Seismic wavefield calculation for laterally heterogeneous earth models—II. The influence of upper mantle heterogeneity

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SUMMARY

There is increasing evidence that the Earth's mantle is laterally heterogeneous on a broad range of scales, but the character of smaller-scale heterogeneity has to be deduced indirectly. The aim of the present paper is to examine the influence of a variety of stochastic representations of heterogeneity on seismic wave behaviour to help constrain the nature of the variations in seismic properties in the upper mantle. For each of the models, the seismic wavefield is simulated using a pseudospectral method in a 2-D cylindrical coordinate system. The presence of stochastic heterogeneity is particularly important for those parts of the seismic wavefield where a significant portion of the propagation path in the upper mantle is close to horizontal, such as the PP and SS phases, and fundamental-mode and higher-mode surface waves. The effects are noticeable traveltime anomalies and waveform changes for the body waves (particularly associated with phase triplications), and significant phase shifts for Rayleigh waves. A variety of styles of stochastic heterogeneity models are compared for the same source and station configurations using wavefield snapshots and the character of the calculated seismograms. The influence of heterogeneity on body waves and on longer-period Rayleigh waves increases as the scale length increases compared to the wavelength of the seismic waves. The aspect ratio of the heterogeneity has a pronounced effect on the coherence and amplitude of traveltime fluctuations and waveform changes across stations at the surface, which depend on the structures encountered along the propagation paths to the specific receivers. The effect of nearly isotropic heterogeneity is to induce small, short-scale variations in traveltime fluctuations and waveform changes. As the heterogeneity becomes more 'plate-like' the fluctuations are on a broader scale and of larger amplitude because the individual patches of heterogeneity have a stronger influence. The effects of broad-scale and stochastic heterogeneity are compared for a model built from a slice through a tomographic model derived from delay-time inversion for the Himalayan region. As would be expected the influence of the deterministic heterogeneity derived from the tomography study has the result of introducing systematic traveltime variations for body waves and noticeable phase shifts for surface waves when compared with the results for the background reference model. The addition of a moderate level of small-scale stochastic heterogeneity, which could not be resolved in the tomography study, has a limited effect on the seismic wavefield at longer periods but is much more significant for periods of less than 4 s when the heterogeneity scale is of the order of 40 km.

Key words: pseudospectral method, random heterogeneities, tomography, upper mantle.

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1 INTRODUCTION

Much effort has been expended to delineate the heterogeneous structure in the Earth's mantle with the aim of obtaining information on dynamic processes such as mantle convection. For example, Woodhouse & Dziewonski (1984) have mapped large-scale heterogeneities of P- and S-wave velocity structure in the upper mantle, expanding mantle heterogeneities in spherical harmonics with a horizontal resolution of the order of 2000 km using the waveforms of very long-period seismic waves. In contrast, Inoue et al. (1990) undertook relatively high-resolution seismic tomography using *P*-wave traveltime data from the International Seismology Centre catalogue and claimed that there are extensive small-scale perturbations in the whole mantle. The presence of such relatively small-scale heterogeneities in the mantle has been confirmed by further traveltime tomography for both P and S wave speeds (e.g. Pulliam et al. 1993; Kennett et al. 1998; Vasco & Johnson 1998). Further evidence for smaller-scale heterogeneity in the mantle has come from regional tomography studies (e.g. Zielhuis & Nolet 1994; Widiyantoro & van der Hilst 1997) and stochastic analysis of global traveltimes (Gudmundsson et al. 1990; Davies et al. 1992).

The models of seismic velocity structure derived from global and regional studies have shown that larger-scale features in the three-dimensional models correlate quite well with surface tectonics such as the presence of continental shields and subducting plates. The amplitude of heterogeneity diminishes as the sampling point moves away from the surface of the Earth, with a minimum in the lower mantle, before rising again as the core-mantle boundary is approached.

For the outer portions of the Earth, the results of global and regional studies are compatible for spatial wavelengths down to about 1000 km. Although the trends are similar, models from regional tomography tend to have somewhat higherpower spectral density at shorter wavelengths that cannot be resolved in global studies (Chevrot et al. 1998a,b). Anomalies in the traveltime and amplitude of short-period body waves recorded at seismic arrays suggests the existence of heterogeneities with smaller scales-less than 300 km-in the uppermost mantle (Kennett 1987; Kennett & Bowman 1990). Furthermore, the characteristics of observed seismic waveforms for distances of less than 30° , notably the nature of P and S coda, clearly indicate the presence of significant scattering in the crust and lithosphere (see e.g. Aki 1981; Kennett 1985, 1987); this scattering must be due to relatively small-scale heterogeneity.

Comparatively few studies have attempted to characterize the nature of the heterogeneity field. For the crust and uppermost mantle, Wu & Flatté (1990) have used data at seismic arrays to derive a model that combines two styles and scales of heterogeneity with overlapping depth intervals. The models comprise isotropic scatterers with small-scale lengths extending to 200 km depth through the whole lithosphere and a second set with broader scales distributed from the lower crust through to the asthenosphere. In contrast, work on the uppermost mantle using observations from longrange refraction profiles has favoured heterogeneity with features that are more plate-like (e.g. Fuchs & Vinnik 1982). Based on observations at greater distances, Kennett & Bowman (1990) considered heterogeneity models with an aspect ratio of about 4 between the horizontal and vertical scales. Recent tomographic models for the upper mantle (e.g. Widiyantoro & van der Hilst 1997) use cellular representations down to 100 km or less but are not able to resolve small-scale variations.

