Scaling laws in granular flows down rough inclined planes

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In this paper, new scaling properties for granular flows down rough inclined planes are presented. In the dense steady uniform flow regime, we have systematically measured the mean velocity of the flow as a function of the inclination of the surface θ and of the thickness h of the layer. The results obtained for different systems of beads corresponding to different surface roughness conditions are shown to collapse into a single curve when properly scaled. The scaling is based on the measurement of the minimum thickness $h_{\mathrm{stop}}(\theta)$ necessary to observe a steady uniform flow at inclination θ. From this experimental observation an empirical description for granular flows down inclined planes is proposed in terms of a dynamic friction coefficient. © *1999 American Institute of Physics.* [S1070-6631(99)00603-0]

I. INTRODUCTION

The flow of granular matters on inclined surfaces is often encountered in engineering applications involving the transport of materials such as minerals and cereals, but also in geophysical situations. Rock avalanches, landslides, and pyroclastics flows are natural events consisting of large-scale flows of particulate solids.[1–3] Besides these important industrial and geophysical applications, granular chute flows down inclines are also of fundamental interest: A layer of granular material flowing on a surface is a simple and well-controlled system which allows a precise study of the rheological properties of particulate systems. The characteristics of the flow being mainly controlled by the balance between the gravity force and the friction force exerted at the surface, one can use the inclined chute flow as a rheometer in order to study how the friction force in a granular layer varies. However, despite this apparent simplicity and the numerous experimental,[4–17] numerical,[18,19] and theoretical works[20–23] devoted to granular chute flows, their description and prediction are still a challenge.

Many chute flow experiments have been carried out and different configurations have been investigated, changing the bed conditions from smooth to rough, using different kinds of materials, and checking the influence of entrance conditions. The main conclusions of these studies are the following.

(a) When the inclined plane is smooth, fully developed uniform flows only exist at a critical inclination angle. Below this angle the material stops, and above this angle the material continuously accelerates along the plane. The system is well described by a constant friction coefficient.[9,10,13]

(b) In the case of a rough bed, accelerating flows are also observed at high inclination angles. In this high velocity regime, direct or indirect measurements of the shear force at the bed[5,9] have shown that the material rheology is well described by a constant coulomb friction coefficient independent of the velocity.

(c) For intermediate value of the inclination angle in the rough bed configuration, steady uniform flows can be observed in a whole range of inclination. In this range the frictional force is able to balance the gravity force indicating a shear rate dependence. However, no clear description emerges from the different experiments. Suzuki and Tanaka[4] have proposed to describe this regime using a Bingham-like constitutive law but this approach is incompatible with the Coulomb-like behavior controlling the onset of the flow. Ancey *et al.*[15] have shown that the shear stress at the bed cannot be written as a function of the mean shear rate in the material. Vallance,[14] in an attempt to quantitatively compare experimental results with Savage theory,[20] has revealed some interesting scaling showing that the mean velocity of the flow varies as the thickness of the granular layer to the power 3/2. A similar scaling law has been observed recently by Azanza.[17] But to our knowledge no constitutive law has been proposed which convincingly describes the experimental observations.

From the theoretical point of view the major difficulty in describing inclined granular chute flows is that they belong to an intermediate flow regime, where both the friction between the grains and the collisions play an important role. Some attempts have been made to incorporate into a kinetic theory which describes the collisional interactions, an empirical rate independent stress tensor in order to take into account the friction.[12,20,22,23] These approaches qualitatively predict the fact that fully developed flows are only possible in a limited range of inclination angles. But unfortunately, a direct quantitative comparison with experiments is difficult since many experimentally inaccessible parameters are introduced in the models, especially in the boundary conditions.

In this paper, a more empirical point of view is adopted in order to describe the intermediate regime of granular chute flows. From precise and systematic velocity measurements, our goal was to experimentally determine the variation of the mean velocity of the flow as a function of the inclination of

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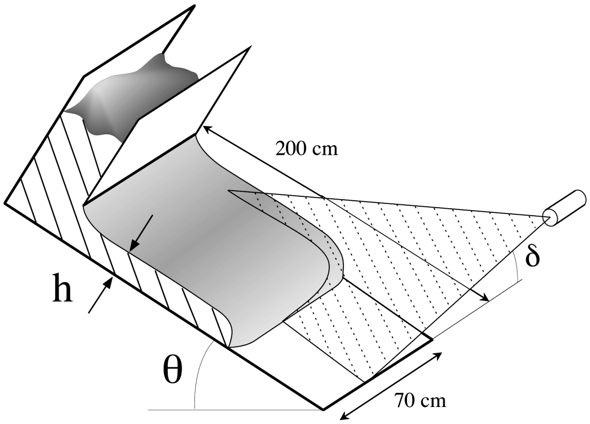

