Geodetic constraints on the shallow magma system at Soufrière Hills Volcano, Montserrat

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[1] Ground surface tilts recorded proximal to a growing lava dome, and sustained through repeated dome collapses, provide unique geodetic constraints on the depth, size, and overpressures of the shallow magmatic system at Soufrière Hills Volcano. Tilt records identify a single pressure-source inflating with periods ~3–30 h. Sustained tilt amplitude ratios for paired stations identify the pressure-source at a centroid depth between ~740 and 970 m. This pressure-source and its robust oscillatory response survived a dome collapse that removed the upper ~200 m of the magmatic system. Shallow overpressures of 1–4 MPa are confirmed over a radius of 200–340 m; this size suggests pressurization of a fluid-saturated fractured rock mass annulus surrounding a narrow magma conduit. **Citation:** Widiwijayanti, C., A. Clarke, D. Elsworth, and B. Voight (2005), Geodetic constraints on the shallow magma system at Soufrière Hills Volcano, Montserrat, *Geophys. Res. Lett.*, *32*, L11309, doi:10.1029/2005GL022846.

1. Introduction

[2] Surface measured tilts have been successfully used to infer pre-, syn-, and post-eruptive characteristics of the magmatic plumbing underlying active volcanoes [*Mogi*, 1958; *Dvorak and Dzurisin*, 1997; *Cayol and Cornet*, 1998; *Voight et al.*, 1998]. This study reports measurements of tilt, recorded unusually close to the summit extrusion, which constrain the interaction of the high-level magma system that underlies the dome, and reacts to dome collapse.

2. Analysis of Tiltmeter Signals

[3] The resurgence of volcanic activity (1995) at Soufrière Hills Volcano (SHV) on Montserrat has provided an important opportunity to observe dome growth, punctuated by episodic destruction. Seismic, petrologic, gas and geodetic signals have contributed to the evolving picture of various processes that control the evolution of the lava dome [*Druitt and Kokelaar*, 2002]. Proximal tilt measurements provide important constraints on shallow processes contributing to dome growth and collapse [*Voight et al.*, 1998, 1999]. These signals are used here to define the depth, size, and overpressure of a pressure-source developed within the underlying shallow magma system.

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2.1. Constraints on Pressure-Source Depth

[4] A unique series of tilt records are available from stations 100–300 m from the crater rim and 500–700 m from the central dome-building conduit of SHV, during the active dome building episodes of December 1996–August 1997 [*Voight et al.*, 1998, see Figure 1]. Three tilt stations (Applied Geomechanics AGI-700), identified as CP1, CP2, and CP3 were deployed on Chances Peak. CP1 was installed 9 December 1996 and was destroyed on 13 January 1997; stations CP2 and CP3 were installed 18 and 22 May 1997, respectively. CP2 lasted until June, whereas CP3 survived several episodes of dome-collapse pyroclastic flows before being destroyed by an explosion on 5 August 1997.

[5] Tilt records from CP2 and CP3 for the period 23–31 May 1997 were resolved into their radial components; these show a pattern of synchronous oscillatory inflation and deflation with a period between 14 and 22 h (Figure 2a). Both instruments exhibit an incremental monotonic irrecoverable inflation of the ground surface with each successive cycle. The sense and magnitude of the tilt increment is the same in each instrument, suggesting that this is not related to random instrument drift. Rather, this permanent deformation, together with the coherence of the cyclic signal recorded between the two instruments, suggests a common pressure-source for the recorded tilts.

[6] The characteristics of the shallow magma system may be recovered by comparing tilt observations with solutions representing the deformation field adjacent to a nucleus of strain [*Mogi*, 1958]. For a spherical nucleus of strain of effective radius a, buried at depth z below the surface of a half-space of shear modulus G, and inflated to overpressure Δp, the radial surface tilt θ is defined as,

$$\theta = \frac{9a^3\Delta p}{4G}\frac{rz}{(r^2+z^2)^{5/2}} \quad (1)$$

where r is the surface trace of the vector joining the nucleus of strain with the recording location. The shear modulus is related to the deformation modulus E and Poisson ratio ν as $G = E/2(1 + \nu)$. Tilt is defined positive for inflation with an increase in magma pressure Δp.