The aim of the present paper is to examine the influence of different styles and scale lengths of heterogeneity in the upper mantle for both P and SV waves so that we can help to provide further information on the spectrum of lateral heterogeneity within the Earth. At present, direct 3-D simulation of the wavefield can be applied for long periods (e.g. Igel 1999) but it is not yet feasible at the relatively fine scales that we would like to investigate. We have therefore generated numerical simulations of the passage of the seismic wavefield through a variety of heterogeneous, anelastic models using the pseudospectral method in a 2-D cylindrical coordinate system (Furumura *et al.* 1998a, hereafter referred to as Paper I).

Our work is complementary to earlier studies of the influence of complex structure in the upper mantle. For example, Kennett & Nolet (1990) have considered the propagation of the shear wavefield through extended heterogeneity using a coupled mode procedure with modal summation to produce synthetic seismograms. Igel & Gudmundsson (1997) have simulated *SH*-wave propagation using a 2-D finite difference technique to examine frequency-dependent traveltime anomalies in a laterally heterogeneous mantle.

Direct numerical simulation allows us to follow the evolution of the seismic wavefield in a heterogeneous medium using sequential snapshots of the behaviour within the Earth, as well as synthetic seismograms for recorders placed at the surface. As a result we are able to examine the effects of different styles of heterogeneity on the major phases such as the direct P and S arrivals, multiply reflected PP and SS phases, and the fundamental-mode Rayleigh wave. We are also able to compare seismic wavefields in the presence of heterogeneity with that for a reference model with only radial variation, so that we can follow the way in which the wavefield is distorted and the consequent change in traveltimes and waveforms.

We first establish the main elements of the seismic wavefield by conducting a numerical simulation of upper mantle propagation through the reference model ak135 (Kennett *et al.* 1995), which depends only on radius. We then superimpose a variety of styles of heterogeneity on the reference model to examine their influence on the seismic wavefield.

A wide range of calculations using different styles of stochastic heterogeneity fields have been carried out. In this paper we illustrate the influence of different styles of heterogeneity on the seismic wavefield with a number of simple models with heterogeneity characterized by different scale lengths and different aspect ratios. We then consider a more realistic model of mantle heterogeneity with a composite model with different styles of stochastic heterogeneity. Finally, we employ a set of models derived from a recent high-resolution tomography study of the Himalayan region (Widiyantoro 1997) with different levels of superimposed small-scale heterogeneity in order to gain further insight into the nature of wave propagation in the upper part of the mantle.

2 THE SEISMIC WAVEFIELD IN LATERALLY HOMOGENEOUS AND HETEROGENEOUS UPPER MANTLES

2.1 PSM simulation for a shallow source

We have carried out numerical modelling of the P-SV seismic wavefield in the mantle using the pseudospectral method (PSM) in a 2-D cylindrical coordinate system, as described in Paper I. The mantle model is represented by 512×256 gridpoints using a uniform grid size 0.088° long by 7 km deep, covering a zone 45° long by about 1792 km deep. An absorbing buffer zone of width 20 gridpoints (Cerjan *et al.* 1985) surrounds the zone of interest in order to minimize artificial reflections from the edges of the segment. We use a seismic source at a depth of 60 km with a source time function that is a pseudo-delta function with a dominant period of 10 s (Herrmann 1979).

As the source lies relatively close to the surface, we modify the calculation scheme used in Paper I and utilize the 'symmetric differentiation' scheme introduced by Furumura & Takenaka (1992) to calculate vertical derivatives. This method provides an alternative approach to the incorporation of the free-surface boundary condition in the simulation of wave propagation. Stress components at the free-surface are then naturally defined (see Furumura *et al.* 1998b). The

modification of the calculation scheme means that we are able to improve the simulation of surface wave propagation along the free surface. For the study of deeper sources such as the 600 km deep source, the 'symmetric differentiation' scheme is not required because the amplitudes of surface waves from such a source are low and the oscillation noise caused by the free surface decreases rapidly as the source depth increases.

In order to demonstrate the effectiveness of the 'symmetric differentiation' scheme for the wavefield calculations with a shallow source, we compare different wavefield simulations. We contrast the seismic waveforms for a model varying only with radius calculated using the PSM in two dimensions with and without the 'symmetric differentiation' and make a comparison with direct 3-D calculations obtained from the direct solution method (DSM, Cummins *et al.* 1994). We partially compensate for the differences in geometrical spreading between the 2-D PSM simulation and the 3-D DSM approach by multiplying the DSM seismograms by a factor $\Delta^{0.5}$, where Δ is the epicentral distance.

We have used the same mantle model with a 60 km deep source and a 10 s dominant period for all three simulations. In Fig. 1, filtered seismograms calculated by the three methods are displayed for five epicentral distances of 3° , 11° , 20° , 28° and 37° from the epicentre; a low-pass filter with a cut-off frequency of 10 s has been applied to each of the records.



Figure 1. Comparison of synthetic seismograms for the IASP91 spherically symmetric earth model, calculated by the PSM without the 'symmetric differentiation' scheme (PSM +SD), and the DSM for epicentral distances of 3° , 11° , 20° , 28° and 37° from the epicentre. On each trace, a low-pass filter with a cut-off frequency of 0.1 Hz is applied. Each trace is normalized by a compensation factor for the seismic geometrical spreading of the line source (PSM) and the point source (DSM) to recover the wave amplitude at larger distances. Major phases are marked.

The implementation of the free-surface boundary in the PSM means that the boundary lies midway between two gridpoints, and therefore the seismograms at the nominal surface stations lie at half a grid below the surface (3.5 km) rather than on the free surface itself. In the DSM scheme (Cummins *et al.* 1994), the basis functions used to build the solutions are constructed to satisfy the free-surface condition directly.