FIG. 1. Experimental setup.

the rough plane, the thickness of the layer, and also of the roughness of the bed. The ultimate purpose is to be able to propose some empirical constitutive laws which could then be useful in the description of more complex configurations closer to geophysical situations such as the propagation of fronts or the spreading of finite mass on irregular relief.[3,24]

The paper begins in Sec. II with the description of the experimental setup and of the measurement methods we have used. In Sec. III the experimental measurements are presented. The range of inclination and layer thickness giving rise to steady uniform flows is first determined. Velocity measurements are then presented and are shown to collapse into a single curve when properly scaled. The scaling properties are discussed in Sec. IV. We show how they lead to an empirical expression for the dynamic friction coefficient describing the interaction between the material and the rough surface. Concluding remarks are given in Sec. V.

II. EXPERIMENTAL METHODS

The experimental setup (Fig. 1) comprises a 2-m-long and 70-cm-wide plane that can be inclined from horizontal up to a maximum of 35°. The choice of such a wide channel (70 cm to be compared with 1 cm, the typical thickness of the granular layer) ensures that the measurements are not affected by the lateral boundaries which are known to dramatically change the flow structure.[7] The granular material flows from the reservoir through a gate whose opening can be precisely controlled. The rough bed condition is obtained by gluing one layer of particles on the inclined surface.

The particles used for the bulk as well as for the rough surface are spherical glass beads. In this paper we present results obtained for four different systems of beads (Table I).

TABLE I. Glass beads used for the bulk material and for the rough surface for the different systems studied; corresponding parameters of fit 3.

	Bulk	Rough surface	θ_1	θ_2	L
System 1	0.5mm±0.04	0.5mm±0.04	20.7°	32.8°	1.96
System 2	1.3mm±0.13	1.3mm±0.13	21.7°	26.4°	1.23
System 3	1.15mm±0.1	1.3mm±0.13	22.9°	30.4°	1.50
System 4	0.5mm±0.04	1.3mm±0.13	20.9°	29.1°	2.9

System 1 consists of 0.50-mm-diam glass beads flowing on a bed made of the same 0.50-mm-diam beads. Systems 2, 3, and 4 are, respectively, 1.3, 1.15, and 0.5 mm glass beads flowing on a rough surface made of 1.3 mm beads. These three systems have different roughness conditions between the bulk and the surface. Notice that systems 1 and 2 correspond to the same configuration at two different scales. Roughness conditions, however, differ slightly: For the bed made of 0.5 mm particles 72% of the surface is covered by the particles whereas 76% is covered for the bed made of 1.3 mm particles. The measurements will show that this slight change in the rough surface conditions affects velocity dramatically.

For the four systems of beads, we have systematically measured the mean velocity u of the granular layer for each inclination angle θ and for each thickness h of the layer. The method we choose to measure the mean velocity consists in tracking the front propagating down the slope using an image processing system.[25] The front is created by suddenly opening the gate, with the material then rushing down the slope. In the range of inclination (20°–28°) and thickness (4–25 particle diameters) studied, the front rapidly reaches a steady state, with a steady shape propagating at a constant velocity u. The front velocity u corresponds simply to the depth averaged velocity of the uniform steady flow that develops behind. Notice that using monodispersed glass beads prevents fingering instability.[26] The front remains straight in the cross-flow direction, which allows precise measurements. This method is very precise for low-velocity flows but becomes less accurate for rapid flows in which the front presents more saltating particles. However, even in the most rapid flows studied, the estimated error is less than 10%.

Another parameter that needs to be experimentally measured is the thickness h of the granular flowing layer. The thickness is well controlled by the opening of the gate but is not equal to the opening. A transient region exists close to the outlet where the thickness decreases before the steady uniform regime is reached. This region typically extends between 20 and 50 cm. The thickness h is measured down slope of this region by illuminating the surface with a sheet laser light slanted from the side with a very low incident angle δ (Fig. 1). When the granular material flows on the plane, the projection of the laser sheet is shifted laterally from its initial position. The lateral shift is proportional to the thickness of the layer and is inversely proportional to the tangent of δ. For low enough incident angle, measurements of h based on the shift of the laser sheet are obtained with a precision estimated to ± 0.2 mm.

