Significant improvements of the space-time ETAS model for forecasting of accurate baseline seismicity

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ETAS model: Ogata(1988, JASA)

$$\lambda(t) = \mu + \sum_{\{j:t_j<t\}} e^{\alpha\{Mj-M_c\}}\nu(t-t_j)$$

where

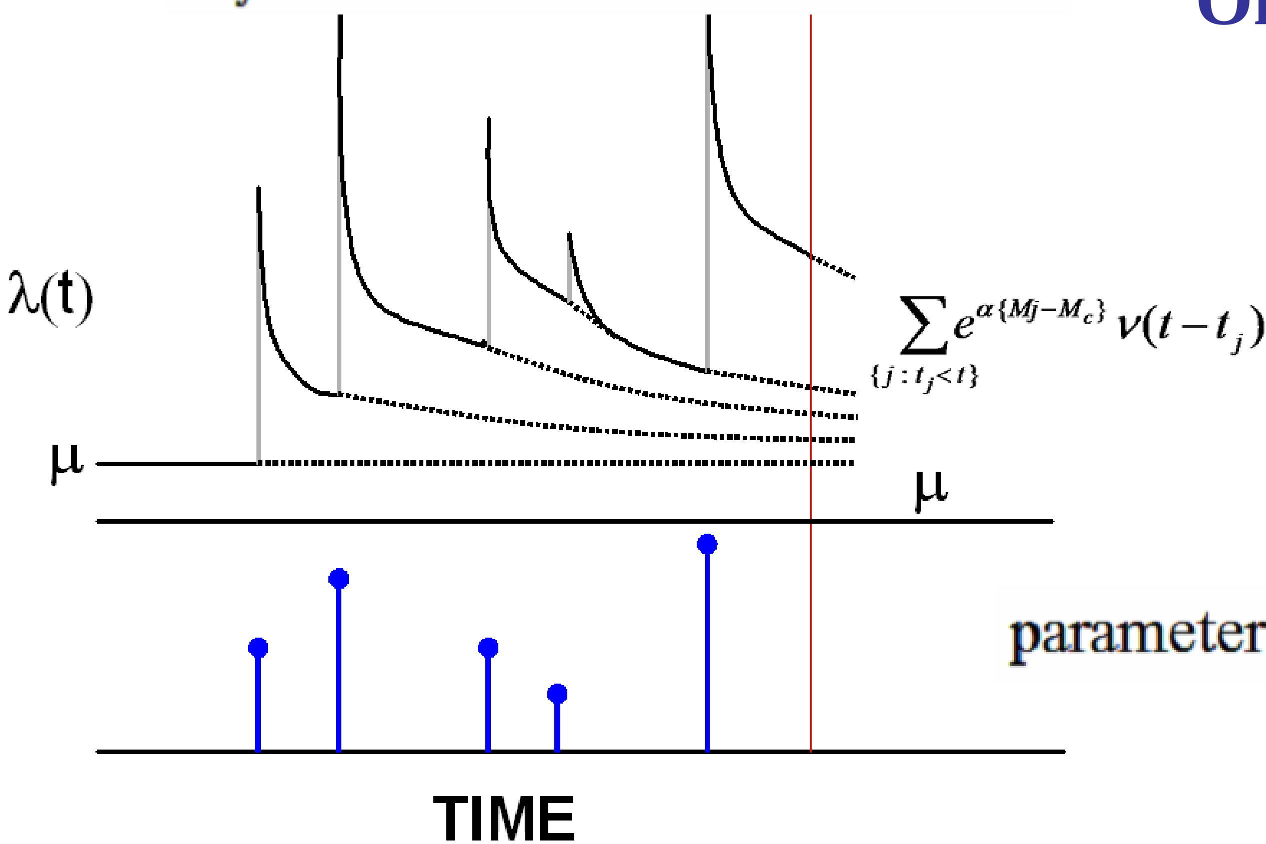
t_j is occurrence time of jth event;

M_j is magnitude of jth event;

$$\nu(t) = \frac{K}{(t+c)^p}$$

Omori-Utsu formula

μ



parameters are $(\lambda_0, K, c, \alpha, p)$.

M.L.E.

