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CAN 'PURE-PATH' MODELS EXPLAIN FREE OSCILLATION DATA?

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Abstract. Several recently published regionalized models of the earth's lateral heterogeneity obtained by 'pure-path' analysis are compared to free oscillation data of Masters et al [1982]. Among these models, Okal's [1977] model shows a strongly dominant pattern of angular order $l = 2$ heterogeneity that is strikingly similar to Masters et al's data. This suggests that Okal's model may explain the free oscillation data as well as the "transition-zone" model of Masters et al, and that shallow lateral heterogeneity may be responsible for the simple pattern of Masters et al. The apparent success of Okal's model suggests the importance of lateral heterogeneity within oceanic regions. The instability of 'pure-path' analysis is also demonstrated by spectral analysis of the lateral heterogeneities of different 'pure-path' models.

Introduction

How the real earth deviates from spherically symmetric models is one of the current major questions of solid earth science. A first order question is how much and how deep the lateral heterogeneity of elastic properties at depth correlates with surface heterogeneity. In 'pure-path' analysis of long-period surface wave phase velocities, which has been the common approach to this question, a *complete* correlation between these two heterogeneities is assumed. A significant, but second order question is determining the extent to which the heterogeneities at depth are not correlated with surface heterogeneity and where these uncorrelated heterogeneities are located.

Recently, from a large number of fundamental spheroidal mode observations, Masters et al [1982] suggested the existence of deep lateral heterogeneity. They proposed a model of lateral heterogeneity of elastic properties with only angular order $l=2$, located in the depth range 420-670 km, which they called the "transition-zone" model.

The purpose of this paper is to examine whether previously published 'pure-path' models, in which the earth is divided into a small number of distinct regions, e.g., stable continents, oceans, tectonic areas, show any correlation with the free oscillation observations reported by Masters et al. Masters et al stated that *"the present schemes of tectonic regionalization are inappropriate at period longer than ~ 200 s"*. We will show that even at periods ~ 300 second some previously published regionalized 'pure-path' models, particularly Okal's [1977] model have a strongly dominant angular order $l = 2$ heterogeneity and produce a very similar pattern to that observed by Masters et al. This suggests that shallow lateral heterogeneity may be mainly responsible for Masters et al's observation and that the proposed deep lateral heterogeneity may be unnecessary.

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Representation of the Lateral Heterogeneity in Models and Data

In 'pure-path' phase velocity analysis, the earth's surface is divided into several distinct regions, in each of which Rayleigh wave phase velocity (referred to as 'phase velocity' in the rest of this paper) is assumed to be given by a single function of frequency. We compare regionalized models obtained by Okal [1977], Nakanishi [1979], Silver and Jordan [1981], Dziewonski and Steim [1982], and Souriau and Souriau [1982] to the data observed by Masters et al [1982].

Masters et al [1982] plotted the frequency difference between the global average and the observed peak of each *unresolved* spheroidal multiplet as a function of the two poles of the great circle which connects the receiver and the source. They found an apparently simple, angular order, $l=2$ dominant pattern of heterogeneity (Figure 1a). For large l's (i.e., shorter periods) there exists an asymptotic equivalence between great-circle phase velocities and multiplet locations of the laterally heterogeneous earth [e.g., Silver and Jordan, 1981]. Therefore in this paper we calculate the average phase velocity along each great circle path for several different regionalized models, which is equivalent to the peak frequency shift, and seek any correlation with their data. The effect of the earth's ellipticity has already been accounted for in the individual papers; i.e., the published 'pure-path' models are for lateral heterogeneity of a spherical earth.

For large l, the eigenfrequency, ω of the free oscillation is related to the phase velocity, c through

$$c = \frac{\omega R}{l + 1/2}, \tag{1}$$

where R is the radius of the earth. Thus the variation of the phase velocity is essentially proportional to that of eigenfrequency.

The average slowness (inverse of phase velocity) over great circle paths can be represented in terms of spherical harmonics as follows,

$$\frac{1}{C(\Theta,\Phi)} = \sum_{s=even}^{\infty} P_s(0) \sum_{t=-s}^{s} f_s^t \, Y_s^t(\Theta,\Phi) \tag{2}$$

where $C(\Theta,\Phi)$, Y_s^t, P_s, Θ and Φ are the average phase velocity of a given great circle path, the fully normalized spherical harmonics, the Legendre function, and the colatitude and longitude of the pole of the great circle, respectively. f_s^t is the spherical harmonic expansion coefficient of slowness as a function of geographical position,

$$\frac{1}{c(\theta,\varphi)} = \sum_{s=0}^{\infty} \sum_{t=-s}^{s} f_s^t \, Y_s^t(\theta,\varphi) \tag{3}$$

where $c(\theta,\varphi)$ is the phase velocity at the point (θ,φ) [Backus, 1964; Souriau and Souriau, 1982].