We also have carried out measurements of the volume fraction occupied by the beads during the flow. The method consists in trapping a known portion of the material during the flow and weighing the trapped material. Knowing the trapped mass M, the density of the glass beads ρ_p, the thickness h, and the surface S of the trapped layer, one obtains the volume fraction $\nu = M/\rho_p Sh$. For all the steady uniform flows studied in this paper, the volume fraction is found to be roughly constant—equal to $\nu = 0.59 \pm 0.03$. No significant variation of ν with the control parameters has been put in evidence. The relatively high volume fraction indicates that

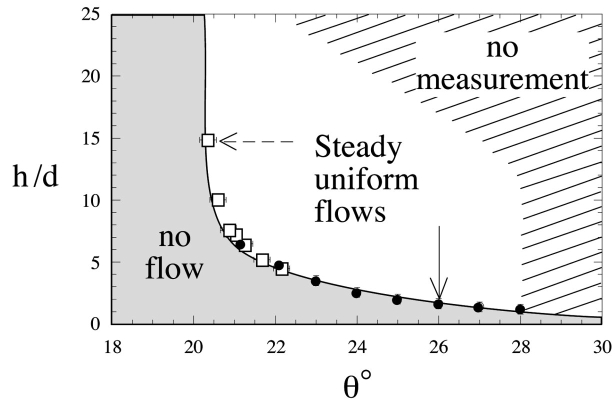

FIG. 2. Region of the parameter space $(\theta, h/d)$ where steady uniform flows are obtained; ●—measurements of $h_{\rm stop}(\theta)$; □—measurements of $\theta_{\rm stop}(h)$. Some error bars are hidden by the symbols. Data obtained with system 1. Solid arrow (respectively, the dashed arrow) represents a typical evolution of the system in the parameter space during the measurement of $h_{\rm stop}(\theta)$ [respectively, $\theta_{\rm stop}(h)$].

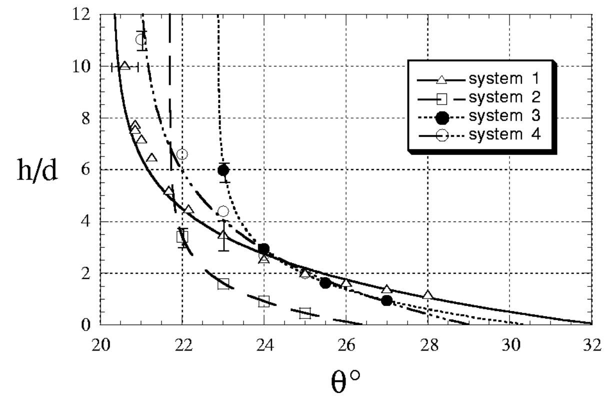

FIG. 3. $h_{\rm stop}$ as a function of θ for the four systems of beads.

we are studying a dense regime in which the frictional interactions between particles are important.

In experiments using granular materials, experimental difficulties often arise from electrostatic interactions or humidity effects. In the experiments presented in this paper the effects of the electrostatic interactions are minimized by sieving the glass particles through a metallic mesh connected to the ground before each experiment. The influence of humidity is much more difficult to control. In order to work at constant humidity, each set of experiments were carried out during a single day in which the ambient humidity varied less than 5%.

III. EXPERIMENTAL RESULTS

A. Existence of steady uniform flows

A first step before studying the dynamics of steady uniform flows is to determine the range of parameters for which they can be observed. These flows, characterized by a thickness and a mean velocity constant in space and time, are not observed for all values of the two controlled parameters h and θ. The limited region of the parameter space $(\theta, h/d)$ where steady uniform flows are observed is presented in Fig. 2 for system 1. The thickness is made dimensionless using the particle diameter d. Similar diagrams were first obtained by Vallance[14] for three-dimensional (3-D) chute flow experiments and more recently by Azanza[17] for two-dimensional (2-D) systems.

The steady uniform flow region is limited in the parameter space by two curves. The upper one corresponds with the transition to nonsteady or nonuniform flow observed when the inclination is too high (the flow then seems to accelerate along the slope) or with too thick a layer (the thickness continuously decreases along the slope) (Fig. 2). This upper limit has not been precisely determined.