Space-Time ETAS model

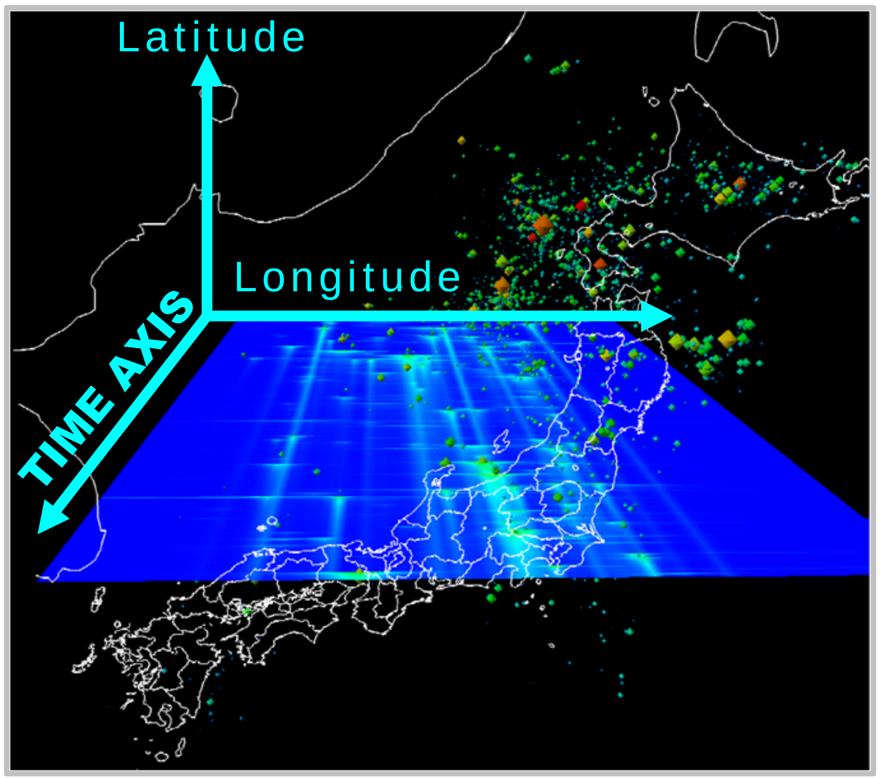
$$\lambda(t,x,y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t-t_j+c)^p} \left\{ \frac{Q_j(x,\, y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

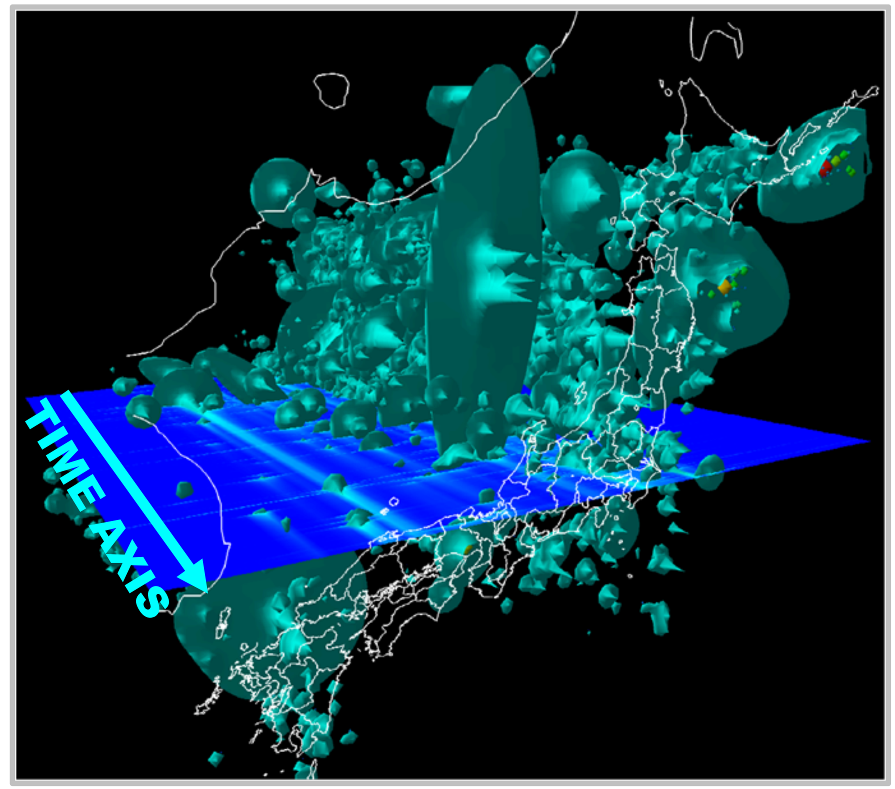
where

$$Q_j(x,\, y) = (x-x_j,\ y-y_j)\ S_j \begin{pmatrix} x-x_j \\ y-y_j \end{pmatrix}$$

Earthquakes (particles)
In and around Japan; depth<100km;
1926 - 1995; magnitude >= 5.0

Space-time ETAS model
Isosurface of an occurrence rate
$\lambda(t, x, y)$





Space-Time ETAS model

$$\lambda(t, x, y) = \mu + \sum_{\{j:\, t_j < t\}} \frac{K}{(t - t_j + c)^p} \left\{ \frac{Q_j(x,\ y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