For body waves the results of the PSM simulations are in good agreement with the results of the DSM simulation. However, there are more significant differences for surface waves, especially at shorter periods. The introduction of the 'symmetric differentiation' scheme gives a significant improvement in the representation of the surface waves but there are still some differences in the phase velocities and waveform from the DSM. The discrepancy between the 3-D DSM results and the 2-D PSM+SD results can be reduced by using a smaller grid spacing below the free surface or a longer-period source. However, the level of grid refinement needed to produce a close match at short periods severely increases the computational costs of each run.

The PSM augmented by 'symmetric differentiation' provides a good representation of the body wavefield and also the general character of the surface wave train. Because the aim of this work is to provide comparisons between different styles of heterogeneity models by comparison with the results for a reference earth model, the PSM + SD approach provides a convenient approach for which multiple computations can be made without excessive cost.

We note that use of the 'symmetric differentiation' scheme can sometimes cause numerical instabilities for long time spans (for example, over 2500 time steps, depending on the model).

2.2 Seismic wave propagation in a laterally homogeneous upper mantle

We first illustrate the character of the seismic wavefield calculated using the PSM + SD approach for a laterally homogeneous reference mantle model, and summarize the characteristics of major phases observed at epicentral distances less than 40° for comparison with later heterogeneous models. We have used the ak135 earth model (Kennett *et al.* 1995) as the radially stratified reference, and have supplemented the model with an anelastic attenuation (*Q*) structure derived from PREM (Dziewonski & Anderson 1981). Various styles of heterogeneities will be superimposed on this reference model, which we designate model AK.

In order to concentrate on the nature of the propagation processes for both P and S waves, we have used an isotropic seismic source composed of a combination of an explosion and a torque source with the same moment, rather than a double-couple-type source.

In Fig. 2(a) we display synthetic velocity seismograms for the vertical and angular components at surface observation points for a source at 60 km depth in the model AK. The maximum amplitude of each trace has been normalized by multiplying by the epicentral distance Δ to the station, so that each seismic phase can be seen clearly. The traveltime curves of the major *P* and *S* waves calculated from ray theory are superimposed on the seismograms. We also show in Fig. 2(b) a sequence of snapshots of the seismic wavefield in the AK model at 100 s intervals, up to the time when the *P* waves have passed through

the right-hand edge of the model. To aid visualization, the total seismic wavefield has been separated into P- and SV-wave contributions by calculating the divergence and the curl of the wavefield (see Paper I): P waves are shown in light-grey and SV waves in black.

2.2.1 Direct arrivals

In the wavefield snapshot at 100 s, we see nearly circular wave fronts for direct P and S and also their reflections from the free surface, pP and sS. The wave front of the body waves is distorted after passing through the upper mantle because of the superimposition of a number of P- and S-wave branches arising from the interaction of the wavefield with the velocity discontinuities at 410 and 660 km (as can be seen in the frame at 200 s). The synthetic seismograms at the surface stations display the interactions of the different branches of the phase triplications for epicentral distances around 20°, which lead to large arrivals and quite complex waveforms for P and S in both vertical and angular components. Beyond 30° the P and S waves return to the surface after interacting with the lower mantle with a somewhat lower amplitude. The simple velocity gradient in the lower mantle does not impose any extra complications on the P and S wave fronts (see the 200 and 300 s frames).

2.2.2 Surface reflections

The large-amplitude *P* and *S* waves arriving at the surface for epicentral distances around 20° are reflected back from the free surface and propagate again through the mantle as *PP* and *pPP* phases or *SS* and *sSS* phases (see the 400 s frame). These multiples return to the surface at epicentral distances around 40° after a second passage through the upper mantle.

2.2.3 Other body waves

The synthetic seismograms at the surface for distances beyond 25° from the epicentre show significant energy arriving between the expected times of pP and PP and sS and SSphases. These phases are mainly composed of reflections, refractions and conversions such as $S_{660}SS$, sSnSS and $S_{660}PP$ arising from the velocity discontinuities in the model.

2.2.4 Surface waves

The synthetic seismograms in Fig. 2(a) clearly display significant fundamental-mode Rayleigh waves and short-period higher-mode Rayleigh waves. The dispersion of the surface waves is clearly seen, leading to a gradually elongation of the wave train as the propagation distance increases. The energy of the fundamental Rayleigh mode is concentrated within the crust and the uppermost mantle, as can be seen in the wavefield snapshots at 500 and 600 s as the sequence of *S* arrivals following the wave front of the *sS* phase.

2.3 Seismic wave propagation in a laterally heterogeneous upper mantle

We have conducted a wide range of tests on different styles of heterogeneity models superimposed on the reference model AK. We examine the nature of the influence of the heterogeneity



Figure 2. Synthetic seismograms and wavefield snapshots from a 60 km deep shallow source of isotropic *P*- and *SV*-wave radiation in the laterally homogeneous reference mantle model AK. (a) Synthetic velocity seismograms at surface stations, reduced by a velocity of 0.083° s⁻¹. Amplitudes are multiplied by the epicentral distance of each station to correct for geometrical spreading. The theoretical traveltime curves for major *P* and *S* phases are shown by grey lines. (b) Snapshots of the *P*–*SV* wavefield. The *P*-wave contribution is shown in light grey, and *S* in black. The velocity discontinuities at 410 and 660 km are superimposed on the snapshots as thin lines.

by comparing the seismic wavefields in the presence of the heterogeneity with the results for the laterally homogeneous reference model AK.