In contrast, the lower curves separating the steady uniform flow region from a region where no flow is possible can be precisely determined and will play a crucial role in the scaling properties presented in Sec. III B. Two methods have been used to measure this flow limit. The first one consists in working at a constant inclination θ and slowly decreasing the thickness h down to the critical thickness $h_{\rm stop}$ when the flow stops (the solid arrow in Fig. 2). This is simply carried out by suddenly closing the gate of the reservoir once a thick flow has developed: The thickness then slowly decreases, the material slows down, stops, and leaves a static layer of material on the bed. The thickness $h_{\rm stop}$ of this layer depends on the inclination θ (filled circles in Fig. 2).

This method for measuring the flow limit—keeping θ constant and decreasing h—is precise for high inclinations when the deposit is thin. However, it becomes less accurate for lower inclinations when the thickness of the deposit strongly varies with the angle. In this region a second method has been used for measuring the flow limit. It consists in keeping the thickness h constant and slowly decreasing the inclination θ (the dashed arrow in Fig. 2) down to the critical value $\theta_{\rm stop}$ when the flow stops. The entire curve $\theta_{\rm stop}(h)$ is obtained by starting with different thickness h of the flow (squares in Fig. 2). For intermediate inclinations, both methods give the same curve, i.e., $\theta_{\rm stop}(h)$ and $h_{\rm stop}(\theta)$ are just inverse functions.

The function $h_{\rm stop}(\theta)$ reveals that the material experiences more resistance to flow near the rough surface than it does further away: This effect has previously been reported by others studies[14,27,28] and is known to be the consequence of nontrivial boundary effects. The scaling properties presented in the following will reveal the important role played by the function $h_{\rm stop}(\theta)$.

The other systems of beads we have studied present qualitatively the same diagram as Fig. 2. However, significant quantitative differences exist as shown in Fig. 3 where the function $h_{\rm stop}(\theta)$ is plotted for the four systems. For example, for $\theta = 23°$, we found $h_{\rm stop}/d = 3.5$, 1.5, 6, and 4.4 for, respectively, systems 1, 2, 3, and 4. This observation shows how sensitive granular chute flows are to slight changes in the roughness conditions. We will show in the following that velocity measurements also are sensitive to the roughness conditions. However, despite this influence of the boundary conditions, scaling properties can be found which reveal the role played by the function $h_{\rm stop}(\theta)$.

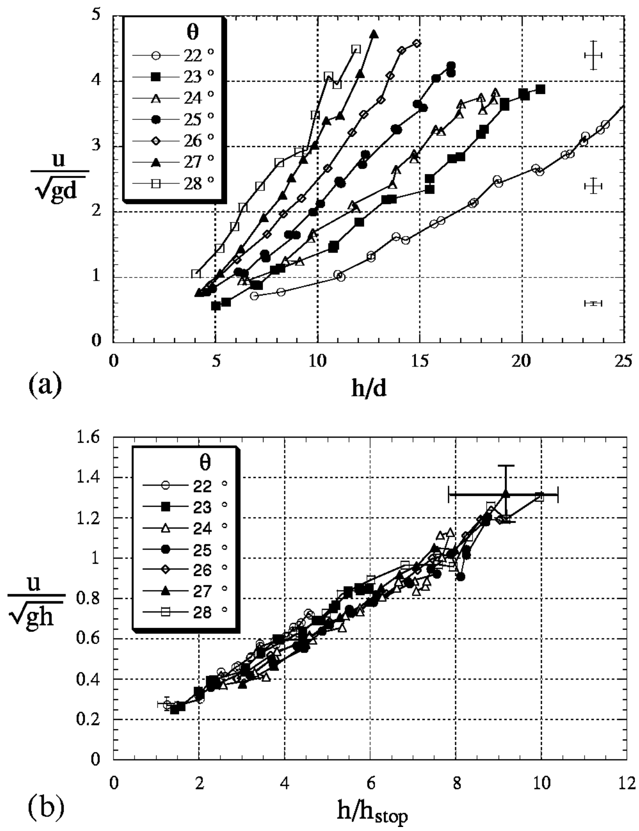

FIG. 4. (a) Dimensionless velocity $u/\sqrt{gd}$ as a function of h/d for different inclination angles. (b) Froude number $u/\sqrt{gh}$ as a function of $h/h_{\mathrm{stop}}(\theta)$ for different inclinations. Data obtained for system 1.