In order to compare the results of the calculation with the raw data of Masters et al (Figure 1a), the effect of the earth's ellipticity is artificially added in Figures 1 and 2 by correcting the length of the great circles. All summations are terminated at angular order $s=40$ for models which use Okal's regionalization

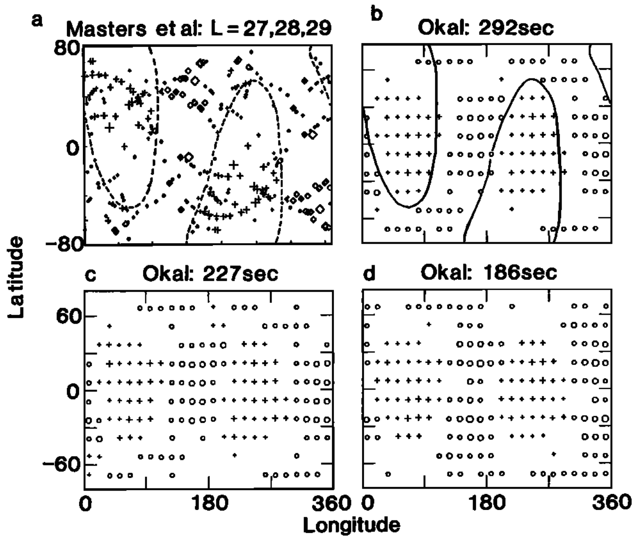

Fig. 1. Average Rayleigh wave phase velocities over great circle paths of Okal's model are plotted at the poles of the great circles at 15° intervals. Symbol type indicates a great circle whose average phase velocity is faster (+) or slower (o) than the average of the whole earth. Symbol size indicates its magnitude (smallest symbols correspond to difference of 0.05-0.15 %; the largest to 0.25-0.35 %; blank spaces to less than 0.05 %). (a) after Masters et al. [1982] for angular order l = 27,28,29. (b) Period T = 292.57 sec, (c) T = 227.55, (d) T = 186.18. Note the similar pattern between (a) and (b). Lines in (a) and (b) correspond to the node of the degree-two pattern obtained by Masters et al. The patterns are very similar even for the shorter periods.

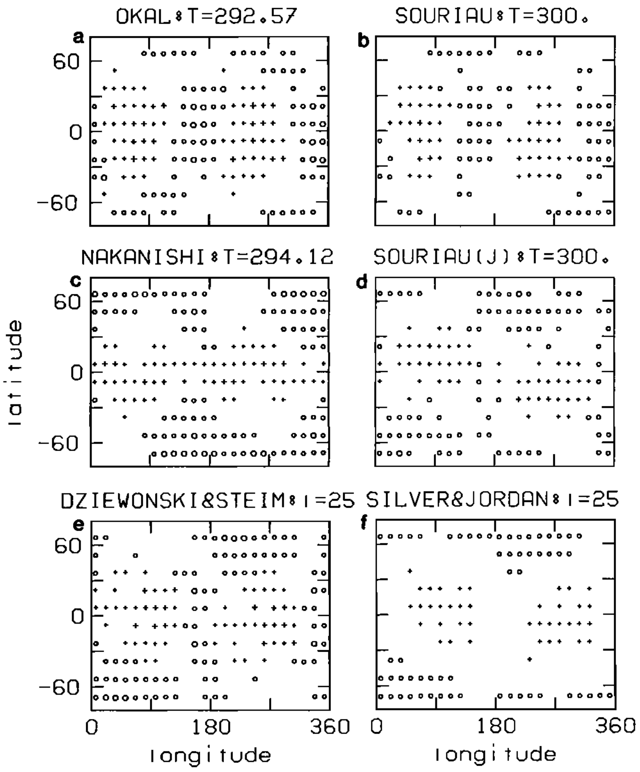

Fig. 2. The same plots as Figure 1 for recently published models at period around 300 sec; (a) Okal [1977], (b) Souriau and Souriau [1982] for Okal's regionalization, (c) Nakanishi [1979], (d) Souriau and Souriau for Jordan's regionalization, (e) Dziewonski and Steim [1982], (f) Silver and Jordan [1981]. Models which use Okal's regionalization (Figures 2a and 2b) show the simple pattern.