Space-Time ETAS model

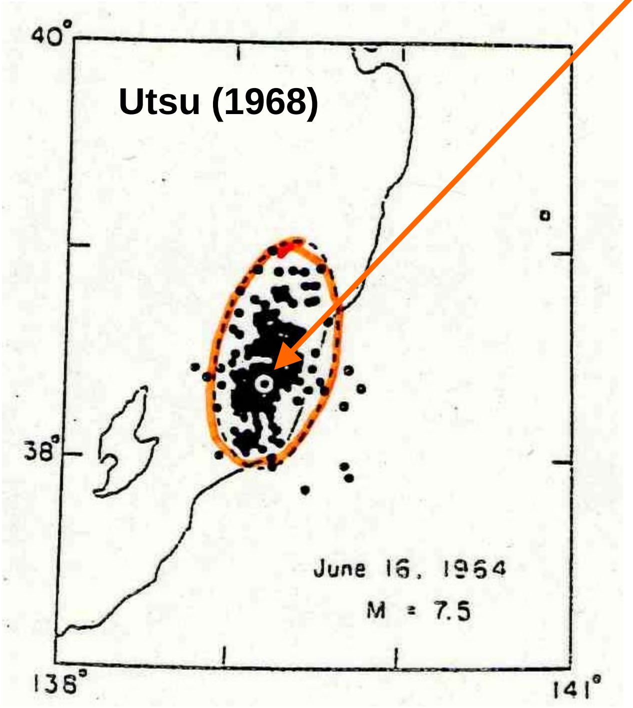
$$\lambda(t,x,y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t-t_j+c)^p} \left\{ \frac{Q_j(x,\ y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

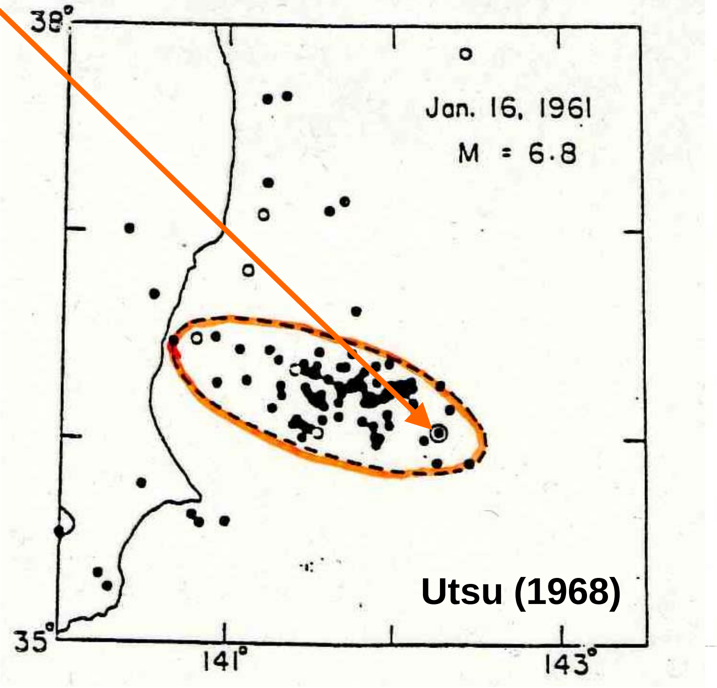
where

$$Q_j(x,\ y) = (x-\bar{x}_j,\ y-\bar{y}_j)\, S_j \begin{pmatrix} x-\bar{x}_j \\ y-\bar{y}_j \end{pmatrix}$$

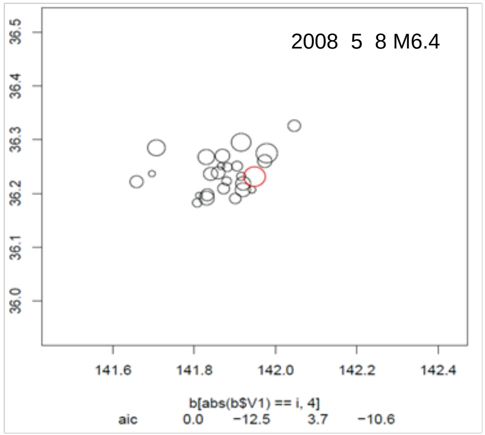
for **ANISOTROPY**:

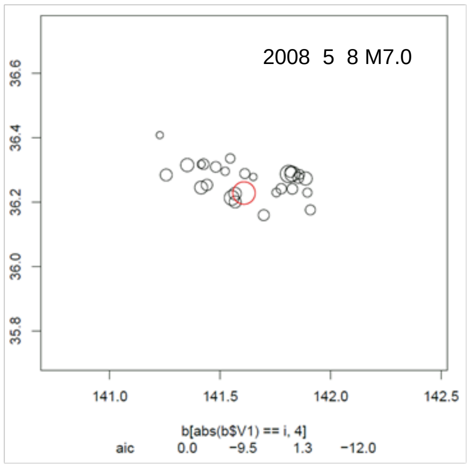
Epicenter

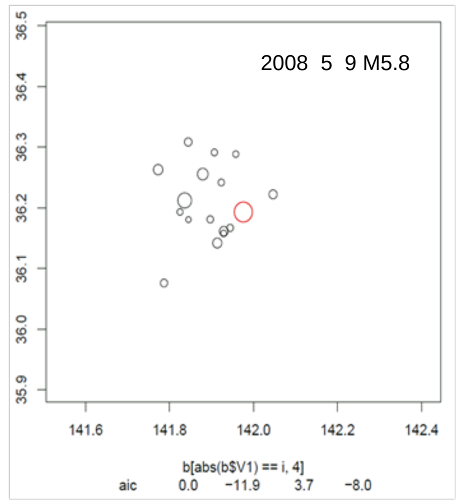


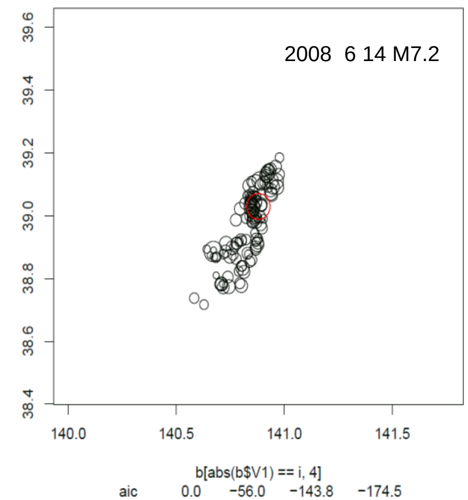


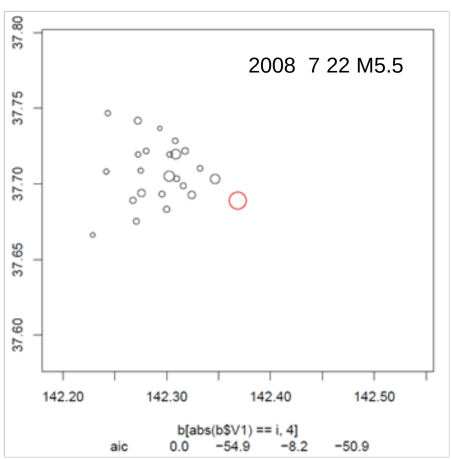
Spatial distributions of the first 1 hour aftershocks (Red circle = mainshock)

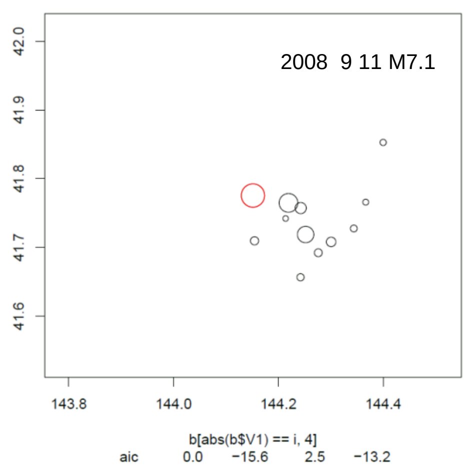


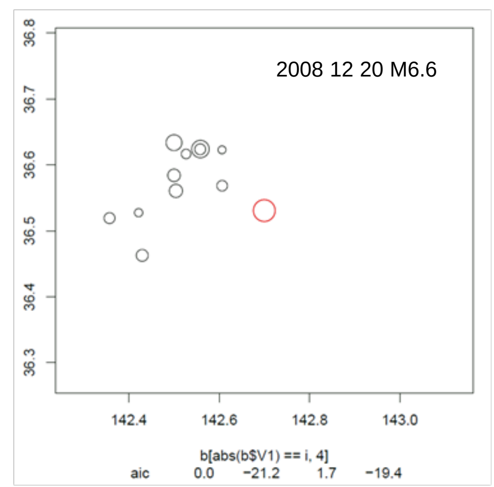












$$\lambda(t,x,y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t-t_j+c)^p}\left\{\frac{Q_j(x,\ y)}{e^{\alpha M_j}} + d\right\}^{-q}$$

Immediately after a large event, we adopt:

Isotropic kernel within 1 hour

$$Q_j(x,y) = (x-x_j)^2 + (y-y_j)^2 = (x-x_j, y-y_j)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x-x_j\\ y-y_j\end{pmatrix}$$

Anisotropic kernel after 1 hour

$$Q_j(x,y) = \frac{1}{\sqrt{1-\rho^2}}\left(\frac{\sigma_2^2}{\sigma_1^2}(x-\bar{x}_j)^2 - 2\rho(x-\bar{x}_j)(y-\bar{y}_j) + \frac{\sigma_1^2}{\sigma_2^2}(y-\bar{y}_j)^2\right)$$

$$= \frac{1}{\sigma_1^2\sigma_2^2\sqrt{1-\rho^2}}(x-\bar{x}_j, y-\bar{y}_j)\begin{pmatrix}\sigma_1^2 & \rho\sigma_1\sigma_2\\ \rho\sigma_1\sigma_2 & \sigma_2^2\end{pmatrix}^{-1}\begin{pmatrix}x-\bar{x}_j\\ y-\bar{y}_j\end{pmatrix}$$

$$Q_j(x,y) = \frac{1}{\sqrt{1-\rho^2}}\left(\frac{\sigma_2^2}{\sigma_1^2}(x-x_j^0)^2 - 2\rho(x-x_j^0)(y-y_j^0) + \frac{\sigma_1^2}{\sigma_2^2}(y-y_j^0)^2\right) = \frac{1}{\sigma_1^2\sigma_2^2\sqrt{1-\rho^2}}(x-\bar{x}_j, y-\bar{y}_j)\begin{pmatrix}\sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2\end{pmatrix}^{-1}\begin{pmatrix}x-\bar{x}_j \\ y-\bar{y}_j\end{pmatrix}$$