Sets of 2-D stochastic heterogeneity models have been constructed using the following procedure:

(1) generate a set of random numbers with a Gaussian probability distribution;

(2) assign the random numbers to 2-D gridpoints in a Cartesian coordinate system and then filter in the wavenumber domain using a 2-D FFT to achieve a Von-Karman-type

correlation function (Frankel & Clayton 1986) corresponding to the desired vertical scale of the heterogeneity;

(3) stretch this perturbation model laterally to achieve a desired aspect ratio between horizontal and vertical scales, project it onto the gridpoints in a cylindrical coordinate system and then superimpose it on the reference model AK.

By this means we have constructed a set of heterogeneous models with scale lengths, a, of 40, 120 and 240 km, and varying aspect ratios, r, between the vertical and horizontal scales of 1, 2, 4 and 8. We keep almost the same area of the heterogeneities for each characteristic scale length. For each model, the vertical scale a_v and horizontal scale a_h are related

to the scale length $a (= \sqrt{a_h a_v})$ and the aspect ratio $r (= a_h/a_v)$. We use this group of models to try to characterize the effect of particular styles of heterogeneity on propagation through the upper mantle. Because of the triplications in the arrivals for the body wave phases returned from the upper mantle discontinuities, the seismograms are built up from quite complex interference patterns comprising waves traversing different parts of the heterogeneity field.

In order to make a direct comparison of the effect of heterogeneity on both the P and SV components of the wavefield, we have adjusted the perturbation models to have a maximum amplitude of 3 per cent for both P and S wave speeds. We also apply the same perturbation pattern for density structure, but



Figure 3. Random perturbation models for velocity structure in the mantle with heterogeneities with scale lengths of 40 (S4), 120 (M4) and 240 (L4) km.

with a slightly smaller maximum amplitude (2.4 per cent). We have not attempted to perturb the *Q* structure.

Because the calculations using the PSM approach are restricted to 2-D heterogeneity models, we cannot include outof-plane scattering effects, which are undoubtedly important in the real mantle. However, as demonstrated in Paper I, a comparison of the seismic wavefields between the laterally homogeneous and heterogeneous mantle models can help to provide an assessment of the influence of heterogeneity.

Because the changes produced by the presence of heterogeneity can be both complex and quite subtle we are presented with a problem in providing effective displays of the influence of the heterogeneity. Differential seismograms, obtained by subtracting the synthetic seismograms for the reference model AK from those for a particular heterogeneity model, are sensitive to changes in both the arrival times of phases and their amplitude pattern with distance, so it can be difficult to separate the different influences. We therefore make a direct comparison of the seismograms recorded at the surface stations for the reference model and different styles of heterogeneity. We also use differential wavefield snapshots calculated by subtracting the P and SVcomponents of the wavefield snapshots for a particular heterogeneity model from those for the model AK. These differential snapshots give a good indication of the wave propagation anomalies due to the velocity and density perturbations because of their sensitivity to small changes in both the traveltime and the waveform of the seismic phases.

2.3.1 Effect of scale length

Using the procedure discussed above we have generated three heterogeneous models characterized by differing scale length but with the same aspect ratio of 4 between the horizontal and vertical scale lengths. The same scale lengths are maintained throughout the mantle segment. The three models are



Figure 4. Snapshots of the seismic wavefield at 300 and 600 s in the reference model and corresponding differential wavefields for the three random heterogeneity models of Fig. 3.

illustrated in Fig. 3. S4 has the smallest scale (a=40 km); the scale length is increased for M4 (a=120 km), and L4 has relatively broad-scale variations (a=240 km). As the scale length is increased the heterogeneity becomes more organized with more pronounced regions of elevated and lowered wave speed.

In Fig. 4 we compare the differential snapshots for the three heterogeneity models at 300 and 600 s after the initiation of the source with the corresponding wavefield snapshots for the model AK. The two snapshot frames have been chosen to illustrate different aspects of the wavefield: at 300 s the P waves are returning to the surface after interaction with the upper mantle, whilst the S wave front shows clearly the influence of the upper mantle discontinuities; at 600 s the major feature is the SS wave and the following chain of higher-order S

multiples, which may alternatively be viewed as the surface wave train.

For our source with a dominant period around 10 s, the dominant wavelength of the seismic waves varies from 61 km in the crust to nearly 115 km at 1000 km depth for P waves, and 36 km in the crust to 64 km at 1000 km depth for S waves. The scale length of variation in model S4 is therefore generally shorter than the dominant wavelength for P waves but of the order of this wavelength for S waves. In both model M4 and model L4 the scale length is longer than the dominant wavelength for both P and S waves.

For the waves reaching the surface at epicentral distances beyond 10° , the width of the first Fresnel zone at the turning point exceeds 300 km for *P* waves and 220 km for *S* waves. In consequence, we can expect scattering effects or diffraction



Figure 5. Comparison of body waves (right) and expanded waveforms of direct *P* and *S* arrivals (left and centre) for the model AK and the three heterogeneity models S4, M4 and L4 at a station interval of 0.7° . A bandpass filter with a cut-off frequency of 0.02 and 0.25 Hz was applied to each trace. The *P* and *S* arrivals are plotted relative to the theoretical traveltimes for the reference model AK and the amplitude of each trace is normalized by its maximum vector amplitude (square root of the sum of the energy on the vertical and angular components) to increase the visibility of the traveltime anomalies and changes in waveforms. Each trace in the window, including many body wave phases, is multiplied by a factor of $\Delta^{0.5}$, where Δ is the epicentral distance of each station, and plotted with a reduction velocity of $0.056^{\circ} \text{ s}^{-1}$.

due to the heterogeneity to be significant in any heterogeneity models considered.