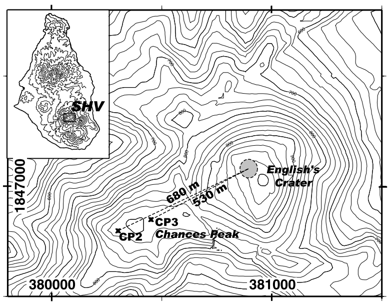

Figure 1. Inset shows location of SHV on Montserrat. Location shown for tilt stations CP2 and CP3 on Chances Peak relative to the active magma conduit at English's Crater.

[7] In addition to the coherence of the synchronous tilt signals recorded on CP2 and CP3, which suggests a common source, the amplitude ratio θ_3/θ_2 of the tilt signals remains in the range $\sim$1.10–1.28 throughout the record, with a mean of $\sim$1.19. These data enable triangulation on the source depth. For a common source, the ratio of tilt θ_3 recorded at radius r_3(CP3) to the tilt θ_2 recorded at radius r_2(CP2) may be recovered from equation (1) as,

$$\frac{\theta_3}{\theta_2} = \frac{r_3}{r_2}\frac{(r_2+z)^{5/2}}{(\mathrm{r}_3+z)^{5/2}} \tag{2}$$

enabling source depth z to be determined from $\theta_3/\theta_2 \sim$ 1.10–1.28. Source depths (z) are estimated to range from $\sim$740–970 m for tilt ratios of 1.10 and 1.28, respectively, and $\sim$850 m for the mean ratio of 1.19. Tilt ratios fix the depth of a shallow pressure-source, but constrain neither source size nor overpressure.

2.2. Constraints on Source Pressure

[8] The tilt record is punctuated by periodic dome collapses, as illustrated by the dome-collapse record of Figure 2b. These events provide an independent mode of environmental loading to the volcano system, and hence enable the active magmatic system to be further constrained. Responses to dome-collapse events were recorded on the single operational tilt meter (CP3) surviving the June 25 and 3 August collapses 1997. The June event had a DRE collapse volume $V = 4.9 \times 10^6$ m^3 [*Calder et al.*, 2002]. The cyclic periodicity observed before the collapse was also maintained afterward, with the post-collapse periods slightly shorter (averages: 9.8 h before, 7.2 h after) and more uniform, and equal tilt amplitudes $\geq$25 μrad. These observations indicate a robust oscillatory mechanism, capable of surviving the unloading of $\sim$200 m of lava dome.

[9] The similarity in periodicity and amplitude of the tilt, both pre- and post-collapse, discounts major structural realignment of the magma transport system. Rather it favors a mechanism of pressure-regulation modulated by the collapse-reduced magmastatic pressure. For a collapse-induced loss of the upper 200 m of the magma conduit (partly within the dome), the stress drop is of the order 5 MPa. This uniform reduction in the conduit pressure enables the magnitude of the oscillatory pressure within the source volume to be defined. This deconvolution is complicated by the concurrent removal of the dome mass, the effect of which is incorporated in the instantaneous deflationary tilt signal of $\sim$22 μrad [Figure 2b]. However, if the mass of the collapse is known, the relative effects of dome removal and depressurization may be deconvolved, as illustrated schematically in Figure 2b to yield the underlying cyclic pressure signal.

[10] The radial surface tilt that results from the loading of a half-space by a point load P applied orthogonal to the surface may be defined from the Boussinesq solution [*Poulos and Davis*, 1974]

$$\theta = -\frac{P(1-\nu)}{2\pi G}\frac{1}{r^2} \tag{3}$$

where r is the separation between the point of load application and the measuring location. Loading is defined positive if directed into the half-space, hence tilts are of opposite sense to those for inflation of an embedded strain nucleus. The surface loading, P, may be replaced by a force equivalent to the mass removed by the mobile dome collapse. For DRE density $\sim$2600 kg/m^3, the equivalent load is $P \sim -127$ GN and for E = 1 GPa and ν = 0.25, the associated inflationary tilt at station CP3 is $\theta_3 \sim 135$ μrad.