(Figure 3): Okal [1977], Nakanishi [1979], and Souriau and Souriau [1982], and at s=20 for other models. '+' or 'o' corresponds to a pole of the great circle whose average phase velocity is greater or smaller than the earth's average, respectively. Of the six models in Figure 2, those which use the Okal's regionalization (Figures 2a and 2b) show a strong angular order l=2 pattern. Okal's model (Figures 1b, 1c, 1d, and 2a) shows a particularly similar pattern to that found by Masters et al (Figure 1a).

"Transition-zone" Heterogeneity vs. Okal's Model

In Figures 1b, 1c, and 1d, the results for Okal's model at three different periods are shown. The solid line in Figure 1b is exactly copied from Figure 1 of Masters et al for the corresponding period (l=27,28,29) (Figure 1a). The basic similarity of these two figures is clear. (Note that plots of the type in Figure 1 will always show symmetry, because each great circle has two poles.)

In Figure 1b, the deviation from the average phase velocity is as much as 0.28 %, which is comparable to that of Masters et al. The distinct similarity between Figures 1a and 1b suggests that the earth's shallow heterogeneity may explain this simple pattern, both because Okal's model has only regionalized heterogeneity (Figure 3) and also because calculations for much shorter period surface waves, which would not sample any deep heterogeneity, show similar patterns (Figures 1c and 1d).

Figure 4a shows the power spectrum of slowness of Okal's model at T = 292.57 sec (see equation (3)) and of its average great circle slowness (equation (2)). The dominance of the l = 2 term in Okal's model can be clearly seen. Because of the nature of the transfer function from slowness in geographical coordinates $c(\theta,\varphi)$ to average great circle slowness $C(\Theta,\Phi)$, after the transformation the l = 2 term becomes more dominant. Figure 5 shows the power spectrum of the transfer function of slowness (see also equation (2)). Thus any great circle data and any figure like Figure 1 will tend to have dominant l=2 heterogeneity.

In terms of the dominance of the l = 2 heterogeneity, both Masters et al's "transition-zone" model

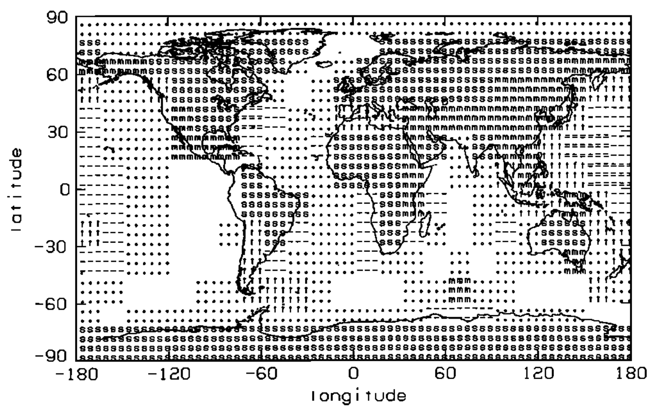

Fig. 3 Okal's regionalized tectonic model digitized in 5°×5° grid (15°×15° in the original) is superimposed on the world map. Symbols are, blank: ocean 0-30 ma, '•': 30-80 ma, '-': 80-135 ma, '=': older than 135 ma, 't': trench and marginal seas, 'm': Phanerozoic mountains, 's': shields.

