

Photo: Entry gate to ERI, 28 Nov. 2025

1 Summary of activities

During my stay at ERI, I completed research into linear and non-linear stability analysis of steady-state sliding on rate-and-state faults under different loading conditions, within two- and three-dimensional media under different loading conditions, a project initiated prior to my visit in close collaboration with Dmitry Garagash. I developed approximate expressions for critical lengthscales for linear and non-linear stages of instability that we previously solved for numerically. The results provide the boundaries between stable creep, slow slip, and seismic slip transients in the model fault parametric space. I drafted, revised, and submitted a manuscript with D. Garagash [*Viesca and Garagash*, 2025] and presented this research at the joint meeting of the Japan and American Geophysical Unions [*Viesca and Garagash*, 2026], and gratefully acknowledge the support from the ERI International Office to do so.

This work provided a fruitful starting point for discussion with my host So Ozawa, from which we identified a number of avenues that naturally stem. We explored the linear stability of circular rate-weakening asperities embedded within a steadily creeping fault, a model geometry of particular interest for repeating earthquakes, creep-driven microseismicity, and potentially low-frequency earthquakes and tremor. We found analytical approximations of critical lengthscales for the emergence of unstable modes, in the limit in which the host rock Poisson ratio $\nu \to 0$, and numerically solved for critical asperity sizes with $\nu \neq 0$. Similarly, we also considered rectangular rate-weakening domains, commonly used in three-dimensional numerical models of the seismic cycle at crustal scale, and found an accurate analytical approximations for critical domain dimensions for loss of linear stability, for an arbitrary aspect ratio of the domain. These results provided conditions for the emergence of linearly unstable modes and served as a starting point with which to examine the emergence of spatio-temporal complexity in rate-and-state models of

two-dimensional planar faults in three-dimensional media, particularly the transition from periodic to chaotic sequences of aseismic and seismic slip under stress- and dispalcement-controlled loadings, investigated by So. So presented this work at the Southern California Earthquake Center (SCEC) Annual Meeting [*Ozawa and Viesca*, 2026a] and has submitted an abstract to be presented at the American Geophysical Union (AGU) Annual Meeting [*Ozawa and Viesca*, 2026b].

Having an office just next door to So's allowed for regular and frequent discussions touching on a number of other related topics, in the office or in group meetings with postdoctoral scholar Xinyu Tan. These include the occurrence or recurrence of seismic and aseismic events in seismic-cycle models involving fluid-injection; the impact of the form of state evolution laws on linear-to-non-linear instability transition; as well as the effect of scale separation between rate-weakening domain size and critical lengthscales, including the spontaneous emergence for low-frequency-like dynamic rupture events. Following our wider discussions on linear stability, So was in position to numerically investigate the linear stability of models with elementary fault complexity (e.g., parallel faults) and compare with analytical calculations predicting critical fault dimensions, finding the latter closely predicting instability onset in numerical solutions.

ERI provided an environment conducive for me to make substantial progress on several other personal projects, including aspects of some peculiar eigenvalue problems and to update myself with the state of earthquake science and familiarize myself with researchers across Japan and from abroad. This was in part facilitated by the great number of colloquia and weekly seminars organized by each research group, touching on just about every aspect of fault operation and observation. In addition to giving an introductory seminar at ERI, hosted by So Ozawa, shortly after my arrival, I was able to travel to a number of institutes in Tokyo and beyond. These include: the Japan Agency for Marine-Earth Science and Technology (JAMSTEC), where I was hosted by Daisuke Sato and able to meet and discuss with Keisuke Aryoshi and Mikito Furuichi; the National Research Institute for Earth Science and Disaster Resilience (NIED) organized by Kuramo Okubo, where stimulating discussions with Angela Meneses-Gutierrez, Nelson Pulido, Tatsuhiko Saito, Futoshi Yamashita, graduate students and postdocs were followed by a live earthquake-rupture experiment conducted by Kurama and Yamashita-san on a remarkably instrumented six-meter long laboratory fault; Hiroyuki Noda organized a seminar and memorable colloquium at the Disaster Prevention Research Institute (DPRI), Kyoto University, Uji campus, with participation by his current and former members of his group showcasing truly novel theoretical work on fault mechanics; this was followed another day by visits to discuss with Eichii Fukuyama, Yoshi Kaneko, and Makiko Ohtani at the Katsura and Yoshida campuses, Kyoto University; professor Ryosuke Ando kindly invited me to give a seminar at the Department of Earth Planetary Sciences, University of Tokyo. Finally, this was all topped by an intense week of introductions and meetings at the joint JpGU-AGU Meeting in Tokyo that are too numerous to mention here.