[11] With the effects of both surface unloading ($\sim$135 μrad) and of decompression ($\sim$157 μrad) of the

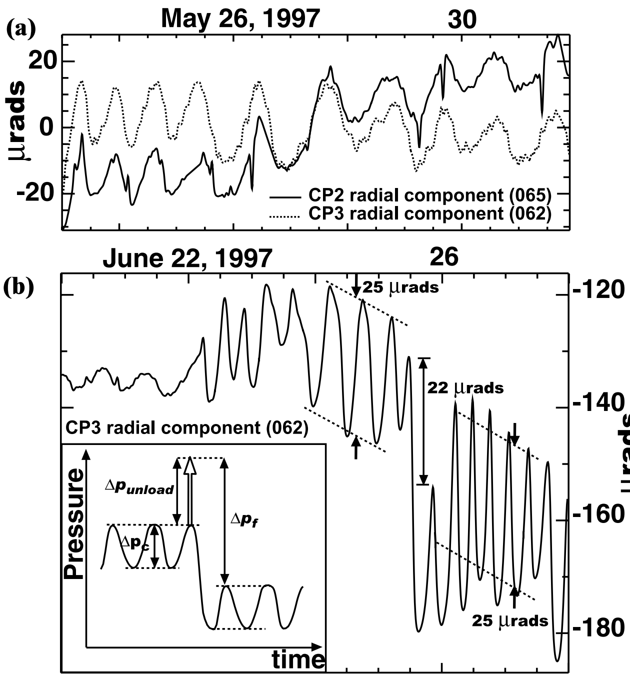

Figure 2. (a) Records of radial tilt for stations CP2 and CP3 for May 23 to 31, 1997. Tilt signals are synchronous with a period of $\sim$14–22 hours, and scale in amplitude in the ratio $\theta_3/\theta_2 \sim$ 1.10–1.28, with a mean of $\sim$1.19. (b) Tilt signal for station CP3 during dome collapse event of June 25 1997. Short-term periodicity has an amplitude of $\Delta p_c \sim$ 25 μrad and the signal offset following collapse is $\sim$22 μrad. The short-term oscillation results from a pressure cycling in the shallow magma system. The post-collapse offset results from the additive effects of unloading of dome surcharge ($-\Delta P_{unloading}$), and pressure drop (Δp_f) in the shallow magma system.

