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On estimation of earthquake magnitude in Earthquake Early Warning systems

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Determination of earthquake magnitude from the initial P-wave portion of ground motion is the key problems for Earthquake Early Warning (EEW). We analyzed performance of scaling relations between one of the frequently used early warning parameters, so-called characteristic period τ_C and moment magnitude M_W in respect of (a) characteristics of datasets accumulated in various regions (earthquake depth, characteristics of network) and (b) variation of initial conditions applying for determination of the parameter τ_C (length of P-wave window, number of used stations). The used data contain strong-motion records from 110 earthquakes (moment magnitude range 4.4–7.6) occurred in Japan and Taiwan. We show that, although the standard error of regression $\tau_C = f(M_W)$ in general becomes smaller with the increase of the length of P-wave window (PL) and the number (N) of averaged observations (stations), the uncertainty in estimation of M_W given observed τ_C does not decrease further for PL $>$ 3–4 seconds and $N >$ 3–4 stations. The information about earthquake depth also plays an important role in reducing the uncertainty of magnitude estimations. The table of confidence limits for estimations of the moment magnitude $\hat{M}_W$ based on observations of the characteristic period τ_C is provided.

Key words: Earthquake early warning, magnitude estimation, characteristic period.

1. Introduction

Rapid estimation of earthquake size in Earthquake Early Warning systems is based on assumption that magnitude of an earthquake can be estimated using the frequency content of the first few seconds on the P-wave arrival. The parameters, which represent the average period of initial portion of the P wave, are the most frequently used. One of these parameters is called "characteristic period" τ_C and it is evaluated as follows (e.g. Kanamori, 2005; Wu and Kanamori, 2005, 2008a, b; Wu *et al.*, 2006, 2007a, b). The ground motion displacement $u(t)$ and velocity $\dot{u}(t)$ from the vertical component are used to compute the ratio

$$r = \int_0^\tau \dot{u}^2(t)dt \Bigg/ \int_0^\tau u^2(t)dt, \qquad (1)$$

where the integration is taken over the time interval $(0, \tau)$ after the onset of P wave (hereafter referred as the P-wave window). The characteristic period τ_C is calculated as $\tau_C = 2\pi/\sqrt{r}$. Application of a high-pass filtering with a cut-off frequency of 0.075 Hz is suggested to remove low-frequency noise.

The scaling relation between characteristic period and magnitude was analyzed on the basis of strong-motion data accumulated in Taiwan (Wu and Kanamori, 2005; Wu *et al.*, 2006, 2007b), southern California (Wu *et al.*, 2007a), and Japan (Shieh *et al.*, 2008; Wu and Kanamori, 2008b). Kanamori (2005) and Wu and Kanamori (2008a) considered the data from Taiwan, California and Japan jointly. The data from shallow earthquakes (focal depth less than 30 km) were used in most studies, however Kanamori (2005) and Shieh *et al.* (2008) selected particular events in Japan with focal depth less than 70 km.

It has been found that the uncertainty in the relation between the period-dependent parameters and magnitude is a function of the number of stations providing P-wave data. Usually, observations of characteristic period at a single station show significant scatter, but once several close-in stations are averaged the scatter is reduced significantly. Different numbers of stations providing P-wave data were used in different studies, e.g. six stations (Shieh *et al.*, 2008; Wu and Kanamori, 2008b) or eight stations (Wu and Kanamori, 2005; Wu *et al.*, 2007b) that are closest to epicenter; one hard-rock station (Wu *et al.*, 2006); all, but at least three, stations recorded given earthquake (Wu *et al.*, 2007a). The possible reasons that stations observe different values of the parameters include factors that may change frequency content of the initial P-wave motion. For example, Yamada and Ide (2008) analyzed influence of complex source rupture and S-wave arrival; the effect of long-period near-field displacement has been studied by Wu and Kanamori (2008b) and Yamada and Mori (2009). It seems that consideration of site effect is also important and Lockman and Allen (2005) suggested using in dense networks station-specific scaling relation. The rupture directivity effect that may affect the measurements at close-in stations should be also mentioned here.

The first three or four seconds after the onset of the P-wave were used in all studies, which is considered to be enough for a good measure of the magnitude of the event up to M_W 7 and even more (Kanamori, 2005; Wu *et al.*, 2007a). The choice of three seconds P-wate time window is based on the numerical experiment for moment-rate functions of Sato and Hirasawa (1973) model. It is reasonably to suggest that the longer the duration, the more reliable is the magnitude estimate. However for the purpose of early warning the speed is important. The longer the time window the later the warning time. However, no comprehensive analysis for determining optimal length of time window for accurate and rapid magnitude estimate has been made based on observational data.

As can be seen, in the studies referred above, the values of characteristic period were determined using the techniques, which are not completely consistent. The authors concentrated their attention on relationship $\tau_C = f(M_W)$ without proper consideration of problems related to the reverse relationship $M_W = f(\tau_C)$. No attempt has been made also to study the influence of characteristics of the input datasets.

In this work we analyzed the performance of the scaling relation between characteristic period and moment magnitude in respect of (a) characteristics of datasets accumulated in various seismic regions and (b) variation of initial conditions applying for determination of the parameter τ_C (the length of the P-wave window and the number of stations used). We selected two regions with developed networks of strong-motion stations that accumulate large datasets of acceleration records, namely: Japan and Taiwan. We used the larger dataset than that used by Wu and Kanamori (2008a) expanding the magnitude range toward lower magnitudes.

The data were divided into two subsets regarding focal depth, namely: shallow events, depth less 30 km, and deep events, depth more than 30 km. For every earthquake we consider various numbers of the closest to epicenter strong-motion stations (up to 10) and various lengths of time window after arrival of the P-wave (from 1 second to 8 seconds). Also for Japanese data we compared the results obtained from the surface records (K-NET and KiK-net networks) and the records obtained in deep boreholes on bedrock approximately 100–3000 m below the surface (KiK-net network).