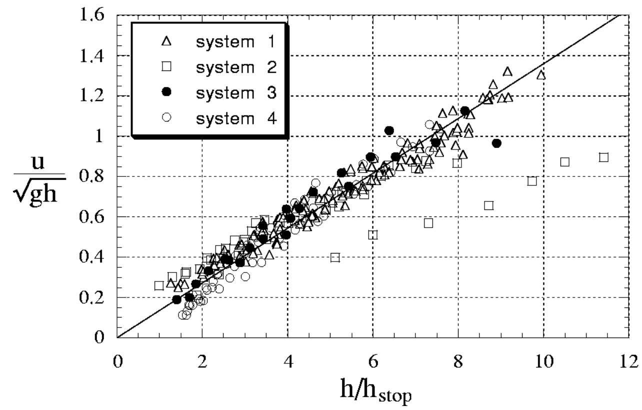

FIG. 5. Froude number $u/\sqrt{gh}$ as a function of $h/h_{\mathrm{stop}}(\theta)$ for the four systems of beads and for different inclination angles.

B. Velocity measurements and scaling properties

The results of the velocity measurements obtained in the steady uniform regime are presented in Fig. 4(a) for system 1. The dimensionless velocity $u/\sqrt{gd}$ where d is the particle diameter is plotted as a function of the dimensionless thickness h/d for different inclination angles θ. The mean velocity of the flow increases with increase of the thickness of the layer and with increase of the inclination [Fig. 4(a)]. Each of the four systems studied behave qualitatively the same way. However, comparing the different velocities obtained for the same dimensionless parameters again shows that the flow dynamics is sensitive to slight changes in the material size or roughness condition: For example at $\theta=24°$ and $h/d=7$, $u/\sqrt{gd}=1.0$, 2.3, 0.9 and 0.7 for systems 1, 2, 3, and 4. Such an influence of the relative roughness has been previously observed and studied in detail in the case of a single bead rolling down a rough surface.[29]

Despite this high sensitivity, we have been able to find a scaling that collapses all data onto a single curve. The idea was to choose a characteristic length scale other than the particle diameter to nondimensionalize the data. A possible choice, in view of the result of Sec. II A, is the critical thickness $h_{\mathrm{stop}}(\theta)$, which gives a θ dependent length scale. In search of relevant scaling we thus have plotted for system 1 the dimensionless velocity $u/\sqrt{gh}$, or the Froude number, as a function of $h/h_{\mathrm{stop}}(\theta)$. This scaling presented in Fig. 4(b) appears to be relevant: Curves obtained for different inclinations collapse onto a straight line. This observation indicates that the variation of the mean velocity u with the inclination θ is correlated to the variation of h_{stop} with θ.

The relevance of the scaling presented here becomes striking when comparing the different systems of beads. Not only does a similar collapse as in Fig. 4(b) occur for each set of beads, but the data for each of the four systems collapse into the *same straight line* (Fig. 5). The Froude number thus appears to be a linear function of h/h_{stop}, independent of the inclination, bead size, and of the roughness condition. The influence of those parameters are taken into account in the single function $h_{\mathrm{stop}}(\theta)$.

The curve in Fig. 5 which does not collapse with the other data, belongs to system 2 and has been obtained for $\theta=25°$, the highest inclination angle studied for this system. No explanation for this discrepancy can be given for the moment. Flows at this angle are rapid but seem to have reached the steady uniform regime. However, a lot of saltating particles are present at the front which could affect the measurements. In conclusion, except for this curve, a scaling has been found which gives rise to the collapse of all the data acquired for four different sets of beads.

IV. DISCUSSION

A. Prediction of the mean flow velocity

This experimental observation leads to several remarks. First of all, the scaling presented here provides an empirical way to predict the velocities of granular chute flows without carrying out velocity measurements (at least for systems of beads which are the only ones presented here). In Fig. 5 the Froude number $u/\sqrt{gh}$ is found to vary linearly with $h/h_{\mathrm{stop}}(\theta)$:

$$\frac{u}{\sqrt{gh}}=\beta\frac{h}{h_{\mathrm{stop}}(\theta)}, \tag{1}$$

with $\beta=0.136$ independent of the inclination, the bead size, and the roughness of the bed (this value of β corresponds to the best fit in Fig. 5 not taking into account the points of system 2 at $\theta=25°$). The variation of the mean velocity u with the control parameters h and θ is thus known as soon as one measures the function $h_{\text{stop}}(\theta)$, which does not involve any velocity measurement. Notice that, for a constant inclination θ, the empirical fit 1 predicts a 3/2 power law for the variation of u with the thickness h which is compatible with other experimental or numerical results obtained by Vallance[14] and Azanza.[17]

Another interesting observation is that the data of Fig. 5 do not converge to 0 when the thickness h/h_{stop} tends to 1. This point seems contradictory with the definition of h_{stop} as the critical thickness when the flow stops in an experiment carried out at a constant inclination and with a slowly decreasing thickness. However, experimentally the velocity does not evolve continuously to zero when approaching h_{stop}: When the thickness h decreases, the flow gently slows down and then suddenly freezes when h reaches h_{stop}. Such behavior is certainly the signature of a velocity weakening friction force[30,31] at the bed, a point which will be discussed later in the paper.