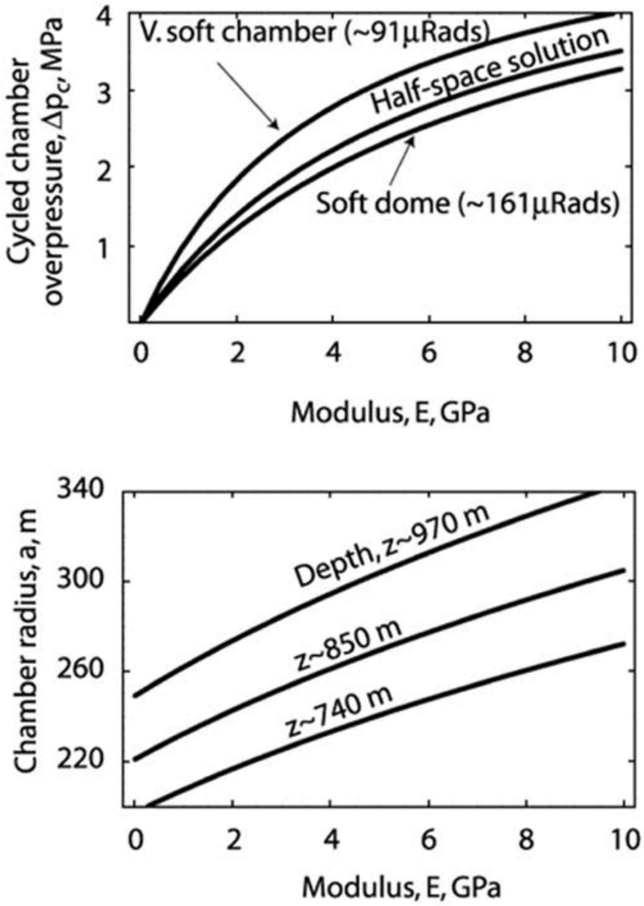

Figure 3. Variation in pressure source (pressure 'chamber') characteristics with host rock modulus. (top) Variation in cyclic pressure for dome-collapse depressurization of 5 MPa. Results are for Mogi and Boussinesq half-space solutions and for true volcano topography (Figure 4) with a z = 850 m soft source region, and a homogeneous volcano capped by a soft dome (properties in Table 1). (bottom) Pressure source (chamber) radius, a, constrained by Mogi source depths predicted from cyclic tilt ratios of 1.28 ($z \sim 740$ m), 1.19 ($z \sim 850$ m), and 1.10 ($z \sim 970$ m).

shallow magma system defined, the physical system may be constrained by the recorded tilt amplitudes recorded at CP3 and the presumed pressure-source decompression magnitude of $\Delta p_f \sim 5$ MPa. The pre-collapse cyclic tilt double-amplitude of $\theta_c \sim 25$ μrad [Figure 2b] is linearly related to the undefined oscillatory pressure change of Δp_c as

$$\theta_c = A\frac{\Delta p_c}{G} \tag{4}$$

where A is defined from equation (1) as $A = \frac{9a^3}{4}\frac{r\,z}{(r^2+z^2)^{5/2}}$. Similarly, as a result of dome collapse, the irrecoverable tilt offset of $\theta_f \sim 22$ μrad (Figure 2b) comprises the additive effects of the collapse-induced unloading and the resulting magma decompression of magnitude Δp_f. This irrecoverable tilt may be defined by a composite of equations (3) and (1) as

$$\theta_f = -\frac{M}{G} + A\frac{\Delta p_f}{G} \tag{5}$$

where A is as defined previously and $M = \frac{2\rho g V(1-\nu)}{4\pi r^2}$ is recovered from equation (3). Note that the magnitude of M is positive for a loss of material (unloading) from the surface, and that θ_f is defined positive for the deflationary change in tilt shown in Figure 2b. Solving equations (4) and (5) simultaneously yields

$$\Delta p_c = \frac{\theta_c}{\left(\theta_f + {}^{M}/_{G}\right)}\Delta p_f \tag{6}$$

to define the oscillatory pressure change Δp_c as constrained by recorded tilt amplitudes, tightly-constrained collapse mass, and presumed collapse-induced decompression of $\Delta p_f \sim 5$ MPa. For a measured intact-rock modulus of E = 9.8 GPa (Poisson ratio of ν = 0.25) and a *rock mass* modulus perhaps an order of magnitude lower [*Goodman*, 1989], the oscillatory overpressure driving surface deformation is calculated in the range Δp_c = 1–4 MPa, as illustrated in Figure 3.

2.3. Constraints on Pressure-Source Size

[12] With the pressure-source depth constrained by the ratio of tilts at CP2 and CP3, and the oscillatory overpressure of the pressure-source defined by the dome-collapse decompression magnitude, source size may also be constrained. Substituting the oscillatory overpressure Δp_c of equation (6) as Δp in equation (1) enables the pressure-source radius, a, to be determined. For a decompression stress-drop of 5 MPa, compatible source radii are in the range 200–340 m, for the bounding centroid depths of z ~ 740–970 m, as illustrated in Figure 3.