Model selection using the AIC

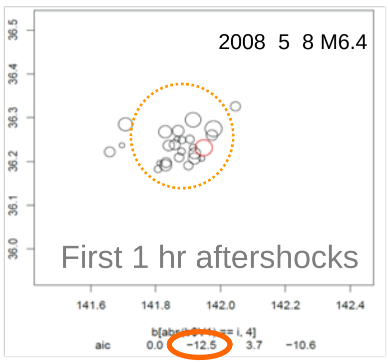
Models

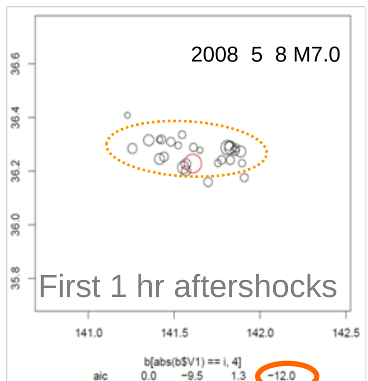
(1) $N\left(\begin{pmatrix}x_1 \\ y_1\end{pmatrix}, \begin{pmatrix}\hat{\sigma}^2 & 0 \\ 0 & \hat{\sigma}^2\end{pmatrix}\right)$

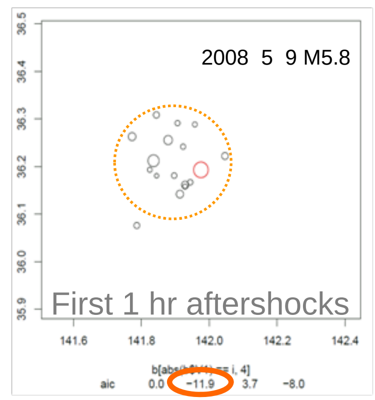
(2) $N\left(\begin{pmatrix}\bar{x} \\ \bar{y}\end{pmatrix}, \begin{pmatrix}\hat{\sigma}^2 & 0 \\ 0 & \hat{\sigma}^2\end{pmatrix}\right)$

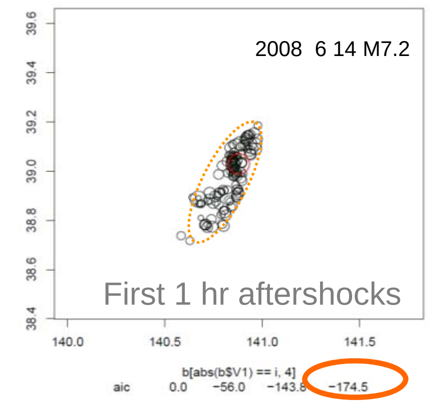
(3) $N\left(\begin{pmatrix}x_1 \\ y_1\end{pmatrix}, \begin{pmatrix}\hat{\sigma}_1^2 & \hat{\rho}\hat{\sigma}_1\hat{\sigma}_2 \\ \hat{\rho}\hat{\sigma}_1\hat{\sigma}_2 & \hat{\sigma}_2^2\end{pmatrix}\right)$

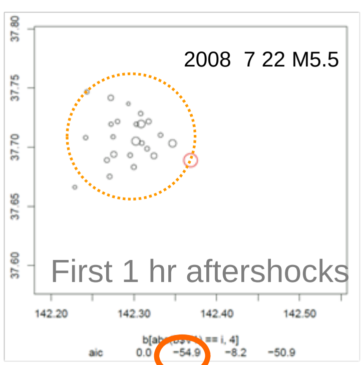
(4) $N\left(\begin{pmatrix}\bar{x} \\ \bar{y}\end{pmatrix}, \begin{pmatrix}\hat{\sigma}_1^2 & \hat{\rho}\hat{\sigma}_1\hat{\sigma}_2 \\ \hat{\rho}\hat{\sigma}_1\hat{\sigma}_2^2 & \hat{\sigma}_2\end{pmatrix}\right)$

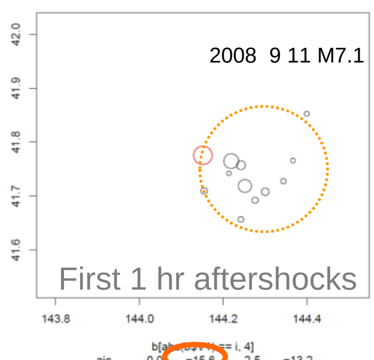


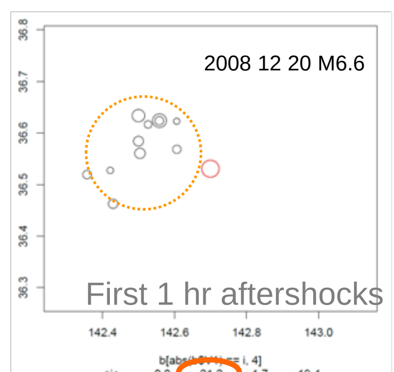












Location Dependent Space-Time ETAS model

Space-Time ETAS model

$$\lambda(t,x,y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t-t_j+c)^p} \left\{ \frac{Q_j(x,\ y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

where

$$Q_j(x,\ y) = (x-x_j,\ y-y_j)\, S_j \begin{pmatrix} x-x_j \\ y-y_j \end{pmatrix}$$