B. Role of the rough bed

From a more fundamental point of view, the scaling properties we have put in evidence underline the role played by the function $h_{\text{stop}}(\theta)$. Not only does this curve limit the flow region in the parameter space, but it also contains all the information about the influence on the flow dynamics of the inclination and of the other parameters like bead size or the roughness of the bed. This function h_{stop}, which has been reported by other studies,[14,27,28] is not yet fully understood but is known to result from complex boundary effects. A qualitative explanation[27] has been given in terms of dilatancy. Near the bed, the medium has to dilate more in order to flow than further away, which could explain the higher resistance to flow close to the rigid rough surface.

As the function $h_{\text{stop}}(\theta)$ is controlled by the boundary conditions, it follows from the scaling presented here that flow dynamics are probably also governed by boundary effects. This means that granular flows down rough inclined planes result not only from intrinsic rheology of the granular materials but also from the complex influence of the rough bed. The importance of the rough bottom could explain the difference between the scaling laws observed here and the scaling laws observed for the granular flow at the free surface of a pile in a rotating drum.[32] In the last case, no rigid boundary is present.

C. Description in terms of a dynamic friction coefficient

From the scaling properties one can extract some information about the friction forces that arise between the flowing layer and the rough surface. In the steady uniform regime, the flow simply results from the balance between the gravity and the shear stress that develops at the bed. A simple depth averaged force balance on an elementary slice of material yields the following equality:

$$\rho g h \sin(\theta) = \tau,$$

where ρ is the density of the granular medium and τ the shear stress at the bed. When divided by $\rho g h \cos(\theta)$, the normal stress on the plane, the force balance can be written in term of a dynamic friction coefficient μ defined as the ratio of the shear to the normal stress:

$$\tan(\theta) = \mu(u,h). \tag{2}$$

It is important to note that the above dynamic friction coefficient $\mu(u,h)$, which is assumed to be a function of the thickness h and of the mean velocity u, is not a property of the bulk material but describes the interaction between the material and the rough surface.

When performing an experiment at a given angle θ and with a thickness h, the granular layer adjusts its velocity in accordance with Eq. (2). In other terms, fixing the inclination is equivalent to fixing the friction coefficient. From the systematic velocity measurements we have carried out, it is thus possible to find the variation of μ with u and h by simply substituting $\tan\theta$ by μ in Eq. (1). In order to do so, we need an analytical expression for the function $h_{\text{stop}}(\theta)$. A good fit (Fig. 3) is given by

$$\tan\theta = \tan\theta_1 + (\tan\theta_2 - \tan\theta_1)\exp\left(-\frac{h_{\text{stop}}}{Ld}\right), \tag{3}$$

where d is the particle diameter, θ_1 corresponds to the angle where $h_{\text{stop}}(\theta)$ diverges, θ_2 to the angle where $h_{\text{stop}}(\theta)$ vanishes, and L is a characteristic dimensionless thickness over which $\theta_{\text{stop}}(h)$ varies. Values of θ_1, θ_2, and L for the different systems of beads are given in Table I. By substituting $\tan\theta$ by μ [Eq. (2)] and h_{stop} by $\beta h\sqrt{gh}/u$ [Eq. (1)] in Eq. (3) one gets the following expression for the dynamic friction coefficient:

$$\mu(u,h) = \tan\theta_1 + (\tan\theta_2 - \tan\theta_1)\exp\left(-\frac{\beta h}{Ld}\frac{\sqrt{gh}}{u}\right). \tag{4}$$

This empirical relation raises several issues. The first one concerns the high velocity regime. Equation (4) predicts that the friction coefficient tends to a limit equal to $\tan\theta_2$ when the velocity tends to infinity. The existence of an upper limit for the friction coefficient implies according to Eq. (2) that no steady uniform flow can be obtained for inclination higher than θ_2. This prediction is compatible with other laboratory experiments[5,9] carried out at high inclinations and which have shown that in this regime the flow accelerates, the friction coefficient between the flowing layer and the rough surface being constant, independent of the velocity. Notice that according to the empirical description, Eq. (4), this maximum friction coefficient $\tan\theta_2$ is simply equal to the inclination angle when h_{stop} vanishes.