At U. Tokyo, I appreciated the opportunity to meet and discuss with professors Nobuki Kame, Masao Nakatani, and Naoyuki Kato of ERI, as well as professor Tetsuo Yamaguchi of the Department of Biomaterial Science, and Chengrui Chang, then a postdoc in his group, and extended discussions following a visit to his laboratory. My office neighbor professor Take Hiraga provided a warm welcome and very useful suggestions. More informally, regular lunches at the Yayoi campus cafeteria, as well as coffee breaks, made for memorable and stimulating discussions with a vibrant and regular group that rotated among ERI assistant professors Yuji Itoh, So Ozawa, Hatsuki Yamauchi, and Lina Yamaya, fellow visiting researchers Kyle Anderson, Lukas Hogan, Harry Linang, Marcos Moreno, and John Wallace, as well as former officemate Natalya Galina and postdocs Pablo

Rey Devesa, Boris Maletckii, Louise Maubant, Darren Tan, Xinyu Tan, Julien Thorin, and Yilin Yang, and grad. student Tarak Yadav. I valued these daily breaks, though will have inevitably left someone off this list in forgetfulness: please accept my apologies.

I am extremely thankful to the ERI International Office, without whose support this visit by myself and family would not have been possible. This began with the warm reception by Yoko Teruhara and office head professor Masa Kinoshita at the AGU Annual Meeting in 2024, shortly after my acceptance in to the program. Office staff Teruhara-san, Yasuko Onozawa, and Yoshie Kimura were essential at every stage of my visit: from visa applications, to finding university housing for myself and my family, patiently navigating me through the Bunkyo Ward Office departmental labyrinth and SMBC branch office, through my exit procedures, and many events in between. I know of no other program that can compare with the quality, proximity, and extent of support that ERI provides its international visitors. I cherished the year at ERI and in Japan on both scientific and personal levels.

In the term of spring 2026, following an invitation by professor Ryosuke Ando and encouragement from So and the ERI International Office, I taught a half-term course in the Department of Earth and Planetary Physics entitled "Semi-analytical methods for problems in earth sciences," with a focus on diffusive phenomena, consisting of 6 lectures, each approximately one hour and 45 minutes. I had not offered such a course before and my time at ERI permitted me to draft original lecture notes. In this course, I introduced dimensional analysis and its role in finding ansatz similarity solutions for the classical diffusion equation on an unbounded domain in one dimension. This was followed by point-source solutions, the introduction of the Green's function, and the use of the Green's function to provide solutions to arbitrary initial conditions. The long-time/far-field self-similar asymptotics of such solutions were discussed. The connection of these asymptotic solutions to a multipole expansion and its relation to the Green's function and its derivatives was developed, including a discussion on the importance of moments of the initial distribution. This was followed by solution of initial value problems for diffusion on a finite domain in one dimension using series solutions. Long-time asymptotics of such solutions were considered and compared with those of the unbounded domain. I introduced of the concepts of separation of scales and "early" and "late" time asymptotic solutions for initial conditions distributed over regions much smaller than the finite domain.