2. The Data

The Japanese dataset includes records from 49 events (magnitude range M_W 4.4–7.4) and the Taiwanese dataset includes records from 61 events (magnitude range M_W 4.3–7.6). Earthquakes in Japan were recorded by the K-NET and the KiK-net nation-wide networks (over 1.800 strong motion instruments), which have been deployed across Japan by the National Research Institute for Earth Science and Disaster Prevention (NIED). The K-NET strong motion network is distributed uniformly along the Japanese Islands at intervals of about 25 km (Kinoshita, 1998). Three-component accelerometers are installed at free field of populated area such as at school yard in elementary and junior high schools and garden of city office. The KiK-net stations are usually placed in quiet place in suburbs, since it is a complex facility with the Hi-net high-gain seismic network. The three-component accelerometers are installed in boreholes of about 100–3000 m deep as well as at free-field. The KiK-net is also distributed across Japan at uniform interval of about 25 km, some of them are placed in the city but most of them are placed at a hill side (Aoi *et al.*, 2000). There are no KiK-net stations at large sedimentary basins such as in the center of Tokyo and Osaka, because the thickness of sediment is too deep. The acceleration records have been obtained from sites http://www.kik.bosai.go.jp/ and http://www.k-net.bosai.go.jp/.

Earthquakes in Taiwan were recorded by the Taiwan Strong Motion seismic network implemented by the Seismological Observation Center of the Central Weather Bureau (CWB), Taiwan, R.O.C. More than 650 digital free field strong-motion instruments are installed in this network. The majority of TSMIP stations are installed on deep plain (Western plain) and alluvium basins (Taipei and Ilan basins).

The criteria for selection of earthquakes and records to be proceeded and analyzed are the following. First, the earthquake should be recorded by least three close-in stations. Second, the record should contain at least 5 seconds of pre-event time window. The visual inspection was used for selection of the moment of P-wave arrival. The high-pass filter with cut-off frequency of 0.075 Hz has been applied after double integration of the accelerograms to remove the drift of the displacement records.

We used moment magnitude in our analysis. Therefore, for the cases when magnitude data contain information only about another type of magnitude, the correspondent estimations were obtained from Harvard seismic catalogue http://www.seismology.harvard.edu/. Also, for Taiwanese data, the regional relationships between seismic moment and local magnitude (Li and Chiu, 1989; Wang *et al.*, 1989) were used.

3. Characteristic Period Versus Moment Magnitude

Here we analyze the relationship between the characteristic period τ_C and the moment magnitude M_W. For every earthquake we consider various numbers of the closest to epicenter strong-motion stations (up to 10 stations) and various time intervals from the P-wave arrival (from 1 second to 8 seconds). The event averages, hereinafter referred to as τ_C, were calculated as $\tau_C = \sum_{i=1}^{n} \tau_{C,i}/n$, where n is the number of used close-in stations.

The parameters of a function that relate two variables usually are estimated using the least squares technique. In ordinary least squares (OLS), the independent (predictor) variables (X) is assumed to be measured without error and all of the errors are in the dependent (response) variables (Y). The OLS procedure, or OLS ($Y|X$), minimizes the sum of the squared deviation (or residual sum of squares) from the observations. The deviation geometrically is the vertical distance between the fitted line and actual y-values.

When analyzing regression of dependent variable τ_C on independent variable M_W, we used the values of seismic moment that were determined independently and we can accept the requirement of a non-error predictor M_W. Otherwise, we would meet so-called error-in-variables problem, when the model errors are distributed over the predictor and the response variables. The technique, which is called the total least squares (TLS) method or orthogonal regression,