Heterogeneity in Space

$$\mu \Longrightarrow \mu\,(x,y);$$

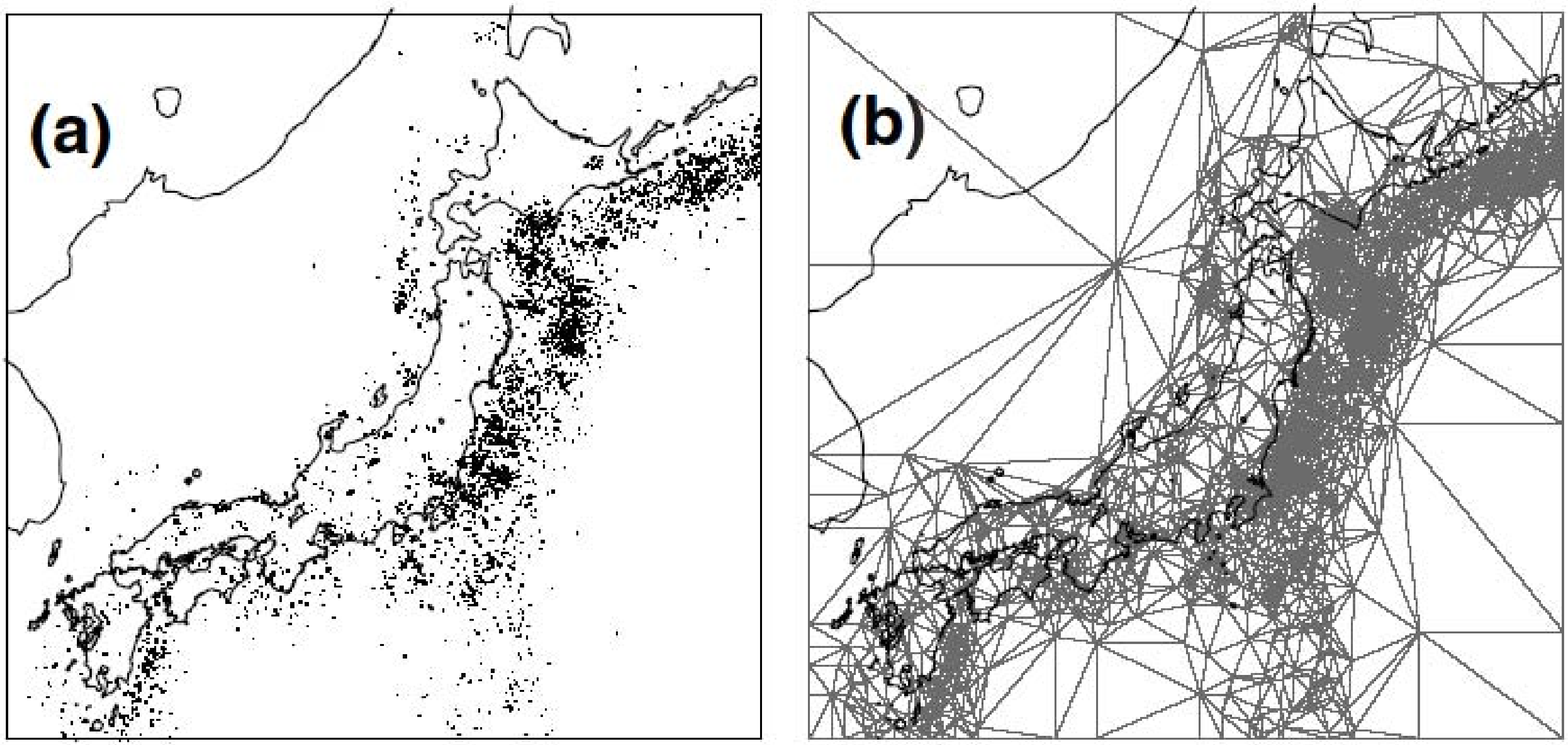
$$K \Longrightarrow K\,(x_j,y_j); \quad \alpha \Longrightarrow \alpha\,(x_j,y_j);$$

$$p \Longrightarrow p\,(x_j,y_j); \quad q \Longrightarrow q\,(x_j,y_j)$$

Hierarchical Space-Time (HIST) ETAS model

estimated an Objective Bayesian method with smoothness constraints

1926 - 1995, M >= 5.0, depth<100km



Space-Time ETAS model

$$\lambda(t,x,y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t-t_j+c)^p} \left\{ \frac{Q_j(x,\,y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