The course finished with problems introducing dominant balance and anomalous diffusion. These problems consisted of classical diffusion occurring along a one-dimensional conduit (e.g., a relatively permeable fault) and comparatively slow leak-off in an orthogonal direction (into a relatively impermeable host rock). The first problem considered a so-called "membrane-diffusion" model for leak-off and the second problem considered leak-off as quasi-one-dimensional diffusion. Both problems permit the introduction of time-dependent dominant balance, in which each problem transitions from early-time behavior in which leak-off is negligible and the transported quantity is stored locally in the one-dimensional conduit to late-time behavior in which leak-off becomes the dominant storage mechanism.

Interest in such problems stem from the issue of fluid flow along faults that are relatively permeable in comparison to surrounding crustal rock. Historically, preliminary models have often neglected permeability beyond the fault [e.g., *Rice*, 1992; *Marck* et al., 2015]. However, numerical models have begun to quantify the effect of a finite- but large permeability contrast [e.g., *Wang and Heimisson*, 2024] and the consequent fault fluid leak-off. This problem provided an opportunity to

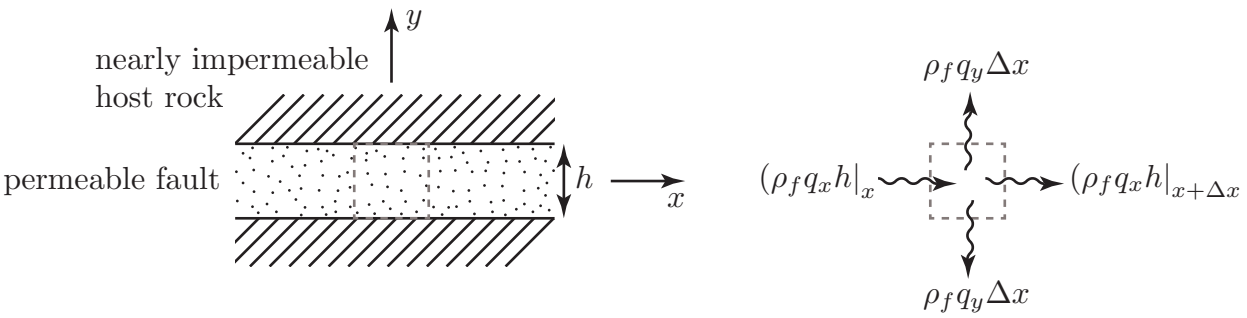


Fig. 1. (left) Schematic of a fault zone of thickness h embedded within a relatively impermeable host rock. A control volume of width Δx is enclosed by grey-dashed lines. (right) Fluid flow across the boundaries of the control volume. q_x and q_y are Darcy fluxes and ρ_f is the fluid density.

present students with a problem not found in textbooks that is a convenient pedagogical introduction to local derivatives versus non-local operators, long- versus short-term memory (the temporal counterpart to long and short-range interactions), anomalous diffusion exponents, and dominant balance. Essentially, the problem for one-dimensional, along-fault diffusion with one-dimensional, diffusive, fault-normal leak-off and we find similarity solution for a particular injection scenario. This problem was relevant to ongoing work of So Ozawa and his research group at ERI. Specifically, prior to my visit, So developed a procedure to numerically solve such coupled one-dimensional diffusion problems and he used these solutions to confirm the similarity solution developed below. In addition, ERI postdoctoral scholar Xinyu Tan in collaboration with So, had developed a numerical solver to consider fluid diffusion along fault networks that is coupled with fully two-dimensional diffusion in the bulk. In the limit of large diffusivity contrast, Xinyu retrieved the similarity solution, demonstrating that the one-dimensional diffusion approximation is appropriate in this limit. Given the potential wider interest, the problem novelty, and the possibility of its concise, self-contained exposition, I have summarized the problem and its solution as presented to the class in the following section.