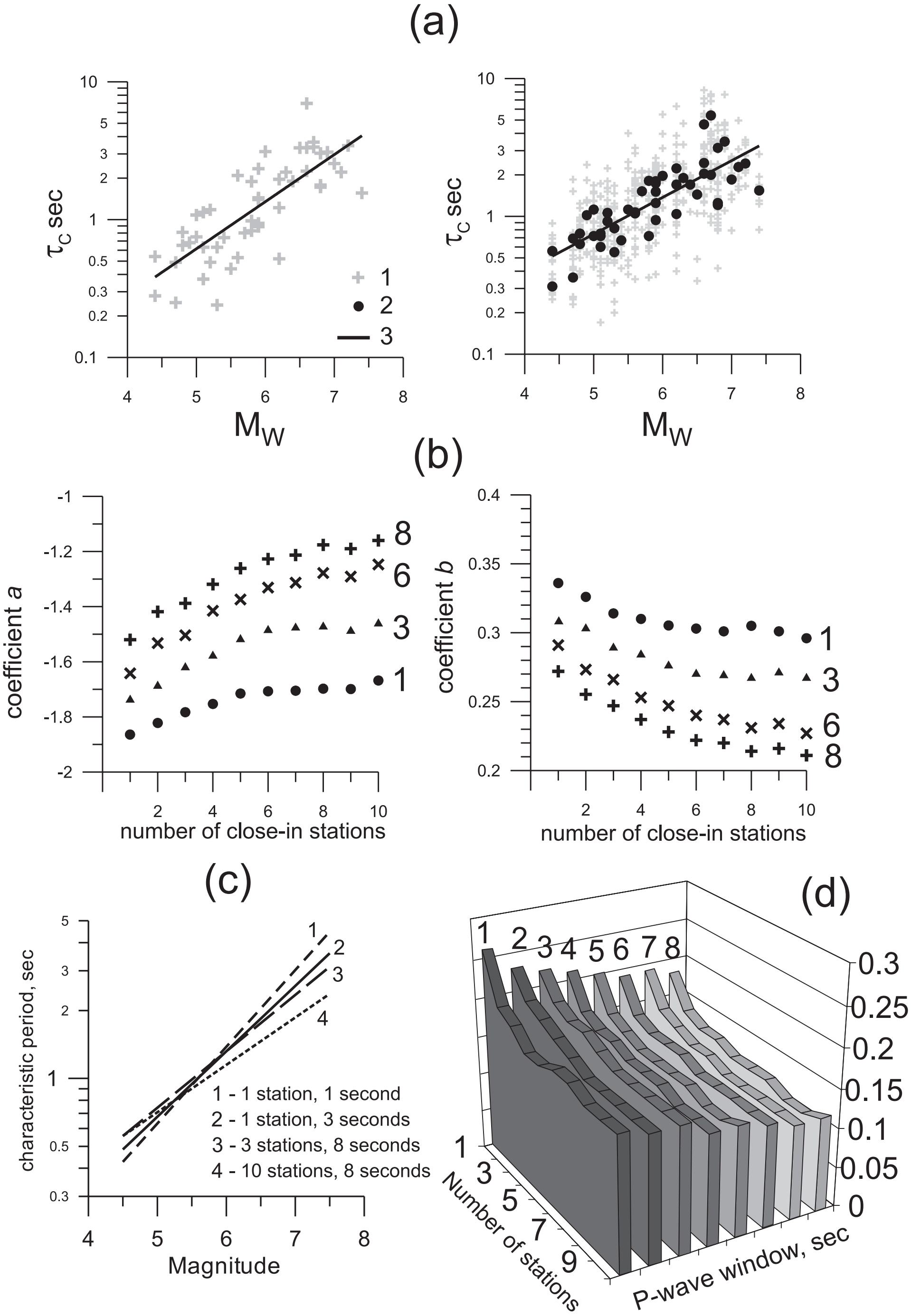

Fig. 1. Scaling relation between the moment magnitude M_W and the characteristic period τ_C, earthquakes in Japan, all data. (a) Initial data and linear relationships $\log_{10}\tau_C = a + bM_W$ obtained when using 1 closest station for every considered earthquake and when using the event averages from 10 close-in stations; first 3 seconds after P-wave arrival. 1—particular observations, 2—events averages, 3—linear regression. (b) Distribution of coefficients of the linear relationship versus the number of close-in stations; different symbols denote various lengths of the P-wave window (1, 3, 6, and 8 seconds). (c) Examples of linear relationships for different numbers of the used stations and lengths of the P-wave window. (d) Distribution of the standard errors of regression versus the number of stations and the length of P-wave window.

requires knowledge of the error variance ratio $\eta = \sigma_R^2/\sigma_P^2$, where σ_R^2 is the error in the response variable and σ_P^2 is the error in the predictor. Moreover, the errors should be relatively similar in magnitude that obviously could not be assumed in the considered case.

Let us, for simplicity, consider a linear regression as $\log_{10}\tau_C = a + bM_W \pm \sigma$. The estimator of regression residuals σ^2, i.e. difference between the observed value and the value calculated by the model, is obtained as $\sigma^2 = \sum(\log_{10}\tau_C - a - bM_W)^2/(n-2)$, where n is the number of observations. The standard error about regression line or standard error of regression σ is obtained as $\sigma = \sqrt{\sigma^2}$.

First we analyzed generalized relationship $\tau_C = f(M_W)$ using the data obtained by K-NET and KiK-net networks in Japan and supposed that initially we have no information about earthquake depth. This is a usual situation during a few seconds after earthquake occurrence. Figure 1(a) shows examples of linear relationships for the Japanese data

Table 1. Ranges of the standard errors of regressions $\log_{10}\tau_C = a + bM_W$ for the boundary cases in the considered range of number N of averaged data (stations) and length L of P-wave window, namely: 1 station—1 second (N1L1); 1 station—8 seconds (N1L8); 10 stations—1 second (N10L1); 10 stations—8 seconds (N10L8).

	All events		Shallow events		Deep events	
Earthquakes in Japan						
All data	0.26(N1L1)	0.18(N1L8)	0.25	0.14	0.25	0.17
	0.17(N10L)	0.12(N10L8)	0.15	0.09	0.19	0.14
K-NET	0.27	0.17	0.25	0.12	0.25	0.17
	0.16	0.13	0.14	0.08	0.16	0.13
KiK-net, surface	0.22	0.15	0.22	0.11	0.25	0.21
	0.18	0.12	0.13	0.08	0.19	0.14
KiK-net, borehole	0.22	0.13	0.22	0.11	0.20	0.13
	0.15	0.09	0.14	0.08	0.17	0.12
Earthquakes in Japan and Taiwan						
All data	0.24	0.19	0.24	0.17	0.26	0.21
	0.17	0.13	0.17	0.12	0.19	0.13

(all earthquakes, 3-seconds P-wave window), which are the follows

$$\begin{aligned} &\log_{10}\tau_C = 0.342M_W - 1.92 \pm 0.223, \\ &\quad \text{one closest station} \\ &\log_{10}\tau_C = 0.267M_W - 1.462 \pm 0.159, \\ &\quad \text{event averages from 10 closest stations} \end{aligned} \qquad (2)$$

Figure 1(b) shows the coefficients a and b of the linear relationships for various numbers of the close-in stations and various lengths of the P-wave window. Distribution of standard errors of regressions σ for the considered ranges of the P-wave windows and the number of the close-in stations is shown in Fig. 1(d). The scaling relation between the characteristic period τ_C and the moment magnitude M_W provides a smaller error of estimation of τ_C given the magnitude when (a) the event averages of individual estimations are used and (b) the number N of averaged values (stations), as well as the length L of the P-wave window, is increased. Table 1 lists the ranges of the standard errors of regressions $\tau_C = f(M_W)$ for particular (boundary) N-L pairs, namely: 1 station—1 second (N1L1); 1 station—8 seconds (N1L8); 10 stations—1 second (N10L1); 10 stations—8 seconds (N10L8).

However, the increase of the number of the closest stations and the length of the P-wave window also leads to a "smoothing" of the relationship $\tau_C = f(M_W)$, namely: the slope (coefficient b) becomes smaller and the intercept (coefficient a) becomes larger (Fig. 1(b)). This means, for example, that the values of characteristic period, which are evaluated using the relatively long P-wave windows, in general becomes larger for small earthquakes and smaller for larger earthquakes than that, which are estimated using the shorter P-wave window (see Fig. 1(c)).

To analyze the phenomenon let us consider also some recent results obtained for another parameter, which also reflects frequency content of the first few seconds after the P-wave arrival, so-called predominant period τ_P. The predominant period τ_P is calculated recursively as a function of time using vertical component of ground motion (Nakamura, 1998). The maximum value $\tau_P^{\max}$ of the function $\tau_P(t)$ is determined within a time window starting from the P-wave arrival. Yamada and Ide (2008) analyzed influence of the length of the P-wave window on the predominant period ($\tau_P^{\max}$)—moment magnitude relation based on numerical simulations. They showed that the parameter $\tau_P^{\max}$ has upper and lower limits. The upper limit depends on the length of the P-wave window, i.e. the longer the window, the higher the upper limit of $\tau_P^{\max}$. At the same time the complexity of waveform due to the effect of complex source and due to the contamination of later S-wave in the P-wave window, on one hand, can prevent values of $\tau_P^{\max}$ from becoming greater for large earthquakes and, on other hand, leads to a large scatter of the $\tau_P^{\max}$ values.