[13] Because these source radii greatly exceed the inferred conduit diameter of ~30 m [*Voight et al.*, 1999; *Melnik and Sparks*, 2002], pressurization of a portion of the magma conduit cannot *simply* account for the observed deformation. Some complexity is apparently involved, causing an "effectively larger" pressurized region at depth. Hypotheses include, (1) local substantial widening of the conduit (in effect, a shallow magma "chamber"), (2) degassing- or dilatation-induced hydrofracturing of the conduit walls to create a densely-fractured, fluid-saturated region surrounding the conduit that greatly expands the zone of pressurization, and (3) a wide zone of plastic deformation surrounding the pressurized conduit, extending

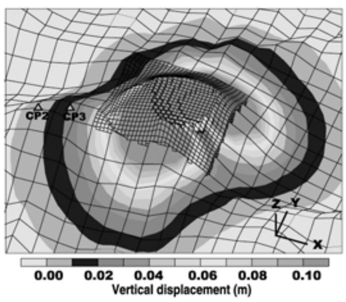

Figure 4. Contours of vertical displacement for a homogeneous volcano (E = 1 GPa, ν = 0.25; ρ = 2600 kg/m^3) in response to removal of a dome volume of 5 × 10^6 m^3. Collapse scar shown in fine mesh. See color version of this figure in the HTML.

Table 1. Tilt at CP3 Due to Unloading-Only of June 25 Dome

	Conditions		Rock Mass Modulus, E, GPa			
Tilt, θ″, μrads	Pressure-Source	Conduit	Volcano	Dome	Pressure-Source	Conduit
161[1]	absent	absent	1	0.01	1	1
153	absent	ultra soft	1	1	1	0.003
137	soft	soft	1	0.5	0.1	0.1
144	soft	very soft	1	0.5	0.1	0.01
130	very soft	soft	1	0.5	0.01	0.1
91[2]	very soft	absent	1	1	0.01	1

[a]Assumed geometry is a vertical conduit, diameter 30 m, linked to vertical cylindrical pressure source (radius 200 m, height 400 m, and centroid at 850 m below summit). Dome extends 400 m from conduit to 400 m below summit. Superscripts on tilts refer to upper[1] and lower[2] bounds of Figure 3 (top).

pressure over a wider annulus [e.g., *Cayol*, 2003]. Of these, plastic deformation seems inconsistent with the elastic-like pressure oscillations, and petrology does not support existence of shallow magma storage [*Rutherford and Devine*, 2002]. A fluid-saturated fractured rock mass transmitting more broadly the oscillating overpressures created within the magma conduit seems a potentially credible explanation of the calculated source dimensions. Hydrofracturing also represents a credible potential trigger mechanism for hybrid seismicity, which displays repetitive waveforms and forms over a restricted depth range [*Miller et al.*, 1998; *Neuberg et al.*, 1998], in clear association with pressure buildup [*Voight et al.*, 1998, 1999].

3. 3-D Model Replicating Tilt History

[14] So far the *Mogi* [1958] idealization has been exclusively used to represent behavior. This solution represents the surface deformation due to a buried strain nucleus under flat ground. The results are known to degrade when the radius of the pressure-source is large in comparison to burial depth (a/z large) [*Dieterich and Decker*, 1975; *De Natale and Pingue*, 1996], and when extensive relief invalidates the assumption of a bounding half-space [*Cayol and Cornet*, 1998]. Both these restrictive assumptions may be avoided where the true size and geometry of the pressure-source, and the true topography of the overlying volcano are represented.

[15] A model of the SHV is represented by a graded mesh of 146,789 3D zones and 156,471 nodes, coarsening radially from the dome summit, as shown in Figure 4. The model assumes linear elasticity (but can also be run as an elastic-plastic medium), includes a pressure-source of radius 200 m at a centroid depth of 850 m below the dome summit, connected to the surface by a vertical conduit of 30 m diameter. The mechanical properties of the host rock mass are E = 1 GPa and ν = 0.25, and those of the pressurized fluid are bounded between an anticipated lower bound magnitude for magma froth (0.003 GPa), and the stiffness of the volcanic host (E = 1 GPa and ν = 0.25).

[16] This numerical model is used to evaluate the fidelity of the previous predictions for source size, depth, and overpressure, recovered from the solution for a nucleus of strain embedded within a flat half-space. Since we approximate the response of the volcano as linearly elastic (a reasonable assumption for the small inflation pressure changes involved), we may invoke linear equations (4) and (5) to solve for the presumed source pressures. This requires that the modeled scaling coefficients renamed A' and M' are determined directly for the true geometry of the magmatic system and topography of the volcano, as represented in the numerical model. Primed variables refer to the numerical model results.