2 Problem of fluid flow along a fault within a relatively impermeable host rock

In Figure 1a we illustrate a fault-zone with thickness h contained by a relatively impermeable host rock. In Figure 1b we illustrate fluid flow around the boundaries of a control volume of an element of the fault zone of width Δx. The density of fluid is ρ_f, and q_x and q_y are the Darcy fluid fluxes, defined as mass fluid flux per unit cross-sectional area (considering a unit length in the out-of-plane direction). The rate of fluid mass m_f contained within the element is given by the conservation equation

$$\frac{\partial m_f}{\partial t} = -\left(\rho_f q_x h|_{x+\Delta x} + (\rho_f q_x h|_x - 2\rho_f q_y \Delta x \right. \tag{1}$$

The fluid mass of the element can be expressed as $m_f = n\rho_f \Delta x h$, where n is the porosity. Neglecting a full poroelastic treatment, including variations of h with x and t, we divide both sides by $\Delta x\, h$ and taking the limit $\Delta x \to 0$, we arrive to

$$\frac{\partial (n\rho_f)}{\partial t} = \frac{\partial (\rho_f q_x)}{\partial x} - \frac{2\rho_f q_y}{h} \tag{2}$$

We linearize the first term on the right hand side and consider $n(p)$ and $\rho_f(p)$, again neglecting poroelastic effects that changes in total stress may have on porosity, such that (2) becomes

$$\rho_f \frac{dn}{dp}\frac{\partial p}{\partial t} + n\frac{d\rho_f}{dp}\frac{\partial p}{\partial t} = -\rho_f \frac{\partial q}{\partial x} - \frac{2\rho_f q_y}{h} \tag{3}$$

The above can be simplified further: defining fluid compressibility β_f as $\beta_f = \rho_f d(\rho_f^{-1})/dp$ and porosity compliance as $\beta_n = dn/dp$ and a composite compressibility $\beta = \beta_n + n\beta_f$, leads to

$$\beta\frac{\partial p}{\partial t} = -\frac{\partial q_x}{\partial x} - 2\frac{q_y}{h} \tag{4}$$

Darcy's law provides an expression for the along-fault flux

$$q_x = -\frac{k}{\mu}\frac{\partial p}{\partial x} \tag{5}$$

In the following sections we consider two models for the leak-off flux q_y: a zero-dimensional elementary membrane-diffusion model (analogous to Newton-cooling) and a one-dimensional fault-normal diffusion model.

2.1 Membrane leak-off solution

As a starting point, we consider the dimensional analysis of fluid flux in the fault-normal direction q_y as it may depend on the host-rock permeability k_o, fluid viscosity μ, fault fluid pressure p and fault-zone thickness h, or

$$q_y = f(k_o, \mu, p, h) \tag{6}$$

Such a dependence suggests two dimensionless groups and a membrane-diffusion model implies a linear relation between q_y and p, requiring the relation between the two groups take the form

$$q_y = \frac{1}{2}\frac{k_o}{\mu}\frac{p}{h} \tag{7}$$

where the constant prefactor is arbitrarily chosen for convenience and considered to be well within the approximation of membrane-diffusion. We substitute the expressions for the fluid fluxes (5) and (7) into (4),

$$\frac{\partial p}{\partial t} = \frac{k}{\mu\beta}\frac{\partial^2 p}{\partial x^2} - \frac{k_o}{\mu\beta}\frac{p}{h^2} \tag{8}$$

We now consider the relative importance of the three terms in (8) via a scaling analysis. With the scalings $t \sim T$, $p \sim T^a$, $x \sim T^b$, the terms of (8) scale in order as T^{a-1}, T^{a-2b}, and T^a, respectively. If the first and second terms dominate, we have a classic diffusion equation and $b = 1/2$. If we consider injection at $x = 0$ at constant pressure Δp, $a = 0$ and consequently the first and second terms scale as T^{-1}, while the third term remains $O(1)$. We are thus justified in neglecting the third term at early times ($T \to 0$), implying classical diffusion in this limit,

$$\frac{\partial p}{\partial t} = \frac{k}{\mu\beta}\frac{\partial^2 p}{\partial x^2} \tag{9}$$

following the solution $p(x,t) = \Delta p \mathrm{erfc}(|x|/2\sqrt{\alpha t})$, where the diffusivity $\alpha = k/\mu\beta$. However, given the T^{-1} scaling resulting from the early-time dominance of the first and second terms, we would be mistaken to neglect the third term at long time ($T \to \infty$) and must revisit the dominant balance.