The parameter $\tau_P^{\max}$ shows dominant period of ground motion within certain time window and the parameter τ_C reflects average period within the time window. Despite of the different definition and measuring techniques used for calculation of these two parameters, the effect of complex source and the influence of S-wave, which were observed for predominant period, may explain the above mentioned "smoothing" of the relationship $\tau_C = f(M_W)$ with the increase of the length of the P-wave window.

The effect of so-called "near-field" term (Wu and Kanamori, 2008b; Yamada and Mori, 2009) may be also considered as a reason of the steep slope (large coefficient b) in the linear relation $\tau_C = f(M_W)$ obtained in our study for the small number of stations (one or two stations) and for the short P-wave windows (one or two seconds) (Fig. 1(c), line 1). The use of data from a single station or from two stations implies considerations of the closest to epicenter records. The records may include long-period components and static displacement, so-called "near-field" term, and relatively large values of characteristic period are obtained in this case. The phenomenon of "steep slope" should be strongly pronounced for shallow earthquakes, and we confirmed it in the next stage of our analysis.

Let us consider shallow (focal depth of 30 km or less) and deep (focal depth more than 30 km) earthquakes in Japan separately. Some deep earthquakes in Japan are characterized by high-frequency ground motion because the subducting plate is an efficient waveguide of very high-frequency signals (e.g. Furumura and Kennett, 2005; Kanno *et al.*, 2006). Large focal depth also implies relatively

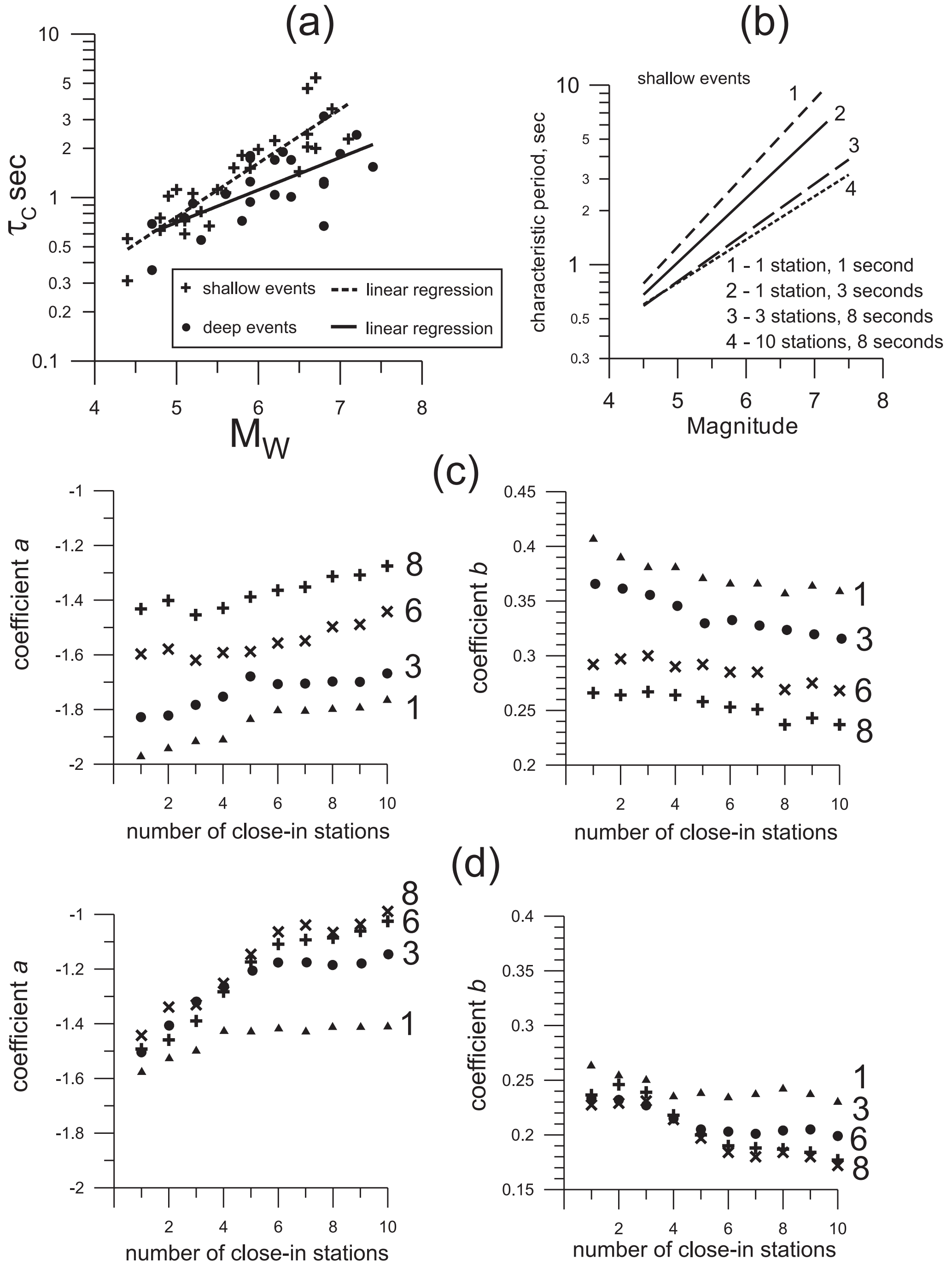

Fig. 2. Scaling relation between the moment magnitude M_W and the characteristic period τ_C, earthquakes in Japan, shallow and deep events. (a) Linear relationships $\log_{10}\tau_C = a + bM_W$. (b) Examples of linear relationships for different numbers of the used stations and lengths of the P-wave window. (c, d) Distribution of coefficients of the linear relationship versus the number of close-in stations; different symbols denote various lengths of the P-wave window (1, 3, 6, and 8 seconds); shallow earthquakes (c), deep earthquakes (d).

large hypocentral distance to the closest station, therefore the influence of propagation path could not be neglected. Distribution of τ_C values (event averages from 10 close-in stations) versus magnitude for shallow and deep events is shown in Fig. 2(a) together with correspondent regression lines. The correspondent regression equations are the following

$$\begin{aligned} \log_{10}\tau_C &= 0.329M_W - 1.766 \pm 0.135, \text{ shallow events} \\ \log_{10}\tau_C &= 0.199M_W - 1.146 \pm 0.167, \text{ deep events} \end{aligned} \quad (3)$$