At 300 s the differential snapshots clearly indicate that body waves travelling through the upper part of the mantle are strongly affected by the mantle heterogeneity (particularly for models M4 and L4). The influence on the body waves is quite significant for S and sS phases, and the wave front of the Swaves in the heterogeneous models is segmented and becomes slightly undulatory. This is caused by interactions of a number of S-wave branches with different propagation paths due to the velocity discontinuities. Compared with the S waves in the upper mantle, the wave fronts of the S waves travelling through the upper part of the lower mantle are not so distorted and have a lower amplitude because of the comparative simplicity of the wave propagation in the simple velocity gradient. The distortion of the P wave front is rather weak, even for model S4, because its wavelength in the upper mantle (about 80 km) is relatively long compared with the scale length of the heterogeneity.

In the snapshots at 600 s, the SS phases are prominent in the differential snapshots at longer epicentral distances. These surface multiples pass twice through the heterogeneous upper mantle, and can acquire significant traveltime anomalies and waveform change during their nearly horizontal passage through the upper mantle. We see relatively small effects of the heterogeneity on the S arrivals at slightly larger epicentral distances because their main propagation paths have been in the lower mantle. The Rayleigh waves behind the wave front of the SS phase are noticeable in the differential snapshots for models M4 and L4.

As the scale length of the heterogeneities increases, the effect of the heterogeneity on both the body waves and the surface waves is more severe, and the scale of variation in the segmented wave fronts of the S waves in the upper mantle and the lower mantle seems to depend on the scale lengths of the heterogeneities. The differential snapshots have larger amplitudes for the larger-scale heterogeneity models M4 (a = 120 km) and L4 (a = 240 km) than for model S4 (a = 40 km). Such



Figure 5. (Continued.)

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effects arise from the build-up of time differences of the wave fronts due to the more organized heterogeneity.

As a means of emphasizing the variations in the wavefield induced by the different styles of heterogeneity, we display seismograms at close spacing for the different models (Fig. 5), both for the full wavefield and for the arrivals following the onset of P and S. In Fig. 5 the surface seismograms for the reference model AK and the models S4, M4 and L4 are displayed with a bandpass filter with corners at 0.02 and 0.25 Hz. The time and amplitude variations due to the heterogeneity are emphasized by plotting 40 s intervals following the theoretical traveltime for P and S for the AK model with the amplitude of each trace normalized by the square root of the maximum energy (from both vertical and angular components). The vertical component is displayed for the P window and the angular component for the S window and the window containing the full seismograms.

In the windows displaying the whole seismograms, we see small changes in the amplitude patterns of the surface multiples SS and PP and also near 20° in the branches of P and S phases (e.g. pP, sS) caused by the mantle heterogeneities. For the direct arrivals careful inspection indicates small differences in the patterns of the waveform and amplitude, even for the direct S waves observed at epicentral distances of less than 20° . The traveltime anomalies of the direct arrivals arising from the heterogeneity are significant, and the fluctuation across the stations increases as the scale length of the heterogeneity becomes larger. The effect of the time fluctuations and waveform changes is most pronounced for epicentral distances around 20° because of the modifications of the triplications associated with the velocity discontinuities in the mantle as different parts of the heterogeneous model.

We have already noted the significant effects of heterogeneity on the higher S multiples, so in Fig.6 we make a direct comparison of the S and Rayleigh wave train between the reference and heterogeneous models at 14 distances. The influence of heterogeneity arises mostly through phase shifts,



Figure 6. Comparison of surface waves for the vertical components for the model AK (thick lines) and the three heterogeneity models (thin lines). Each bandpassed trace is multiplied by a factor of $\Delta^{0.5}$, where Δ is the epicentral distance of each station. The reduction velocity is $0.0417^{\circ} \text{ s}^{-1}$.

For periods of 40 s or more the fundamental Rayleigh mode penetrates below 100 km, which is much larger than the vertical scale of the small-scale heterogeneity, so the influences of the small-scale features will average out to give a very small net effect on the phase. In contrast, higher-mode Rayleigh waves with shorter periods have amplitudes extending substantially deeper than the fundamental mode with the same period, so that phase shifts can be induced by the smaller-scale heterogeneities. In this study, the calculations are restricted to 2-D heterogeneity models, so the dominant influence is from the relative timing of arrivals. In 3-D heterogeneity models, multipathing of the surface waves will occur with consequent amplitude anomalies as well as phase shifts.

2.3.2 Effect of aspect ratio

Next we consider a group of heterogeneous models with a constant scale length, a, of 240 km and varying aspect ratios (Fig. 7). We consider model L1 with isotropic heterogeneity (i.e. an aspect ratio of 1) and L2 with an aspect ratio of 2 and thus more rapid vertical variation. The third model, L8, has an aspect



Figure 7. Random perturbation models for velocity structure in the mantle with aspect ratios of 1 (L1), 2 (L2) and 8 (L8).

ratio of 8, so the heterogeneity has a more plate-like character. A comparison of the seismograms for these three heterogeneity models is presented in Figs 8 and 9 in a comparable form to that in Figs 5 and 6. The intermediate case, L4, with an aspect ratio of 4, has already been discussed in the previous section.

As the aspect ratio of the heterogeneities increases, the fluctuation of the traveltime of the *P* and *S* arrivals changes, reflecting the character of the heterogeneity models (Fig. 8). For the heterogeneity model with large aspect ratio (model L8), the horizontal scale is large (672 km), so the traveltime anomalies can become quite large during the almost horizontal passage of the body wave fronts in the upper mantle, and the traveltime fluctuations along the wave front occur over relatively long distances. The corresponding vertical scale is small, which leads to fluctuations in traveltime over relatively short distances for the wave fronts of body phases with a relatively steep path. Significant waveform changes of sS compared with S at large epicentral distances are likely to arise from the plate-like

heterogeneity. The differences in the propagation paths of the various S-wave branches lead to substantial traveltime anomalies for model L8, and the waveforms of the S waves formed by superimposition of these branches are changed noticeably from those of the AK reference model.