It is not possible for the first and third terms to balance under the condition that $a = 0$ as the first term always decays. So at long time, the only remaining possibility is that the second and third terms come to balance each other and dominate

$$\frac{k_o}{\mu\beta}\frac{p}{h^2} = \frac{k}{\mu\beta}\frac{\partial^2 p}{\partial x^2} \tag{10}$$

which we can rewrite as

$$\frac{k_o/k}{h^2}p = \frac{\partial^2 p}{\partial x^2} \tag{11}$$

implying that we may non-dimensionalize along-fault distances as

$$\bar{x} = x/(\sqrt{k/k_o}h) \tag{12}$$

where we expect $k/k_o \gg 1$ to follow from an orders-of-magnitude contrast in permeability between the permeable fault and the host rock. Since the governing equation is now time-independent, the fluid pressure distribution along the fault at late times takes on a steady-state solution of the form $p(\bar{x})$ to the equation

$$p = p'' \tag{13}$$

with the general solution

$$p(\bar{x}) = Ae^{-\bar{x}} + Be^{\bar{x}} \tag{14}$$

Since the first and second terms diverge unphysically for $\bar{x} < 0$ and $\bar{x} > 0$, respectively, the solution must be defined piecewise, and considering the boundary condition of injection at constant pressure Δp at the origin, the solutionis

$$p(\bar{x}) = \Delta p\, e^{-|\bar{x}|} \tag{15}$$

What separates early from late times? An answer arises naturally from considering that the solutions in these two regimes have two apparent lengthscales: at early times, the solution exhibits self-similar spreading of fluid pressure along the fault with the lengthscale $\sqrt{\alpha t}$ while at late times the steady-state distribution varies over the lengthscale $\sqrt{k/k_o}h$. A characteristic transition time follows from considering when self-similar spreading approaches the membrane-diffusion lengthscale. We define the transition time t_c as that satisfiying $\sqrt{\alpha t_c} = \sqrt{\frac{k}{k_o}}h$. Assuming the compressibility β is similar within the fault zone and host rock, we may equate the permeability ratio k/k_o with the diffusivity ratio α/α_o, implying that the transition time for appreciable flux into the host rock is

$$t_c = h^2/\alpha_o \tag{16}$$

which could have been anticipated by dimensional analysis. The early- and late-time solutions are thus asymptotically valid for $t \ll t_c$ and $t \gg t_c$.

2.2 One-dimensional leak-off solution

The proceeding problem was relatively straight forwared: introducing "membrane" leak-off eliminated long-time self-similarity and gave rise to a steady-state solution; leak-off slowed along-fault fluid migration to a halt. However, this model is a simplification. Fluid pressure in the host rock obeys a two-dimensional diffusion equation, letting p_o denote pressure outside of the permeable fault zone,

$$\frac{\partial p_o}{\partial t} = \frac{k_o}{\mu\beta_o}\nabla^2 p_o \tag{17}$$