Distribution of the coefficients a and b of the linear relationships versus the number of close-in stations and the length of P-wave window is shown in Fig. 2(c) and 2(d) for shallow and deep earthquakes correspondingly. For shallow earthquakes, the coefficients, showing the dependence on the length of P-window and on the number of close-in stations, reveal the importance of possible influence of complex source and S-wave (see discussion above). Note a high discrepancy between the relations $\tau_C = f(M_W)$, which were evaluated for shallow earthquakes using the data from various number of stations and various lengths of the P-

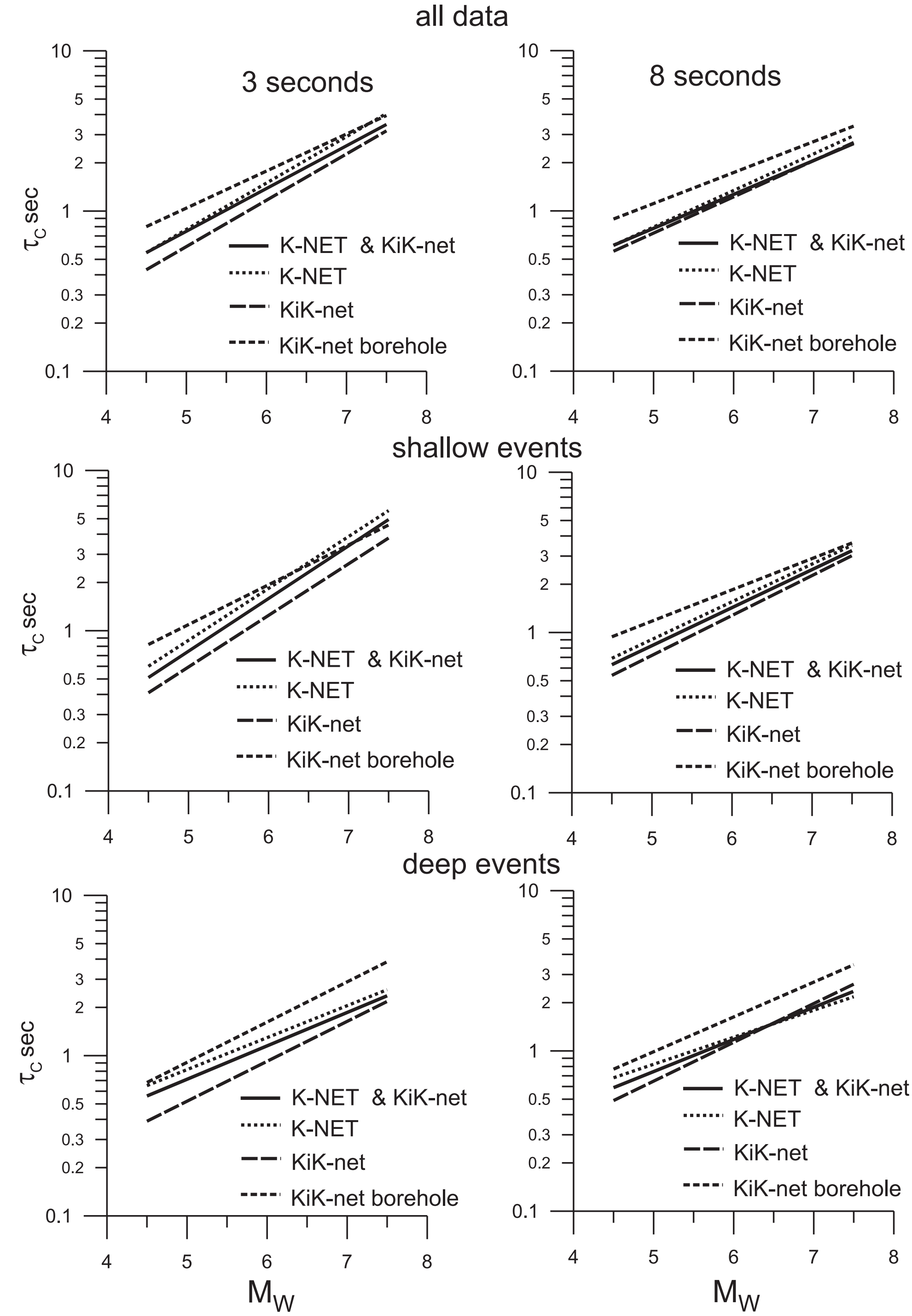

Fig. 3. Linear relationships $\log_{10} \tau_C = a + bM_W$ obtained for earthquakes in Japan using the considered datasets. The event averages from 10 close-in stations were used.

wave window (Fig. 2(b)). It seems that the near-field term is predominant in our estimations of characteristic period for large shallow earthquakes, when using a single and closest to epicenter station of the dense strong-motion networks in Japan. The influence of long period components is being reduced when a longer time window is used and when the additional data from stations located at relatively large distances from epicenter are considered.

For deep earthquakes, the number of the close-in stations is more important than the length of P-window (at least for duration more than 3 seconds), which may be considered as an evidence of the influence of whole propagation path including local site effect. It seems that deep events are characterized by the smaller values of characteristic period than shallow events, at least for magnitudes more than 5.5–6.0 (Fig. 2(a)). This phenomenon, besides the influence of the near-field term, may be also explained by a nature of rupture process during large deep earthquakes, e.g. by a generally higher stress drop than that for large shallow events.

Analysis of data obtained separately by the K-NET, KiK-net (surface) and KiK-net (borehole) networks did not reveal a prominent difference between the estimations of the parameters of scaling relation $\tau_C = f(M_W)$ (Table 1). The largest standard error of regression is observed when the data from a single close-in station and P-wave window of 1 second is used; while the data from 10 close-in stations and P-wave window of 8 seconds provide the lowest standard error of regression.

The phenomena analyzed above using three different data of the K-NET free-field stations and the KiK-net free-field and borehole stations show that the influence of factors, which disguise the characteristic period-magnitude relation changing frequency content of initial P-wave motion (e.g. influence of S-wave especially for small epicentral distances, effects of complex source and near-field term), depends on variation of initial conditions applying for determination of the parameter τ_C (length of the P-wave window and the number of used stations).