On the other hand, the low velocity regime is not well described by Eq. (4). According to this empirical relation, steady uniform flows can be observed for any thickness as soon as the inclination is higher than θ_1. This is in contradiction with the existence of the function $h_{\text{stop}}(\theta)$. However, one has to keep in mind that expression (4) for the friction coefficient derives from expression (1) for the velocity, which is only valid for $h > h_{\text{stop}}$ or equivalently for $u/\sqrt{gh} > \beta$. Consequently, Eq. (4) is also only valid for Froude number greater than β. Below this limit, the friction coefficient certainly presents some velocity weakening variations, as suggested by the sudden stop of the material observed in experiments carried out with a decreasing thickness. Such a behavior, which has been put in evidence in other configurations,[31] cannot be captured from the study of the steady uniform flows.

V. SUMMARY AND CONCLUSIONS

This paper has presented experiments on the flows of glass beads down rough inclined surfaces. By systematically studying the variation of the mean velocity with the inclination of the plane and with the thickness of the granular layer it has been possible to spotlight some interesting scaling properties. All the data obtained for different systems of beads corresponding to different surface roughness conditions collapse into a straight line when expressed in terms of the Froude number as function of $h/h_{\text{stop}}(\theta)$:

$$\frac{u}{\sqrt{gh}}=\beta\frac{h}{h_{\text{stop}}(\theta)},$$

with $\beta=0.136$. The function $h_{\text{stop}}(\theta)$, which contains all the information about the influence of the inclination, the bead size, and the roughness of the bed, is simply obtained by measuring the thickness of the layer of material remaining on the surface when a flow created at inclination θ stops.

The interpretation of this scaling is far from being straightforward. What exactly controls the function $h_{\text{stop}}(\theta)$, and why its measurement is enough for predicting the mean flow velocity, are complex questions. More detailed information about the internal structure of the flow (velocity profiles, internal stresses, etc.), which is not accessible in our experiment, will certainly be helpful. However, the role played by the function $h_{\text{stop}}(\theta)$ suggests that the influence of the rough boundary is important. Theoretical approaches should then take into account the boundary conditions in a proper way.

Although there is no precise interpretation of the scaling laws, we have shown that the experimental measurements can yield information about the frictional force mobilized during the flow. An empirical relation has been derived for the friction coefficient between the granular layer and the rough bed as a function of the mean velocity and the thickness. This formulation, characterized by two critical angles and a length scale, could then be used in more complex configurations. It can, for example, be introduced in models of avalanches like that of Savage and Hutter.[3,24] However, one has to be cautious in using the result presented in this paper. The scaling laws as well as the empirical friction law have been derived for granular material comprised of quasi-monodispersed glass beads. The relevance of those scaling properties in the case of angular particles or for polydispersed material such as the ones encountered in geophysical situations is an open question which needs further investigation.