A highly permeable fault zone ($k \gg k_o$) implies variations of fluid pressure will stretch along x much faster than along y; fluid flow in the fault-normal y direction will predominate in the host rock and $\partial^2 p_o/\partial x^2 \ll \partial^2 p_o/\partial y^2$, such that fluid pressure in the host rock locally follows a one-dimensional diffusion equation

$$\frac{\partial p_o}{\partial t} = \frac{k_o}{\mu \beta_o} \frac{\partial^2 p_o}{\partial y^2} \tag{18}$$

and variations along x are only introduced by imposing fluid-pressure continuity between the fault zone and host rock: $p(x,t) = p_o(x,0,t)$. Implicitly, we assume the fluid pressure within the finite-thickness fault zone is approximately uniform in the y-direction: $p(x,y,t) \Rightarrow p(x,t)$. This appears reasonable in the asymptotic limit $k \gg k_o$ as fluid pressure within the fault zone will equilibrate along y on timescales much shorter than for appreciable variations in the host rock. Thus leak-off in this case will depend on the local history of the fault-zone pressure p.

The problem is to now determine the amount of leak-off as a solution to the semi-infinite boundary value problem for $p_o(x,y,t)$ for some history of $p_o(x,0,t) = p(x,t)$. To do so, we recall the solution for a sudden imposition of fluid pressure of the amount Δp starting from $t = 0$ at a boundary $y = 0$: $p_o(y,t) = \Delta p \operatorname{erfc}(y/2\sqrt{\alpha t})$. This implies, by superposition, that the solution for the host-rock pore pressure $p_o(x,y,t)$, given a fault zone pore pressure history at some position x $p(x,t) = p_o(x,0,t)$ is [e.g., *Carslaw and Jaeger*, 1959]

$$p_o(x,y,t) = \int_0^t \frac{\partial p}{\partial t'} G(y, t-t') dt' \tag{19}$$

where

$$G(y,t) = \operatorname{erfc}\left(\frac{y}{2\sqrt{\alpha t}}\right) \tag{20}$$

and $y > 0$ is taken to be the upper-half of the host-rock domain (and we may use this result, by symmetry, to determine the fluid pressure in the lower-half). Using this result, we may calculate the gradient of p_o along y, and hence the leak-off flux across the host-rock/fault-zone boundaries

$$\frac{\partial p_o}{\partial y} = \int_0^t \frac{\partial p}{\partial t'} \frac{\partial G}{\partial y} dt' \quad \text{with} \quad \frac{\partial G}{\partial y} = -\frac{e^{-y^2/4\alpha_o(t-t')}}{\sqrt{\pi \alpha_o (t-t')}} \tag{21}$$

which at the host-rock boundary is

$$\left.\frac{\partial p_o}{\partial y}\right|_{y=0} = -\int_0^t \frac{\partial p/\partial t'}{\sqrt{\pi \alpha_o (t-t')}} dt' \tag{22}$$

such that Darcy flux at the boundary is

$$q_y = -\frac{k_o}{\mu} \left.\frac{\partial p_o}{\partial y}\right|_{y=0} \tag{23}$$

Substituting this into the local fluid mass conservation equation (4), and rearranging, we find that the fault-zone fluid pressure p satisfies

$$\frac{\partial p}{\partial t} + \frac{k_o}{\mu \beta} \frac{2}{h} \int_0^t \frac{\partial p/\partial t'}{\sqrt{\pi \alpha_o (t-t')}} dt' = \frac{k}{\mu \beta} \frac{\partial^2 p}{\partial x^2} \tag{24}$$

where, if we presume $\beta = \beta_o$, the first prefactor of the second term is identically α_o. If we let

$$x = \sqrt{\frac{\alpha}{\alpha_o}} h \bar{x}, \quad t = \frac{h^2}{\alpha_o} \bar{t} \tag{25}$$

then we may non-dimensionalize the problem, up to a dimensional pressure p, as

$$\frac{\partial p}{\partial \bar{t}} + 2\int_0^{\bar{t}} \frac{\partial p/\partial \bar{t}'}{\sqrt{\pi(\bar{t}-\bar{t}')}} d\bar{t}' = \frac{\partial^2 p}{\partial \bar{x}^2} \tag{26}$$

We note that the lengthscale here reduces to $\sqrt{k/k_o}h$, which we recall is that also found for the membrane leak-off problem. We identify the terms in order as representing (1) the storage of fluid in a permeable fault zone, (2) the storage of fluid in the host rock, and (3) the along-fault gradient of fluid flux. The first term is a local operator, implying a short-range interaction in time (or more simply, short-term memory), whereas the second term is a non-local operator providing for long-range interaction (long-term memory).