To analyze, even if roughly, the influence of local site conditions on the relation $\tau_C = f(M_W)$, we consider the datasets obtained separately by the K-NET, KiK-net (surface) and KiK-net (borehole) networks. Figure 3 shows the relations evaluated using the particular considered datasets. The event averages from 10 close-in stations were used in this demonstration to show the general features of the relations obtained from various networks and their combination. The influence of local site conditions on the relationship between the characteristic period and the moment magnitude can be observed here. The values of characteristic period obtained from surface records are generally smaller than those from borehole KiK-net data. Small τ_C values imply large values of the ratio r in Eq. (1) that, in turn, suggests the larger velocity integral, or the larger ground motion amplitudes at intermediate frequencies. It seems that the phenomenon reflect the influence of site amplification in this frequency range, which becomes more prominent with the increase of length of the P-wave window. The difference between the surface and the borehole data diminishes with the increase of magnitude for shallow earthquakes when the source factors, especially for the close-in stations, become predominant.

Note that the results obtained for a random sample of n variables (observations) from a statistical population of concern. Population refers to a set of potential measurements, including both the case actually observed and those that are potentially observable. Since estimates of regression parameters (coefficients a and b, and squared residuals σ^2) are obtained from a random sample, their values are not fixed, and the estimates are also random variables. If had taken different random samples by chance and calculated a and b we wouldn't always find the same values. In the considered case, when using a certain number of the triggered close-in stations, different stations may be selected in different samples. In every particular sampling, the close-in station, which has been selected in the previous sampling for a given earthquake, may be considered as being not triggered. To analyze statistical characteristics of the possible regression coefficients a and b that we could calculate based on many samples, a set of samples should be created, say through Monte-Carlo simulation, selecting possible combinations from the available data.

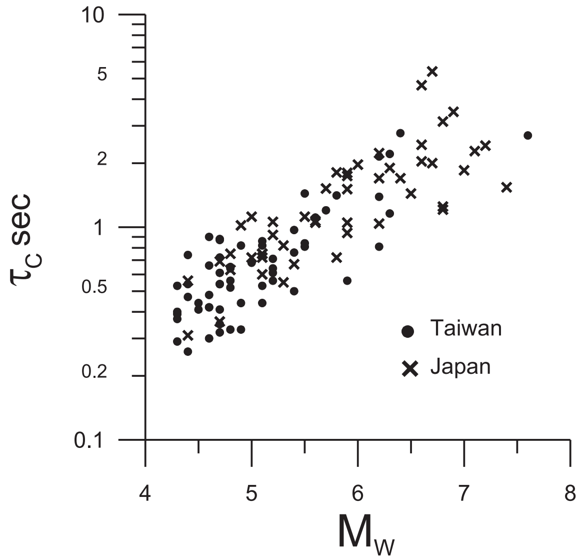

Fig. 4. Distribution of the values of characteristic period τ_C versus moment magnitude M_W; the event averages from 10 close-in stations; first 3 seconds after the P-wave arrival.

Before considering jointly the data obtained from earthquakes in Japan and in Taiwan, we need to test if these independent samples come from the same population or from different populations. The t-test showed that the null hypothesis (the mean of one group equals the mean of another group) could not be rejected with probability 99% for the Japan-Taiwan pair of datasets. Distribution of τ_C values versus moment magnitude (event averages from 10 close-in stations) for Japan and Taiwan is shown in Fig. 4. The correspondent equations evaluated for the joint database are the following

$$\begin{aligned}
\log_{10}\tau_C &= 0.293M_W - 1.644 \pm 0.152, \text{ all events}\\
\log_{10}\tau_C &= 0.343M_W - 1.896 \pm 0.131, \text{ shallow events}\\
\log_{10}\tau_C &= 0.224M_W - 1.332 \pm 0.165, \text{ deep events}
\end{aligned} \tag{4}$$

As can be seen from Table 1, both datasets provide approximately the same characteristics of scaling relation $\tau_C = f(M_W)$.

4. Moment Magnitude Versus Characteristic Period

Till now we considered an observation τ_C, which was related to a true value M_W, and we evaluated a linear relationship $\tau_C = f(M_W)$. In Earthquake Early Warning we would like to estimate the true value M_W, given an observed τ_C. This so-called "calibration procedure" is neither a logically nor a statistically trivial problem. Strictly speaking, we cannot solve this problem by obtaining the regression M_W on τ_C using normal OLS procedure, i.e. minimizing the vertical distance between the fitted line and actual M_W values. The solution of the problem is application of so-called reverse or inverse regression (e.g. Maddala, 1992; Rao *et al.*, 2007) to estimate regression line $M_W = a_R + b_R \log_{10}(\tau_C) + \gamma$, where γ is the random error. The variable on vertical axis (M_W) is presumed to be measured without error, whereas