For model L1, the waveforms of the S and sS phases for distances beyond 20° show little change in shape from AK but are shifted in time (Fig. 8). The large vertical scale of model L1 in the lower mantle means that the wave front for turning S waves in the lower mantle does not suffer much distortion. However, there is a distinctive high-velocity area in the centre part of the lower mantle of L1 (see Fig. 7), and thus the arrival time of body waves passing through that feature is faster than that for the reference model.

The heterogeneous structure in the uppermost mantle controls the way in which the Rayleigh wave train is affected, so the phase shifts for the Rayleigh waves differ significantly among the three heterogeneity models (Fig. 9). The effect of



Figure 8. Comparison of body waves (right) and expanded waveforms of direct *P* and *S* arrivals (left and centre) for the model AK and the three heterogeneity models L1, L2 and L8 in the same form as in Fig. 5.



Figure 8. (Continued.)

heterogeneity is most significant for model L8, especially for the longer-period portion of the fundamental Rayleigh mode and the higher Rayleigh modes. This arises because heterogeneities are stretched laterally in L8 and the lateral variation of the perturbed amplitude is quite small, with the result that there is a significant change in the average structure from that of the AK reference model.

The aspect ratio of the heterogeneity has a pronounced effect on the coherence of traveltime fluctuations and waveform change across the surface stations, depending on the structure along the paths. From Figs 8 and 9 we see that nearly isotropic heterogeneity displays small, short-scale variations in the traveltime fluctuations and waveform change for waves with near-horizontal travel paths, whereas more plate-like heterogeneity (L8) has a large, broad-scale variation. Aspect ratios of 2–4 would seem to be reasonable for a general representation of stochastic heterogeneity in the upper mantle since they are in accord with observed patterns of fluctuation.

2.3.3 Composite random model

Having gained some insight into the effect of models in which the style of heterogeneity remains constant throughout the mantle segment, we now construct an alternative style of upper mantle model in which the scale lengths and perturbation amplitudes of heterogeneities vary with depth, with the object of providing a better representation of the likely class of stochastic heterogeneity in the Earth.

From the modelling of traveltime and amplitude anomalies of short-period body waves seen on portable array experiments with variable station spacing, Kennett & Bowman (1990) have proposed a class of heterogeneous mantle models with a depthdependent scale length, in which the maximum amplitude of the perturbation model does not vary with depth through the upper mantle. However, the results of seismic tomography studies suggest that the perturbation is stronger near the surface of the Earth and decreases towards the lower mantle



Figure 9. Comparison of surface waves for the vertical components for the model AK (thick lines) and the three heterogeneity models (thin lines). Each bandpassed trace is multiplied by a factor of $\Delta^{0.5}$, where Δ is the epicentral distance of each station. The reduction velocity is 0.0417° s⁻¹.

(e.g. Gudmundsson *et al.* 1990; Vasco *et al.* 1994). We have therefore constructed a composite heterogeneity model that incorporates the relatively short horizontal scales suggested by Kennett & Bowman (1990), with a perturbation that both increases in scale length and decreases in amplitude with depth (Fig. 10). This model, designated C4, is composed of four heterogeneous subdomains with a fixed aspect ratio of 4 in the following depth ranges:

(1) 0–210 km, scale length 40 km, maximum perturbation amplitude 4 per cent;

(2) 210–410 km, scale length 60 km, maximum perturbation amplitude 3 per cent;

(3) 410–660 km, scale length 120 km, maximum perturbation amplitude 2 per cent;

(4) 660–1792 km, scale length 240 km, maximum perturbation amplitude 1 per cent.

The perturbation in density is set at 0.8 times the velocity perturbation.

In Figs 11 and 12 we compare the synthetic velocity seismograms for the reference model AK and the composite heterogeneity model C4. Once again the S waves in the upper mantle that travel nearly horizontally are most strongly affected by the strong heterogeneities in the shallow part of the model. Because the upper mantle is characterized by small-scale heterogeneities, the effects of heterogeneities on the S and SS phases and the Rayleigh waves in model C4 resemble those for the uniform small-scale heterogeneous model S4 (see S4 in Figs 5 and 6).

On the other hand, model C4 involves larger-scale heterogeneity in the lower mantle, so we might expect a significant fluctuation in traveltime and changes in waveform of the S and sS phases observed at epicentral distances over 25°, as seen in the previous experiments such as for model L8 (Fig. 8). However, the reduced amplitude of the perturbation in model C4 in the lower mantle means that the influence of the lower part of the model is much reduced.

With this composite model, we are able to draw together the results of the previous simulations to provide insight into



Figure 10. A composite random perturbation model for velocity structure in the mantle. The scale length of the heterogeneity increases and perturbation amplitude decreases with depth.



Figure 11. Comparison of body waves (right) and expanded waveforms of direct *P* and *S* arrivals (left and centre) for models AK and C4 in the same form as Fig 5.



Figure 12. Comparison of surface waves for the vertical components for model AK (thick lines) and model C4 (thin lines). Each bandpassed trace is multiplied by a factor of $\Delta^{0.5}$, where Δ is the epicentral distance of each station. The reduction velocity is 0.0417° s⁻¹.

the effects of realistic stochastic heterogeneity models. Our assumption of an aspect ratio of 4 for the heterogeneity produces reasonable results but is not a well-constrained feature of the model.