Location Dependent Space-Time **ETAS model** and occurrence data

$\{(t_i, x_i, y_i, M_i); \quad i = 1, \cdots, n\}$ in $[0, T] \times A$ are given. Then **Log Likelihood** is

$$\log L(\theta) = \log L\left(\mu_{\theta_1}, K_{\theta_2}, \alpha_{\theta_4}, p_{\theta_5}, q_{\theta_7}\right)$$
$$= \sum_{i=1}^{n} \log \lambda_\theta(t_i, x_i, y_i) - \int_0^T \iint_A \lambda_\theta(t, x, y) dt dx dy.$$

where $\theta = (\theta_1, \theta_2, \theta_4, \theta_5, \theta_7)$

Penalized Log Likelihood

$$Q(\theta | \; w_\mu, w_K, w_\alpha, w_p, w_q)$$
$$= \log L(\theta) - penalty(\theta | \; w_\mu, w_K, w_\alpha, w_p, w_q)$$

where the $penalty$ is

$$\int\int_A dxdy \left\{ w_1(\mu_x^2 + \mu_y^2) + w_2(K_x^2 + K_y^2) + w_3(\alpha_x^2 + \alpha_y^2) + w_4(p_x^2 + p_y^2) + w_5(q_x^2 + q_y^2) \right\}$$

$\rho = (\; w_\mu, w_K, w_\alpha, w_p, w_q \;)$

$$posterior(\theta \,|\, \rho) = \frac{L(\theta) \cdot prior(\theta \,|\, \rho)}{\Lambda(\rho)}$$

Normalizing factor of Posterior

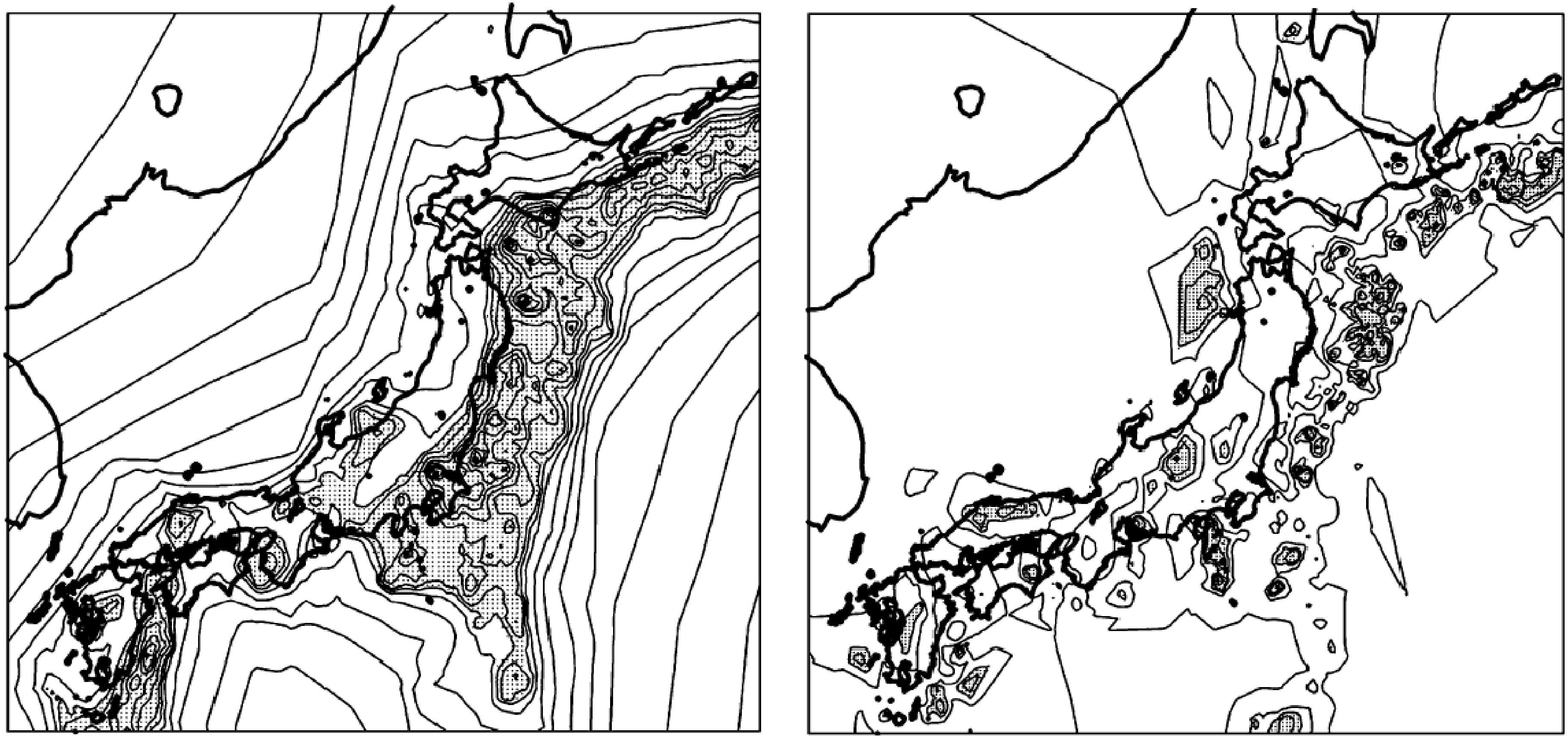
$$\Lambda(\rho) = \int \ldots \int L(\theta) \cdot prior(\theta \,|\, \rho) d\theta$$

(**Likelihood** of a Bayesian model)