We may repeat an exercise of dominant balance of the three terms, taking again an example of a constant-pressure source. In this case, the first and second terms scale as T^{-1} and $T^{-1/2}$, respectively. The slower decay of the second implies it will come to dominate at long times. The dominant balance is thus between the second and third terms in the large t limit. if $x \sim T^b$, this balance implies $b = 1/4$, representing sub-diffusive spreading of fluid pressure along the fault zone (an example of anomalous diffusion). This scaling implies a similarity variable

$$\eta = \frac{\bar{x}}{\bar{t}^{\,1/4}} \quad \text{or} \quad \eta = \frac{x}{\sqrt{\sqrt{\frac{\alpha}{\alpha_o}} h \sqrt{\alpha t}}} \tag{27}$$

where, looking at the latter, dimensional representation, we remark that x is scaled by the geometric mean of the two lengthscales $\sqrt{\alpha/\alpha_o}\,h$ and $\sqrt{\alpha t}$.

A constant-pressure source implies a solution of the form $p = \Delta p f(\eta)$ with $f(0) = 1$ and $f \to 0$ as $\eta \to \infty$. Such a solution must satisfy

$$2\int_0^{\bar{t}} \frac{\partial p/\partial \bar{t}'}{\sqrt{\pi(\bar{t}-\bar{t}')}} d\bar{t}' = \frac{\partial^2 p}{\partial \bar{x}^2} \tag{28}$$

Upon substitution of the self-similarity ansatz,

$$\frac{\partial^2 p}{\partial \bar{x}^2} = \Delta p\, f'' \left(\frac{\partial \eta}{\partial \bar{x}}\right)^2 = \frac{\Delta p}{\bar{t}^{1/2}} f'' \tag{29}$$

and since $\partial p/\partial \bar{t}' d\bar{t}' = \Delta p f'(\lambda) d\lambda$, where $\lambda = \bar{x}/\bar{t}'^{1/4}$,

$$2\int_o^{\bar{t}} \frac{\partial p/\partial \bar{t}'}{\sqrt{\pi(\bar{t}-\bar{t}')}} d\bar{t}' = 2\Delta p \int_\infty^{\eta} \frac{f'(\lambda)}{\sqrt{\pi\left[(x/\eta)^4 - (\bar{x}/\lambda)^4\right]}} d\bar{\lambda} = \frac{2\Delta p}{\bar{t}^{1/2}} \int_\infty^{\eta} \frac{f'(\lambda)}{\sqrt{\pi[1-(\eta/\lambda)^4]}} d\lambda \tag{30}$$

and rearranging we arrive to the equation governing the similarity solution

$$-2\int_\eta^\infty \frac{f'(\lambda)}{\sqrt{\pi(1-(\eta/\lambda)^4)}} d\lambda = f''(\eta) \tag{31}$$

We solve (31) numerically using a piecewise constant approximation for $f(\lambda)$ on the left hand side, such that $f'(\lambda)$ is approximated by a set of Dirac deltas occurring at a discrete set of points λ_j; on the right hand side, we use a second-order finite-difference approximation for $f''(\eta)$, with