3 SEISMIC WAVE PROPAGATION IN THE HIMALAYAN REGION

The previous studies have employed heterogeneity models derived from stochastic processes with prescribed scale lengths and aspect ratios but do not include the systematic variations characteristic of the velocity variations in the mantle. We therefore consider a model derived from a recent high-resolution study using seismic tomography from traveltimes. We use a slice through the *P*-wave velocity perturbation model beneath the Himalayan region (Widiyantoro 1997). The Himalayan orogenic belt formed as a result of the collision between the Indian and Eurasian plates and has abnormally thick lithosphere, so we can expect complications in the seismic wavefield.

3.1 Heterogeneity model of the mantle beneath the Himalayan region

The heterogeneity model is derived from a 45° long and 1582 km deep region in the *P*-wave velocity structure model along an east–west profile at 27.5° N that crosses the Himalayas and lies above the Makran subduction zone (Widiyantoro 1997). The slice comprises a relatively strongly heterogeneous uppermost mantle, involving two high-velocity areas correlated to the subsurface structure, a low-velocity zone below the lithosphere, and a quite weakly heterogeneous lower mantle.

The cut through the tomographic model of the mantle has been chosen to lie in a zone where the velocity variation is relatively small perpendicular to the profile (north–south direction), so the 2-D simulation model should be a reasonable approximation of the actual 3-D heterogeneous seismic velocity structure.

The damping employed in the tomographic inversion has the effect that the inferred velocity perturbation is likely to somewhat underestimate the variations in the real mantle structure. We have therefore modified the amplitude of the P-wave velocity perturbation to have a maximum value of about 3 per cent (T3 in Fig. 13). We constructed an S-wave velocity structure by using a multiplier of 1.6 (to give a maximum perturbation of about 5 per cent) according to the results from recent P- and S-wave simultaneous inversion studies using global data (e.g. Vasco *et al.* 1994; Su & Dziewonski 1997). We do not apply any perturbation to the density structure or the Q structure.

The tomographic model was constructed using $1^{\circ} \times 1^{\circ}$ cells with layers of the order of 75 km thick and so would not be able to represent the smaller-scale heterogeneity we have examined in the previous sections. We have therefore constructed further heterogeneous mantle models by the addition of small-scale random heterogeneities (a = 40 km) with an aspect ratio of 4 to the velocity structure of model T3 (Fig. 13). We consider three different models (models T3+R1, T3+R2 and T3+R3), with the maximum amplitude of the additional perturbations restricted to 1, 2 and 3 per cent, respectively. The resulting model T3+R1 displays the principal features of the original heterogeneity model T3 but these features are broken up and modified by the presence of the small-scale heterogeneity. For model T3+R3 the amplitude of the additional small-scale heterogeneities is almost the same as the amplitude of the larger-scale perturbations of the *P*-wave velocity in model T3, so that the features of model T3 are significantly modified.

We use a double-couple source, which is placed at 63 km depth, 6° into the model near the western edge of a high-velocity area. We have used a rather short-period seismic source with a predominant period of 5 s in order to simulate realistic seismic wave behaviour within the complex structure. To ensure the stability of the calculation of the short-period seismic waves, the grid size in both the angular and the vertical direction was reduced by half compared with the previous cases. Such a small grid size requires a small time step (0.0625 s) and many time steps to cover the full time range of interest (1500 s needs 24 000 time steps), so we use the original PSM code to avoid the long-time instability associated with the symmetric differentiation scheme. We therefore concentrate on the body wave portion of the wavefield to examine the influence of the different styles of heterogeneity.

3.2 Seismic wave propagation in the Himalayan model

Fig. 14 displays snapshots of the seismic wavefields for model T3 with a 200 s interval between frames. This is accompanied by a display of the relative time lag of the different classes of seismic waves derived from cross-correlation of the wave amplitudes in the snapshots with that for the reference model AK. The cross-correlation is calculated for a 0.7° wide window at each depth (16 gridpoints) using the amplitudes of the Pand SV waves between the snapshots for models T3 and AK. The distance lag in the angular direction is obtained by maximizing the cross-correlation. The time lags of the phases are then derived by dividing the distance shift by either the Por the S-wave velocity of the reference model. Phases with an advance in time compared with the reference model are indicated by shades of turquoise and those with a delay compared with the references by shades of pink. The relatively narrow spatial window at each depth enhances the effects for body waves travelling nearly horizontally but means that the display in Fig. 14 is relatively insensitive to vertically travelling body waves and Rayleigh waves.

As might be expected, the P and S arrivals are initially advanced because the source lies in a faster region than the reference model (100 s snapshot). Because the amplitude of the perturbations in S wave speed is larger than for the P wave, the traveltime anomaly of the S wave is much more significant.

At later times the advance in the traveltimes of the body waves is gradually reduced during the passage through the lowvelocity zone in the lower part of the upper mantle. In the 300 and 500 s frames, the S waves that travel long distances in the low-velocity zone are significantly delayed. The heterogeneity in the lower mantle is weak, but the seismic velocities in the eastern region are slightly higher than for AK, so the traveltime anomaly of the S waves is again gradually enhanced during the passage through the lower mantle.