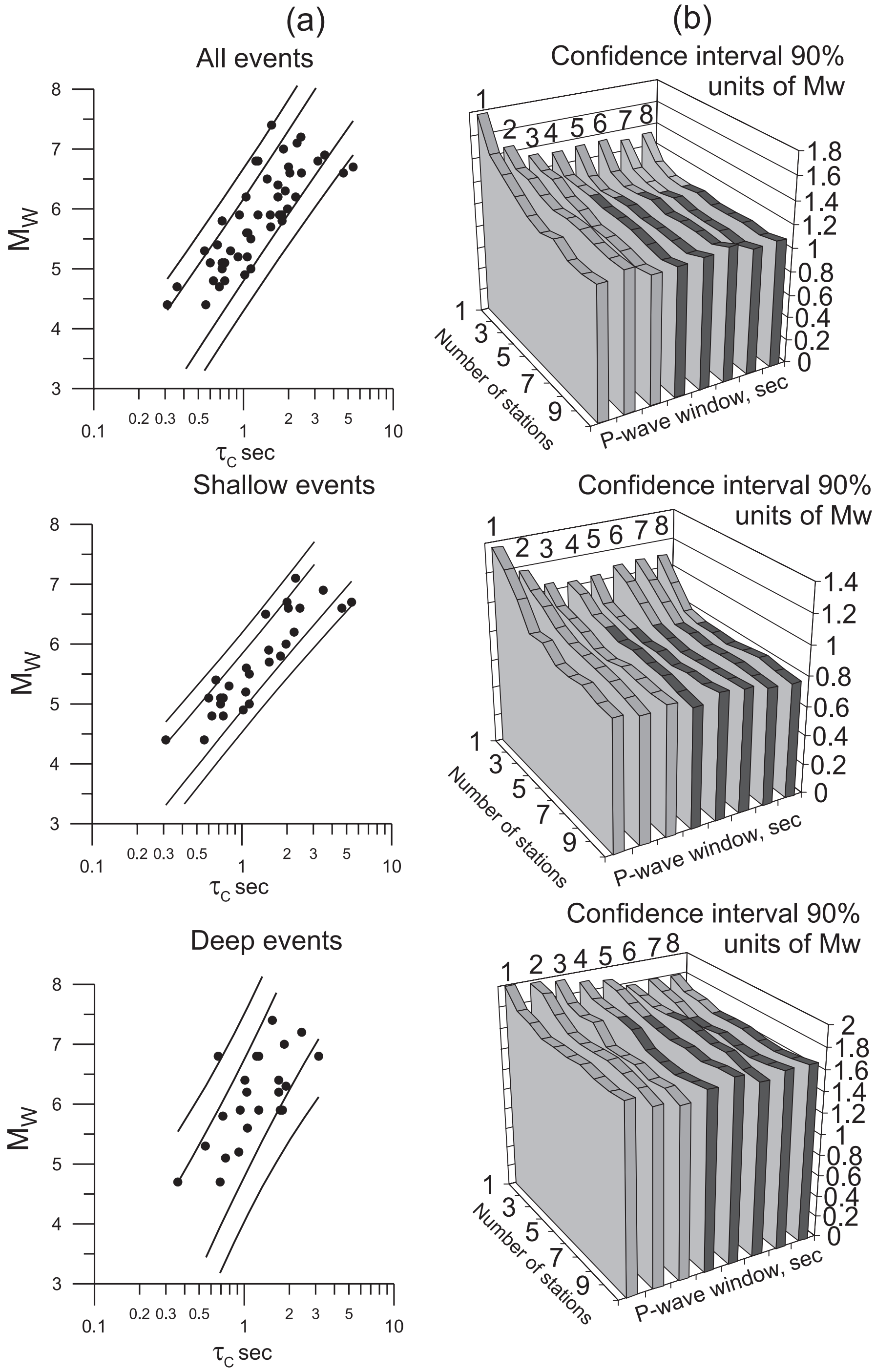

Fig. 5. Relationship between the moment magnitude M_W and the characteristic period, τ_C earthquakes in Japan. (a) Confidence intervals for 90% and 50% probability, the events averages from 10 close-in stations, first 3 seconds after P-wave arrival. (b) Distribution of the half-widths of confidence intervals (90% probability) versus the number of close-in stations and the length of the P-wave window. The dark gray area shows the range within which the particular values were averaged to provide the generalized estimations shown in Table 2.

the variable on the horizontal axis (τ_C) is presumed to be stochastic. In this case, the regression line should now be fit by minimizing the squared horizontal distances between the data and the line.

The obtained reverse regression line $M_W = f(\tau_C)$ produces an estimator $\hat{M}_W$ of true value M_W given an observed τ_C. The statistical properties of the estimator for normally distributed y-values (τ_C in our case) is widely discussed in statistical literature (see, for example, review given by Osborne, 1991). Here, following Lieftinck-Koeijers (1988), we only mention that (1) the estimator $\hat{M}_W$ in general can not be considered as the mean of possible M_W values and (2) classical calculation of the regression residuals in vertical direction, i.e. the departures of each point M_{Wi} from a line as $\sigma_R^2 = \sum(M_{Wi} - a_R - b_R\tau_{Ci})^2/(n-2)$, could not be used for assessment of accuracy of M_W estimations. The expressions for mean and mean squared errors of the estimator can be found in Lieftinck-Koeijers (1988).

Instead of estimating the parameter M_W by a single value using the relation $M_W = f(\tau_C)$, the estimation of confidence intervals of M_W for a given τ_C should be used. Confidence intervals (CI) are used to indicate the reliability of an estimate (e.g. Sprent, 1969; Graybill, 1976). How likely the interval is to contain the parameter is determined by the confidence level or confidence coefficient. Increasing the desired confidence level will widen the confidence interval. The selection of a confidence level for an interval determines the probability that the confidence interval produced will contain the true value of parameter. The width of confidence interval (CI width) for possible M_W values depends on characteristics of the correspondent direct regression $\tau_C = f(M_W)$, namely: the number n of used variables (observations), the standard error σ of regression, and coefficient b in the correspondent linear relationship. The width is smaller for the centre of x (M_W in our case) distribution ($\overline{x}$) and it increases as x moves away from $\overline{x}$. Here, when referring to the width of confidence interval, we mean the value averaged for the whole considered range of variables. The technique described by Sprent (1969, chapter 7) has been used in our analysis for estimation of confidence interval for M_W.

Figure 5(a) shows, as an example, the relation between the moment magnitude and the characteristic period, parameters of which were evaluated for the dataset that contains data from all earthquakes in Japan (shallow and deep) averaging estimations from 10 closest stations and using 3 seconds interval from the P-wave arrival. Distribution of the CI width (the half values, 90%) versus various lengths of the P-wave windows and various numbers of the close-in stations are shown in Fig. 5(b). Thus, if the value of characteristic period averaged, for example, from 10 close-in stations using 3-seconds P-wave window was estimated as 1.0 second, we can expect that the real value of magnitude would be constrained between M_W 4.3 and M_W 6.7 with probability 90% and between M_W 4.8 and M_W 6.2 with probability 50%. However, if we can assume that this particular earthquake is a shallow event (focal depth less than 30 km), the limits for expected value of magnitude are reduced, namely: M_W 4.6–M_W 6.2 for probability 90% and M_W 4.9–M_W 5.8 for probability 50%.

Comparison of Figs. 1(d) and 5(b) shows the small values of the standard errors σ of regression in relation $\tau_C = f(M_W)$ may not correspond to the small values of the CI width of the M_W estimations. In the considered cases, while the value of the standard error becomes smaller, the coefficient b becomes smaller too (see Figs. 1(b) and 2(b)), that may lead even to the increase of the correspondent CI width. Thus, it seems that the event averages from 3–4 stations and the time window of 3–4 seconds after the P-wave onset would be enough for the magnitude estimation. Lockmann and Allen (2007) studying the predominant period had come to the same conclusion that a reasonably accurate magnitude estimate can be obtained using four close-in seismic stations. However, the technique that they used for analysis of uncertainty of the magnitude estimations can not be considered as a faultless from statistical point of view.

Analysis of the joint database compiled using the dataset from Japan and Taiwan showed approximately the same confidence intervals for the magnitude estimations as that for the case of Japanese earthquakes only. Table 2 summarizes the estimations of the CI width for the considered cases.