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[1] T. Takahashi, ''Debris flow,'' Annu. Rev. Fluid Mech. **13**, 57 (1981).
[2] C. S. Campbell, P. W. Cleary, and M. Hopkins, ''Large-scale landslide simulations: Global deformations, velocities and basal-friction,'' J. Geophys. Res. **100**, 8267 (1995).
[3] M. Naaim, S. Vial, and R. Couture, ''Saint Venant approach for rock avalanches modelling,'' in *Multiple Scale Analyses and Coupled Physical Systems: Saint Venant Symposium* (Presses de l'École Nationale des Ponts et chaussées, Paris, 1997).
[4] A. Suzuki and T. Tanaka, ''Measurement of flow properties of powder along an inclined plane,'' Ind. Eng. Chem. Fundam. **10**, 84 (1971).
[5] D. A. Augenstein and R. Hogg, ''Friction factors for powder flow,'' Powder Technol. **10**, 43 (1974).
[6] D. A. Augenstein and R. Hogg, ''An experimental study of the flow of dry powders over inclined surfaces,'' Powder Technol. **19**, 205 (1978).
[7] S. B. Savage, ''Gravity flow of cohesionless granular materials in chutes and channels,'' J. Fluid Mech. **92**, 53 (1979).
[8] M. Ishida, H. Hatano, and T. Shirai, ''The flow of solid particles in an aerated inclined channel,'' Powder Technol. **27**, 7 (1980).
[9] O. Hungr and N. R. Morgenstern, ''Experiments on the flow behaviour of granular materials at high velocity in an open channel,'' Geotechnique **34**, 405 (1984).
[10] J. S. Patton, C. E. Brennen, and R. H. Sabersky, ''Shear flows of rapidly flowing granular materials,'' ASME J. Appl. Mech. **52**, 172 (1987).
[11] T. J. Drake, ''Structural features in granular flows,'' J. Geophys. Res. **95**, 8681 (1990).
[12] P. C. Johnson, P. Nott, and R. Jackson, ''Frictional-collisional equations of motion for particulate flows and their application to chutes,'' J. Fluid Mech. **210**, 501 (1990).
[13] H. Ahn, C. E. Brennen, and R. H. Sabersky, ''Measurements of velocity, velocity fluctuation, density, and stresses in chute flows of granular materials,'' ASME J. Appl. Mech. **59**, 119 (1991).
[14] J. W. Vallance, ''Experimental and field studies related to the behavior of granular mass flows and the characteristics of their deposits'' thesis, Michigan Technological University, 1994.
[15] C. Ancey, P. Coussot, and P. Evesque, ''Examination of the possibility of a fluid-mechanics treatment of dense granular flows,'' Mech. Cohesive-Frictional Mater. **1**, 385 (1996).
[16] E. Azanza, F. Chevoir, and P. Moucheront, ''Experimental study of rapid granular flows in a two-dimensional channel,'' in *Powder and Grains 97*, edited by R. P. Behringer and J. T. Jenkins (Balkema, Rotterdam, 1997).
[17] E. Azanza, ''Ecoulements granulaires bidimensionnels sur un plan incliné'' thèse de l'École Nationale des Ponts et Chaussées, 1998.
[18] C. S. Campbell and C. E. Brennen, ''Chute flows of granular material: Some computer simulations,'' ASME J. Appl. Mech. **52**, 172 (1985).
[19] O. R. Walton, ''Numerical simulation of inclined chute flows of monodisperse, inelastic, frictional spheres,'' in *Advances in Micromechanics of Granular Materials*, edited by H. H. Shen *et al.* (Elsevier, Amsterdam, 1992).
[20] S. B. Savage, ''Granular flows down rough inclines—review and extension,'' in *Mechanics of Granular Materials: New Models and Constitulive Relations*, edited by J. T. Jenkins and M. Satake (Elsevier, Amsterdam, 1983).
[21] S. B. Savage, ''The mechanics of rapid granular flows,'' in *Advances in Applied Mechanics*, edited by T. Y. Wu and J. Hutchinson (Academic, New York, 1984).
[22] K. G. Anderson and R. Jackson, ''A comparison of the solutions of some proposed equations of motion of granular materials for fully developed flow down inclined planes,'' J. Fluid Mech. **241**, 145 (1992).
[23] K. Hutter and K. R. Rajagopal, ''On flows of granular materials,'' Continuum Mech. Thermodyn. **6**, 81 (1994).
[24] S. B. Savage and K. Hutter, ''The motion of a finite mass of granular material down a rough incline,'' J. Fluid Mech. **199**, 177 (1989).
[25] R. Wayne, public domain NIH-Image program available by anonymous ftp from wippy.nimh.nih.gov.

[26]O. Pouliquen, J. Delour, and S. B. Savage, ''Fingering in granular flows,'' Nature (London) **386**, 816 (1997).
[27]O. Pouliquen and N. Renaut, ''Onset of granular flows on an inclined rough surface: Dilatancy effects,'' J. Phys. II **6**, 923 (1996).
[28]S. Douady and A. Daerr, ''Formation of sandpiles, avalanches on an inclined plane,'' in *Physics of Dry Granular Media*, NATO ASI Series, edited by H. J. Hermann, J. P. Hovi, and S. Luding (Kluwer Academic, Dordrecht, 1998).
[29]F. X. Riguidel, R. Jullien, G. H. Ristow, A. Hansen, and D. Bideau, ''Behavior of a sphere on a rough inclined plane,'' J. Phys. I **4**, 261 (1994).
[30]F. Heslot, T. Baumberger, B. Perrin, B. Caroli, and C. Caroli, ''Creep, stick-slip, and dry-friction dynamics: Experiments and a heuristic model,'' Phys. Rev. E **49**, 4973 (1994).
[31]S. Nasuno, A. Kudrolli, and J. P. Gollub, ''Friction in granular layers: Hysteresis and precursor,'' Phys. Rev. Lett. **79**, 949 (1997).
[32]J. Rajchenbach, ''Continuous flows and avalanches of grains,'' in Ref. 28.