In Fig. 15 we compare synthetic velocity seismograms produced by the same double-couple source for the reference model (AK) and four heterogeneity models (T3, T3+R1, T3+R2 and T3+R3) recorded at surface stations at 6°, 13°, 20°, 27° and 34° from the epicentre. As expected from the time-lag snapshots in Fig. 14, the traveltime anomalies of the body waves due to the heterogeneities along the propagation paths are quite clear. The interactions of the triplications in the body waves returned from the heterogeneous mantle cause considerable change in the waveform of the body waves. The strong heterogeneity in the uppermost mantle also causes a significant phase shift of the fundamental Rayleigh mode, although the waveform is unchanged.

Because the maximum perturbation amplitude of the smallscale heterogeneities added in model T3+R1 is small compared with the large-scale heterogeneity, it is very difficult to identify any direct influence of the small-scale heterogeneity from the seismograms. Such small-scale heterogeneities should mainly affect turning body waves such as the *S* and *SS* phases. However, as the amplitude of the additional small-scale heterogeneity increases, the waveform change of the *S* phases is quite noticeable (see e.g *S* and *sS* at 27° and 34°).

The influence of the small-scale heterogeneity increases with frequency as can be seen when we filter the seismograms with a high-pass filter with a cut-off frequency of 0.25 Hz (Fig. 16). Comparing the reference model AK with the heterogeneity models, we first notice significant traveltime anomalies and waveform changes in the body waves due to the large-scale heterogeneities; in addition, we observe changes in the waveform of the body waves between the heterogeneity model T3 and the heterogeneity models with increasing small-scale heterogeneities (T3 + R1, T3 + R2 and T3 + R3). The waveform changes due to the additional small-scale heterogeneities are particularly significant for the pP phase observed at 27°, the PPphase at 34° and the S phase at 27° and 34° and are comparable to the changes due to the large-scale heterogeneities. The addition of pervasive small-scale heterogeneity can therefore have a substantial influence on the character of the wavefield, and when its amplitude approaches that of the larger-scale heterogeneity the waveform of the direct arrivals is significantly modified and we can also see waveform changes in the broad-band seismograms (Fig. 15). Although it is difficult to place a direct constraint on the level of small-scale heterogeneity, it is clear that the nature of heterogeneities with a scale length of the order of 40 km is very important for understanding short-period (< 4 s) body wave propagation.



Figure 13. *P*-wave velocity models beneath the Himalayan region. Model T3 is based on the *P*-wave velocity structure derived from a tomography study (Widiyantoro 1997), and models T3 + R1, T3 + R2 and T3 + R3 are created from model T3 by adding small-scale heterogeneities with maximum amplitudes of 1, 2 and 3 per cent, respectively.



Figure 14. (a) Snapshots of the seismic wavefield and (b) time-lag snapshots for model T3 with a double-couple source.

4 DISCUSSION AND CONCLUSIONS

We have simulated seismic wave propagation in various styles of heterogeneous mantle structure using the 2-D pseudospectral method, and we have shown how sequences of snapshots and synthetic seismograms for surface stations can help us to understand the nature of the seismic wavefield in complex media.

Those parts of the seismic wavefield where propagation is close to horizontal such as body waves turning or reflected at wide angles in the upper mantle and the fundamental and higher modes of Rayleigh waves are most affected by the character of the heterogeneity in the mantle. Both the overall scale length of the heterogeneity and the relative scales of horizontal and vertical variation (aspect ratio) have a significant effect on the nature of the interactions.

As the scale length of stochastic heterogeneities increases, the effects on body waves and longer-period surface waves (period >40 s) become more significant. The distortion of waveforms, shifts in time or phase increase compared with a reference but there is a compensating greater coherence amongst the influences at nearby stations. The influence of the heterogeneity becomes more apparent when parts of the waveforms are built from the superimposition of branches with different propagation paths, as in the upper mantle triplications, for example. Such effects are stronger for S than for P at the same scale length because of its shorter wavelength.

For surface waves we see systematic patterns in the phase shifts induced by the heterogeneity; small scale lengths influence the higher-frequency waves, whereas the large-scale features are also seen in lower-frequency surface waves. The influence of heterogeneity is stronger for the higher modes than the fundamental mode because of their greater depth penetration.

The aspect ratio of heterogeneity has major effects when either small (around 1) or large (around 8), but similar behaviour is seen for moderate values (2–4). Such a form of stochastic heterogeneity with a shorter vertical than horizontal scale appears to provide a reasonable representation of likely medium- to small-scale mantle structure superimposed on the broad-scale features determined from seismic tomography.



Figure 15. Synthetic velocity seismograms at five surface stations at 6° , 13° , 20° , 27° and 34° for the reference model AK and various heterogeneity models.

From the simulation of the seismic wavefield for the slice of the tomographic model beneath the Himalayan region, we see the systematic effects of this model on the seismic wavefield. However, the results are not noticeably changed by the addition of moderate levels of heterogeneity with a scale length of 40 km. This implies that the velocity structure derived from the seismic tomography should be a good representation of the longer spatial wavelength components of mantle structure, even in the presence of some amount of smaller-scale heterogeneity. However, care should still be taken with the effect of the additional small-scale heterogeneities, which can change the waveform and coherence of short-period body waves (less than 4 s).

In this study we have restricted attention to 2-D simulations of the seismic wavefield because full 3-D simulations of shorter-period seismic waves in the upper mantle are still too

expensive, even with current high-performance computers. As a result we are unable to include out-of-plane scattering effects, and the influence of heterogeneity on the seismic waves is likely to be somewhat enhanced compared with that in the actual 3-D heterogeneous mantle. However, our results on the sensitivity of the different seismic phases to the various scales of mantle heterogeneities should be directly transferable to the 3-D case.

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Figure 16. Filtered synthetic seismograms of Fig. 15 using a high-pass filter with a cut-off frequency of 0.25 Hz. The traces are amplified to enhance the visibility.

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