Table 2. Averaged width of confidence intervals (90%/50%) for estimation of the moment magnitude from the observed characteristic periods. The values were calculated by averaging of the particular data obtained for at least 4 stations and at least 4 seconds of P-wave window (see Fig. 5).

	All events	Shallow events	Deep events
Earthquakes in Japan			
All data	1.14/0.66	0.82/0.46	1.47/0.85
K-NET	1.09/0.63	0.76/0.43	1.38/0.77
KiK-net, surface	1.09/0.63	0.76/0.43	1.26/0.70
KiK-net, borehole	1.08/0.62	0.75/0.43	1.31/0.73
Earthquakes in Japan and Taiwan			
All data	1.02/0.59	0.78/0.45	1.50/0.89

5. Conclusion

We investigated the performance of scaling relation between the characteristic period and the moment magnitude, which have been obtained and proposed recently for Earthquake Early Warning systems, in respect of (a) characteristics of datasets accumulated in various seismic regions and (b) variation of initial conditions applying for determination of the characteristic period (the length of P-wave window and the number of used stations). For this purpose we analyzed ground motion database containing records that were obtained from 110 earthquakes (magnitude range 4.3–7.6) occurred in Japan and Taiwan.

We have found that the scaling relationship between the characteristic period τ_C and the moment magnitude M_W in general allows predicting earthquake magnitude within ± 1.0 units for 90% and ± 0.60 units for 50% confidence limits, when (a) using the time interval of at least 3–4 seconds from P-wave arrival and (b) averaging the data from at least 3–4 close-in stations. However, if we can assume that the considered particular earthquake is a shallow event (focal depth less than 30 km), the limits for expected value of magnitude are reduced, namely: ± 0.78 units for 90% and ± 0.45 units for 50% confidence limits. Therefore, the rapid estimation of the focal depth is necessary in EWW systems. It seems also that for the networks located mostly on rock sites, the magnitudes of small earthquakes ($M_W < 5.0$) may be underestimated when using the generalized relationship (Eq. (4)). Note that our results for shallow earthquakes are close to those reported by Wu and Kanamori (2008a) (see Table 3). Table 4 shows the values of confidence intervals for estimations of moment magnitude based on the scaling relation between τ_C and M_W, which was obtained from the generalized dataset (earthquakes in Japan and Taiwan).

Bearing in mind the necessity to consider the confidence interval instead of a single value of M_W, every attempt to reduce the errors in the direct relationship $\tau_C = f(M_W)$ (e.g. Shieh *et al.*, 2008) should be accompanied by analysis of the confidence intervals for correspondent reverse estimations of $\hat{M}_W$. The problem of joint consideration of several input parameters, for example both period-dependent pa-

Table 3. Characteristics of initial datasets and the scaling relations between the characteristic period (τ_C) and the moment magnitude (M_W).

	Wu and Kanamori (2008a)	This study
	Number of used earthquakes and magnitude range	
Japan	17 (6.0–8.0)	49 (4.4–7.4)
Taiwan	11 (6.2–7.6)	60 (4.3–7.6)
North America	26 (4.1–7.4)	—
Regression* $\log_{10}(\tau_C) = a + bM_W$	$\log_{10}(\tau_C) = 0.296M_W - 1.462 \pm 0.122$	$\log_{10}(\tau_C) = 0.286M_W - 1.632 \pm 0.148$
Reverse regression* $M_W = a + b\log_{10}(\tau_C)$	$M_W = 5.787 + 3.373\log_{10}(\tau_C)$	$M_W = 5.644 + 3.49\log_{10}(\tau_C)$

*The results were obtained for the dataset, which contains only shallow earthquakes (event averages from 6 close-in stations, P-wave window of 3 seconds). See in text the notes regarding the accuracy of estimation of the moment magnitude based on the observed τ_C.

Table 4. Confidence limits for estimations of the moment magnitude $\hat{M}_W$ based on observations of the characteristic period τ_C, which were averaged from at lest 4 stations using at least 4 seconds of the P-wave window (earthquakes in Japan and Taiwan).

τ_C	−90%	−50%	$\hat{M}_W$	50%	90%
	General data				
0.36	2.75	3.25	3.96	4.66	5.17
0.45	3.14	3.64	4.33	5.03	5.53
0.57	3.53	4.03	4.71	5.39	5.89
0.72	3.92	4.41	5.09	5.77	6.26
0.91	4.30	4.79	5.47	6.14	6.63
1.15	4.68	5.17	5.84	6.52	7.01
1.45	5.05	5.55	6.22	6.90	7.40
1.83	5.41	5.92	6.60	7.28	7.79
2.30	5.77	6.29	6.98	7.67	—
2.90	6.13	6.65	7.35	—	—
3.66	6.48	7.01	7.73	—	—
	Shallow earthquakes				
0.37	3.21	3.61	4.14	4.67	5.07
0.47	3.54	3.93	4.46	4.98	5.37
0.59	3.87	4.26	4.77	5.29	5.67
0.74	4.19	4.58	5.09	5.60	5.98
0.93	4.51	4.89	5.40	5.91	6.29
1.16	4.83	5.21	5.72	6.23	6.61
1.46	5.13	5.52	6.04	6.55	6.94
1.84	5.44	5.83	6.35	6.87	7.27
2.31	5.73	6.14	6.67	7.20	7.60
2.91	6.03	6.44	6.98	7.52	—
3.66	6.32	6.75	7.30	7.85	—

rameters τ_C and τ_P^{max}, also is not a trivial task (e.g. Klepper and Leamer, 1984; Cready *et al.*, 2001). One cannot simply average the magnitude estimations $\hat{M}_W$ obtained from the reverse regressions on τ_C and on τ_P, but should consider overlapping of the correspondent confidence intervals (e.g. Merkle, 1983).

Among the future tasks for analysis of scaling relation between the characteristic period and the moment magnitude, we can mention the following: (a) increase of the database, which is used for analysis of the scaling relations, by consideration of suitable strong motion records obtained by seismic networks worldwide (e.g. Italy, Greece, New Zealand) in order to create a uniform and statistically reliable sampling, (b) comprehensive analysis of the possible influence of rupture effects based on numerical modeling and observed data; (c) application and testing of the technique for the areas with a lack of strong motion data but with rapidly developing seismic networks (e.g. Switzerland or Romania).

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