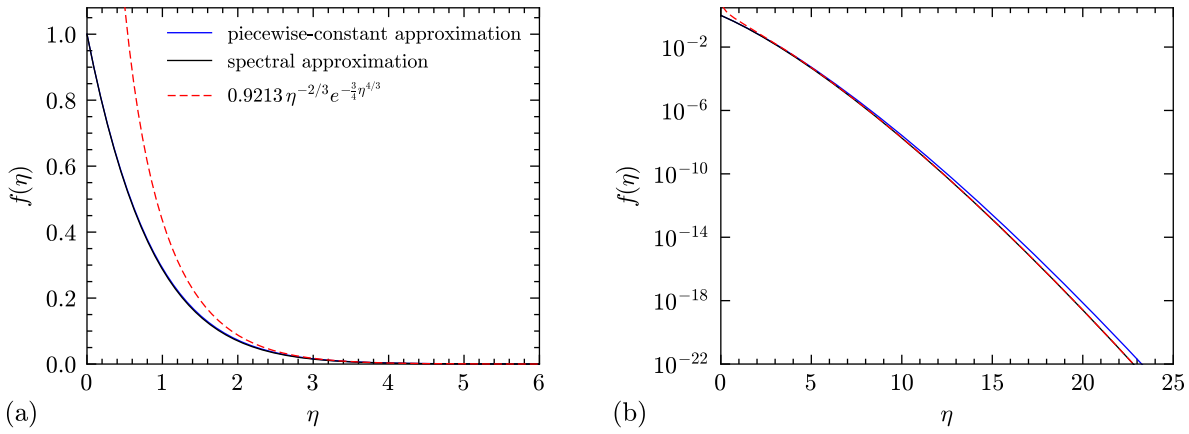


Fig. 2. Plot of the self-similar solution for pore pressure evolution along a fault zone of thickness h, with locally one-dimensional leak-off into the host rock. The solution is given in terms of the similarity variable $\eta = x/(\sqrt{h\sqrt{\alpha t}}\sqrt{\alpha/\alpha_o})$ on linear (left) and semi-logrаithmic (right) scales. In blue, the solution found by a piecewise-constant approximation of $f(\eta)$ discretized with $N = 32000$ elements of width $\Delta\eta = 0.00625$. In black is the solution found using a spectral method involving Gauss-Chebyshev integral quadrature and Chebyshev differentiation. This spectral numerical solution was implemented by Claude. The red-dashed curve shows the leading-order asymptotic behavior of f at a large η, the numerical prefactor was determined by comparisons with the spectral solution. The piecewise-constant approximation does not explicitly include this asymptotic behavior in its solution, while the spectral solution does.

the evaluation of $f(\eta)$ occurring at the midpoints η_i of the piecewise constant approximation. In Figure 2 we plot this numerical solution on linear and semi-logarithmic scales in blue. For comparison, we plot more accurate numerical solutions in black, found using a spectral method implemented using Claude with two key features: Gauss-Chebyshev quadrature chosen on the basis of the kernel singularity and the separation from the solution variable of the exponential component of its asymptotic decay in the form $A\exp(-3\eta^{4/3}/4)$, anticipated partly on the basis of the piecewise-constant numerical solutions and similar decay found in the error-function solution for classical diffusion, which decays asymptotically as $\mathrm{erfc}(\eta/2) \sim 2/\sqrt{\pi}\exp(-\eta^2/4)/\eta$.

We revisit the dominant balance of the governing equation (26) under presumption of constant pressure injection: the terms scale in order as T^{-1}, $T^{-1/2}$, and T^{-2b}. We see that in the early-time limit ($T \to 0$) that the first term surpasses the second, implying a balance of the first and third terms. In this limit, like for the problem in which leak-off was approximated by membrane diffusion, we retrieve classical diffusion in which $b = 1/2$ and the self-similar solution for $p(x,t) = \Delta p\,\mathrm{erfc}(|x|/2\sqrt{\alpha t})$. The transition from the early- to late-time solutions applies when the late-time lengthscale $(\sqrt{\alpha/\alpha_o}h\sqrt{\alpha t})^{1/2}$ and early-time lengthscale $\sqrt{\alpha t}$ are comparable. Not surprisingly, this occurs when t is $O(h^2/\alpha_o)$, the timescale for appreciable diffusion into the host rock and also the transition timescale found for the problem with the membrane-diffusion approximation.

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