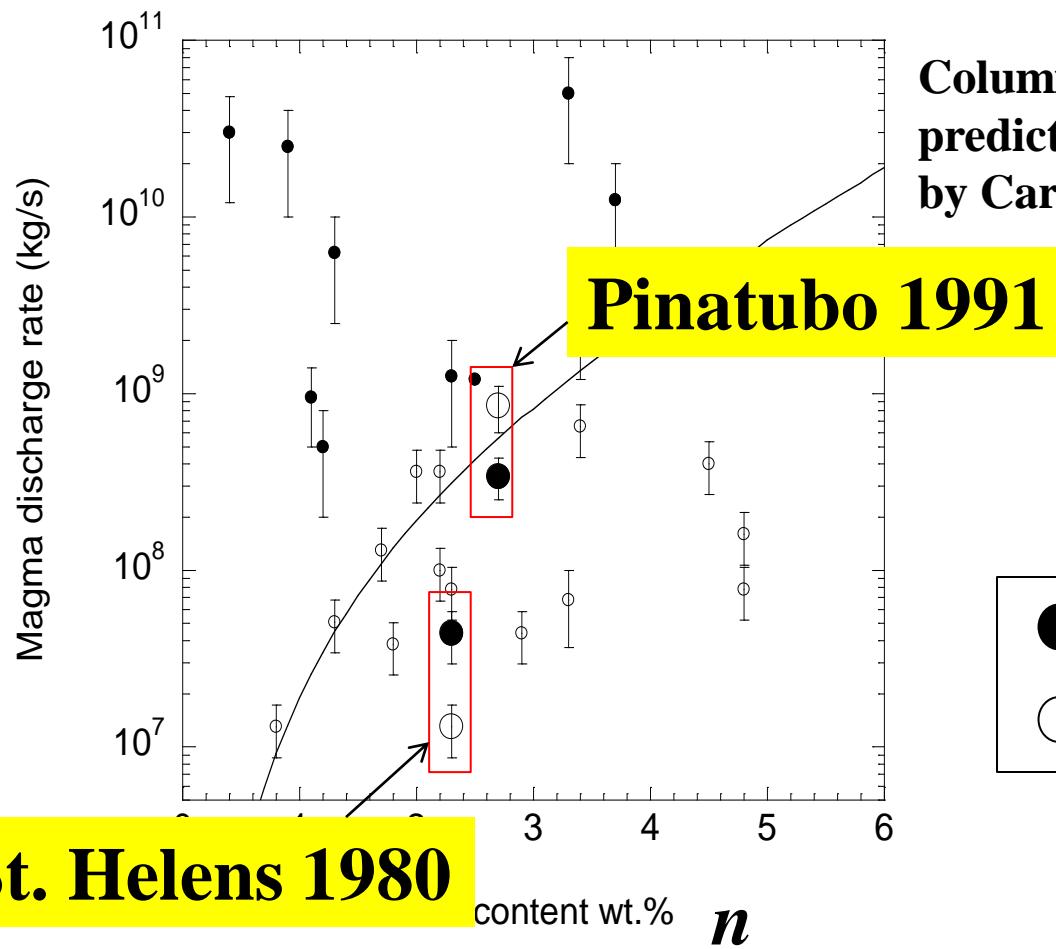


St. Helens 1980



Pinatubo 1991

# Observations and previous prediction



## Observations

- Pyroclastic flow
- Buoyant plume

# Conclusions and future direction

- The column collapse condition depends on entrainment coefficient, crater shape, and magma chamber pressure.
- The entrainment coefficient may be approximated by
  - $k=0.04 \sim 0.07$  near the exit
  - $k=0.10 \sim 0.15$  far from the exit.
- The effects of crater shape and magma chamber pressure on the column collapse condition can be systematically analyzed using the magma discharge rate vs  $r_c$  diagram.
- In order to confirm the present conclusions, 3-D simulations of eruption cloud are in progress, particularly focusing on more quantitative evaluation of the effects of compressible flow inside and just above the crater.