[17] Correspondingly, we run the model to yield the tilt θ' at location CP3, for an arbitrarily defined shear modulus G'. The model is run in two modes: First, with an applied source overpressure $\Delta p'$ to uniquely define the transfer function A' that links source overpressures to tilts via equation (4) as $\theta' = A'\frac{\Delta p'}{G'}$. Then second, with the pressure-source infilled but unpressured, and then removing the collapse volume of 5×10^6 m^3 (with DRE density 2600 kg/m^3) from the summit, to uniquely define the transfer function M'' of the truncated form of equation (5) as $\theta'' = -\frac{M''}{G''}$. Again the selection of G'' is arbitrary. With A' and M'' defined, substituting A' for A in equations (4) and (5), and M'' for M in equation (5), eliminates the need for the geometric transfer function A'. This peculiar outcome results since tilt records are available for twin pressurization cycles (θ_c and θ_f) of the assumed same pressure-source, and where one tilt reading (θ_c) involves source pressurization absent other loading modes. The resulting expression is a revised form of equation (6) as

$$\Delta p_c = \frac{\theta_c}{\left(\theta_f + {}^{M''}/_{G}\right)}\Delta p_f = \frac{\theta_c}{\left(\theta_f + {}^{\theta'' G''}/_{G}\right)}\Delta p_f \qquad (7)$$

where θ'' is the tilt at CP3 recovered from the numerical model of shear modulus G'', relative to the field measured tilts θ_c and θ_f (Figure 2b) recorded on the volcano of true but unknown modulus, G. This enables corrected magnitudes of the cyclic inflation pressure Δp_c to be determined where the effects of source geometry and topography are rigorously incorporated. The influence of topography on the predictions of chamber pressure is shown in Figure 4 for a variety of volcano stiffnesses, defined in Table 1. Modeled unloading tilt θ'' varies only by a factor of two (91 to 161 μrads) for modulus contrasts of the volcano to magma ($1 > E_{fluid}/E_{volcano} > 0.003$) and to dome rocks ($1 > E_{dome}/E_{volcano} > 0.01$) of two-orders-of-magnitude. These define the relative insensitivity of the unloading response to component moduli, and result in relatively robust predictions of source pressure, bounded with host modulus in Figure 3.

4. Conclusions

[18] Near-summit measurements of surface tilt are used to define the depth, size, and oscillatory overpressure response of a shallow pressure-source at Soufrière Hills volcano. Source depth and size are defined by the ratio of oscillatory tilt signal amplitudes, measured at two different locations.

These ratios constrain a spherical source depth in the range 740–870 m, and source radius in the range 200–340 m. These dimensions are conditioned by the relative magnitudes and distribution of the deformation moduli of the volcano. However, where tilt signals are simultaneously recorded from dual locations, the estimates of source depth and size are particularly robustly defined – they are independent of source pressure, as the same chamber pressure acts to induce tilts at the dual measuring locations. We have also explored cylindrical shaped sources, and have arrived at similar conclusions. The calculated dimensions greatly exceed the estimated conduit size inferred from magma flux and fluid dynamical considerations, but a fluid-saturated fractured rock mass transmitting more broadly the oscillating overpressures created within the magma conduit, seems a potentially credible explanation for the calculated source dimensions.

[19] Where the response to a subsidiary, quantifiable, but independent forcing to the magmatic system is available, further constraint of the system is possible. This is provided here by the unloading of the volcano resulting from the removal of the lava dome. Since the magnitude of the unloading is known, and the resulting change in pressure of the decapitated system assumed, the pre-collapse oscillatory overpressures of the pre-failure system may be determined. In this case, pre-eruptive pressure oscillations in the shallow deformation source region are of the order of 1 to 4 MPa for volcano moduli in the range 1 to 10 GPa, and are apparently little influenced by the effect of the true topography of the edifice.

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