Stress dependence of b-value in a discrete spring-block model of earthquake

The variations on the b-value in the earthquake statistics has received substantial interests recently. Recent observations have found a common dependence on stress for b (Schorlemmer et al. Nature 437, 539 (2005)) and c (Narteau et al. Nature 462, 642 (2009)) values of aftershock sequences. We attempted in explaining these variations through simple statistical models in order to understand the physical mechanism.

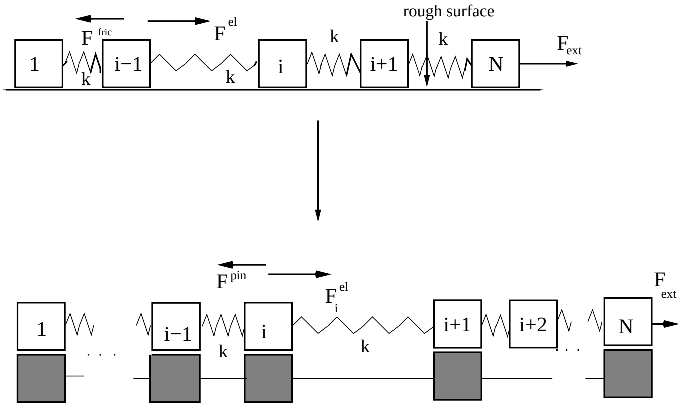

Figure 1: The conventional spring-block model is shown in the upper figure, while its discretization is shown in the lower figure. The usual velocity dependent friction law for the conventional model is now replaced by pinning forces at each site. Another important difference from the conventional model is in the definition of time, which is now taken as a single sweep of the lattice. The number of blocks moving in a single sweep is called an avalanche.

The statistical aspects of earthquake data have been attempted to be modeled by several ways (see Kawamura et al. Rev. Mod. Phys. 84, 839 (2012) for a review). Among them a popular approach is the Burridge-Knopoff spring-block type models. In these models a set of massive blocks, connected by linear springs, are slowly driven over a rough surface. The velocity dependent frictional force between the blocks and the rough surface is the only source of non-linearity in the model. The stick-slip motion of the blocks and the bursts of energy releases associated with it satisfy the GR law of earthquakes giving power-law distribution of energy releases. The Omori law for aftershock statistics is not seen in the original model.

Recently we proposed a simplified spring-block model (Biswas et al. Eur. Phys. J. B 86, 388 (2013)) in which the surface over the system moves have also been made discrete (see Fig. 1) and the velocity dependent friction is replaced by random pinning. While preserving the statistics, this simplifies the simulation immensely. We worked with the variations of this model in addressing the changes of b-value with stress.

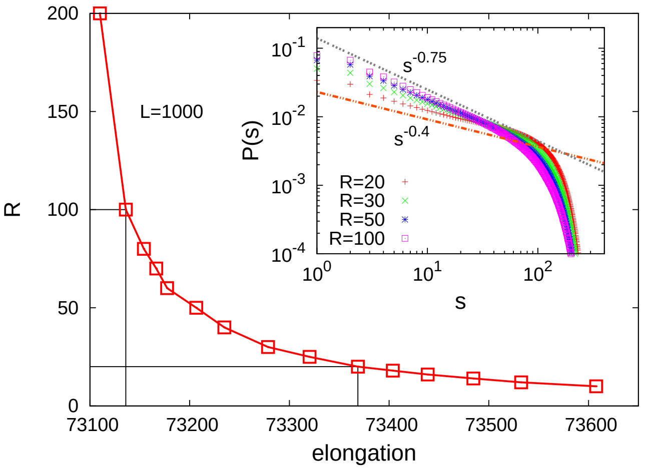

Figure 2: When the train model is driven at a finite rate (one step in every R time step), the saturation elongation depends on the rate of drive, as can be seen in the main figure. The inset shows the GR law for different driving rate. The b-value decreases with decreasing R. Therefore, the b-value decreases as stress (elongation) is increased.

Particularly, we have driven the spring-block system with a finite rate, as opposed to the quasi-static driving mechanism usually followed. In other words, the right-most block is moved one step in every R time steps (while one time step is a full scan of the system), irrespective of the state of the system (i.e. whether it is in a stable configuration or not). Driving in this manner keeps the system in a state which slightly away from the self-organized critical state of interface depinning. In particular, the total elongation of the system, which is a measure of the stress, increases as the drive rate is increased (R is decreased). Fig. 2 depicts the situation clearly.

The main observation from this study is the change in the b-value of the GR law (shown in inset of Fig. 2). As can be seen, the b-value decreases as R is decreased. In other words, the b-value decreases as the stress is increased. This supports the well known observation along this line (as mentioned before).

Apart from that we have studied the finite size scaling of the size distribution of the avalanches. It appears that the largest size scales as $s_L \sim L^{\alpha}$, with $\alpha < 1$. The reason for the sub-linear scaling could be the fractal nature of the area covered by the moving blocks in an avalanche.

We have also looked at the Omori-law scaling. Although an Omori-law with finite c value appears for the statistics of the model, the dependence of the c value on stress is not appreciable.