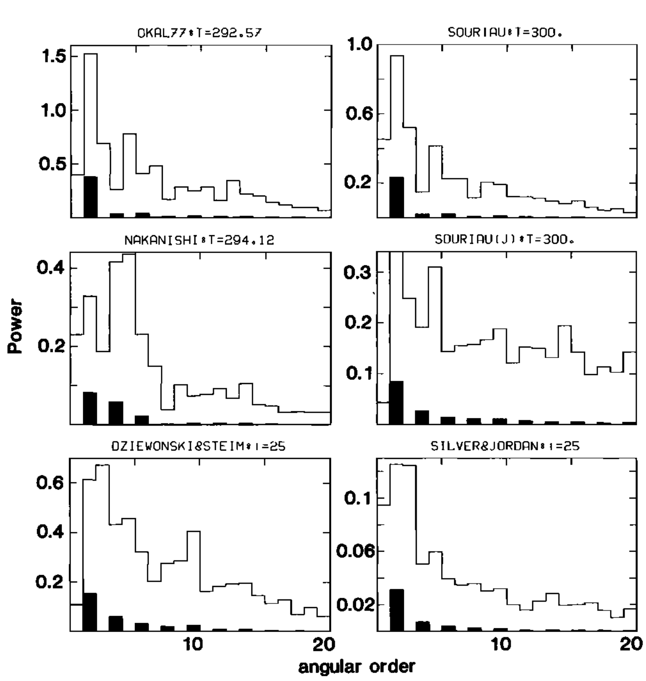

Fig. 4. Power spectrum of published 'pure path' models for periods of about 300 sec. White is power spectrum of heterogeneity represented in geographic coordinate and black is one in great circle pole position. Heterogeneities are expressed in terms of slowness (inverse of velocity) for (a), (b), (c) and (d) and of local eigenfrequency for (e) and (f). The eigenfrequency data are scaled to allow comparison with the phase velocity data. The large variation of the power spectra (even for the lowest angular order numbers) shows the lack of resolution of 'pure-path' method. The unit of the vertical axis is $10^{-6}\,(sec/km)^2$. The sources of the models are same as Figure 2.

and Okal's model generate the simple $l = 2$ pattern and in that sense both models are equally reasonable candidates as explanations for the earth's lateral heterogeneity. It seems, however, unnatural to prefer the deeper lateral heterogeneity to the shallower heterogeneity, when the latter also explains the major part of the data and has a reasonable physical basis inferred from the surface geological features. There

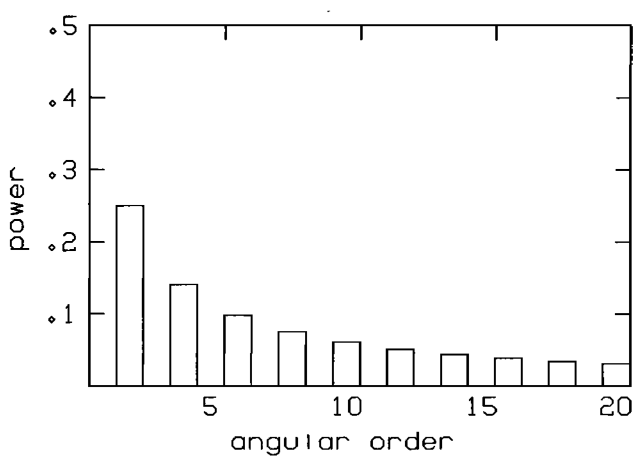

Fig. 5. Power spectrum of the transfer function from geographical coordinate to 'great circular pole position' coordinate (i.e., $P_l(0)^2$). Zero values for odd angular order result in the loss of information on the odd order lateral heterogeneity in the great circle data. Also note that the lowest order (l=2) heterogeneity is always weighted most.

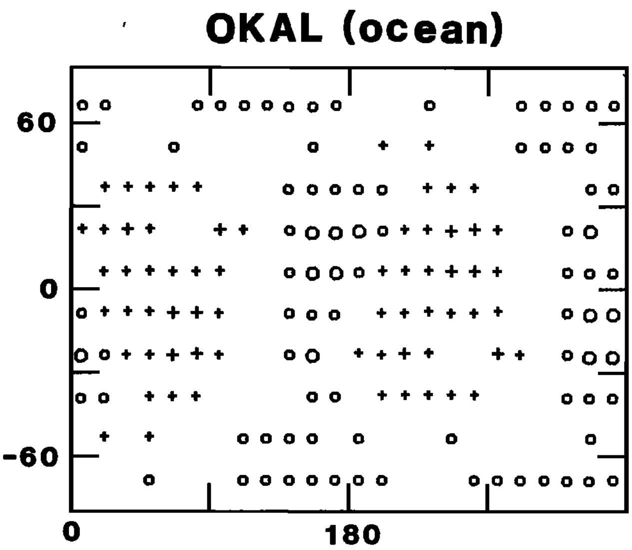

Fig. 6. Effect of the heterogeneity in the ocean of Okal's model at T = 292.57 sec. Phase velocities of continents are fixed at average value. The basic l = 2 pattern is apparent.

is, of course, some deviation between the data (shown in Figure 1a) and the calculations for Okal's model (Figure 1b), which could be attributed to possible deep lateral heterogeneity. Those residuals, however, would show a very different pattern and amplitude from that of Figure 1a, and therefore the resulting deep heterogeneity would be very different from Masters et al's. The residual also can be due to either the inapplicability of geometrical optics or incompleteness of Okal's regionalization.

The foregoing argument seems to imply the simple conclusion that it is questionable for Masters et al to have inferred the existence of deep lateral heterogeneity without first having taken shallow (down to ~ 250 km) heterogeneity into account.

Tectonic Regionalization

Where does the simple pattern of Masters et al come from, and what does the fact that Okal's model generates a similar pattern mean? Figure 6 shows the effects of the lateral heterogeneity in the ocean regions. The phase velocities of continents are fixed at the average phase velocity of the whole earth. The basic feature of the l = 2 pattern can be seen, which demonstrates the importance of the detailed regionalization in the oceanic regions.

Souriau and Souriau [1982] tested the adequacy of three different regionalized models: Okal's [1977], Leveque's [1980], and Jordan's [1981], with regression analysis, and concluded that Okal's model explained the surface wave data best. Souriau and Souriau [1982] attributed Okal's success to the separation of 'trench and marginal sea' areas from other regions. We also note that Okal's model has four different ocean regions (plus 'trench and marginal seas') in contrast to Leveque's and Jordan's two and three, respectively. As Okal [1977] mentioned, the variation of phase velocity within the ocean regions is as significant as the difference between shields and the average ocean, and it is therefore essential for such analyses to separate the ocean into many regions.

Both Nakanishi (Figure 2c) and Souriau and Souriau (Figure 2b) use Okal's regionalized model. The result for Nakanishi's (Figure 2c) does not show the simple pattern. We attribute this partly to his having combined three of Okal's oceanic regions into only one in his analysis. The result for Souriau and Souriau's model (Figure 2b) basically shows a similar pattern with the maximum deviation 0.23 %. In his analysis, Okal *a priori* specified the phase velocities of the four oceanic regions on the basis of the shallow lateral heterogeneity (above depth 180 km) obtained from the analysis of shorter period (< 150 sec) surface waves [e.g., Leeds, 1975]. If there is significant lateral heterogeneity below this depth beneath the ocean basins, Okal's approach would not count this deep lateral heterogeneity and would have tended to attribute them to the other regions. However, the basic agreement between the two different analyses (Okal's and Souriau and Souriau's) seems to indicate the adequacy of Okal's method and the efficacy of Okal's regionalization.

Instability of 'Pure-path' Analysis

Figure 4 shows the power spectrum of lateral heterogeneity for the models which are used to obtain Figure 2. The effect of the earth's ellipticity is not added here. The large variation between the spectra (even for the lowest order coefficients) suggests an apparent lack of resolution of odd order heterogeneities and a resulting instability of the 'pure path' technique. The instability also can be easily found in Souriau and Souriau's [1982] result. They performed 'pure-path' analyses for the same data set with Okal's regionalization (Figures 2b and 4b) and with Jordan's regionalization (Figures 2d and 4d). Both the power spectra and the great circle phase velocity plots differ greatly for the two regionalizations.

The cause of these difference is the inherent instability of 'pure-path' analysis: from smoothed and low-angular-order weighted great circle data, which contain only the even part of the whole heterogeneity, the odd heterogeneity is extrapolated with a constraint *a priori* given by the chosen regionalized model. As Souriau and Souriau [1982] noted, the results of this process are thus strongly dependent on the choice of regionalized model.

Because of the simplicity of our experiment, nothing quantitative about the possible errors or biases of the 'pure path' technique can be stated within the scope of this short note. It is, however, essential for our understanding of the earth's lateral heterogeneity to know the limit of those simpler theories and this understanding can be only obtained in the light of more accurate theoretical work [e.g., Woodhouse and Girnius, 1982] and forward modeling experiments [e.g., Morris et al, 1982].

Conclusion

A simple calculation of average phase velocities along great circle paths using Okal's model shows the basic fit of his model to the data of Masters et al. This suggests that there is a large correlation between the earth's surface heterogeneity (represented by Okal's regionalization, Figure 3) and Masters et al's observations, and that it is therefore questionable for Masters et al to infer the existence of deep lateral heterogeneity localized between 420-670 km without first having taken the effect of shallower heterogeneity into account. The success of Okal's model in generating Masters et al's pattern also demonstrates the importance of the variation of phase velocity within oceanic regions. It is also shown that the results of 'pure-path' analysis are strongly dependent on the choice of regionalized model and that the averaging property of great circle data reduces the resolution of the heterogeneity in the result.

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