Choose ρ that maximize the $\Lambda(\rho)$, **and then minimize**

$$ABIC = (-2) \max_{\boldsymbol{\rho}} \{\log \Lambda(\boldsymbol{\rho})\} + 2 \times \dim(\boldsymbol{\rho})$$

Akaike Bayesian information criterion (Akaike, 1980)



Space-Time ETAS model

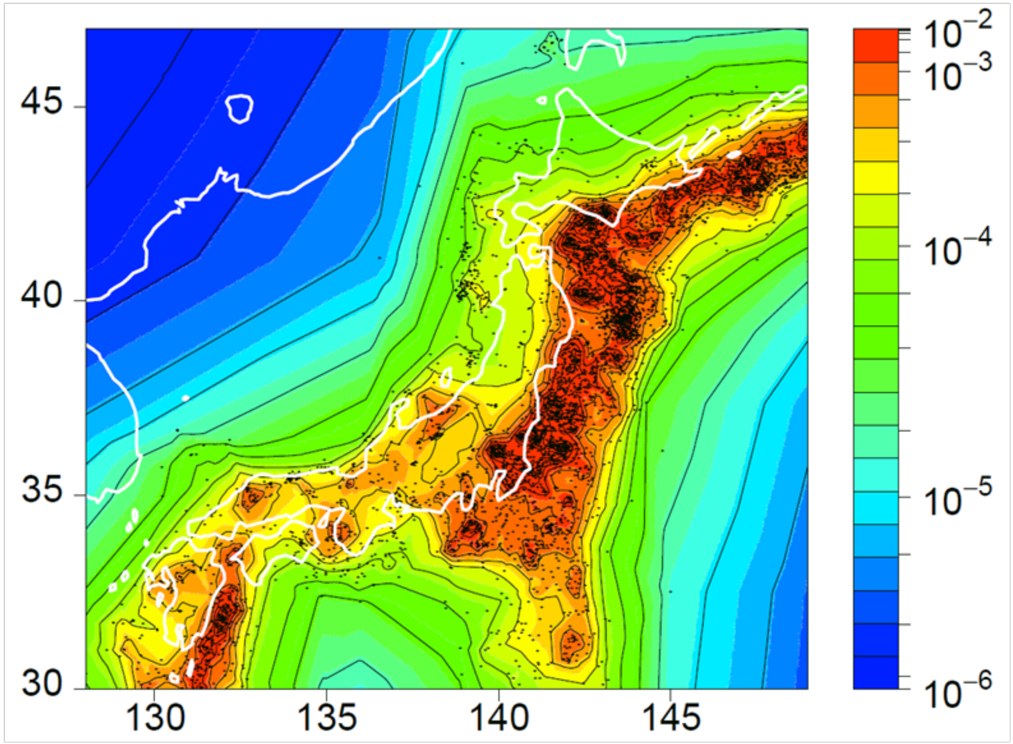
$$\lambda(t, x, y) = \mu + \sum_{\{j:\ t_j<t\}} \frac{K}{(t - t_j + c)^p} \left\{ \frac{Q_j(x,\ y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

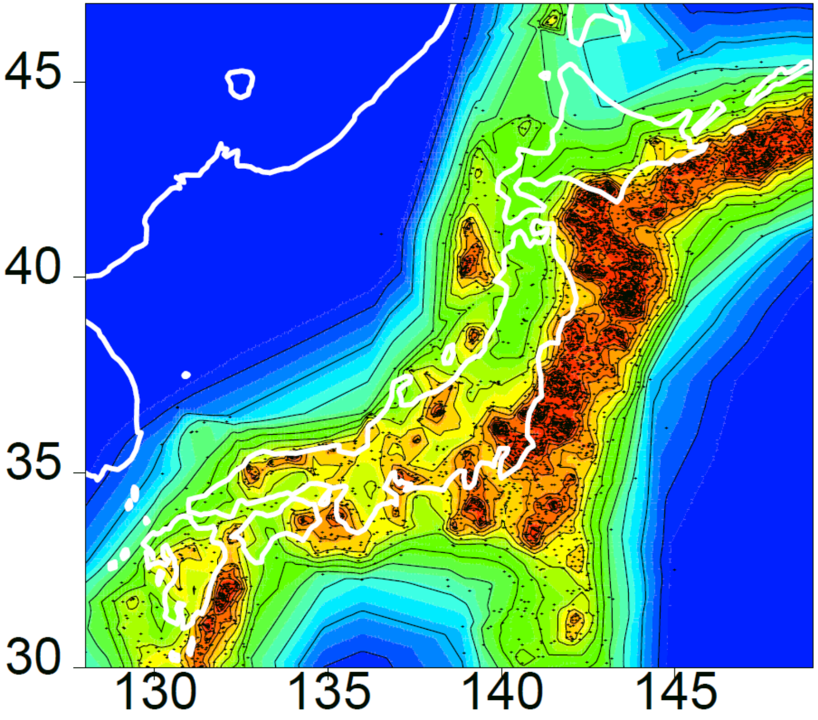
Long term forecast of large earthquakes, in and around Japan

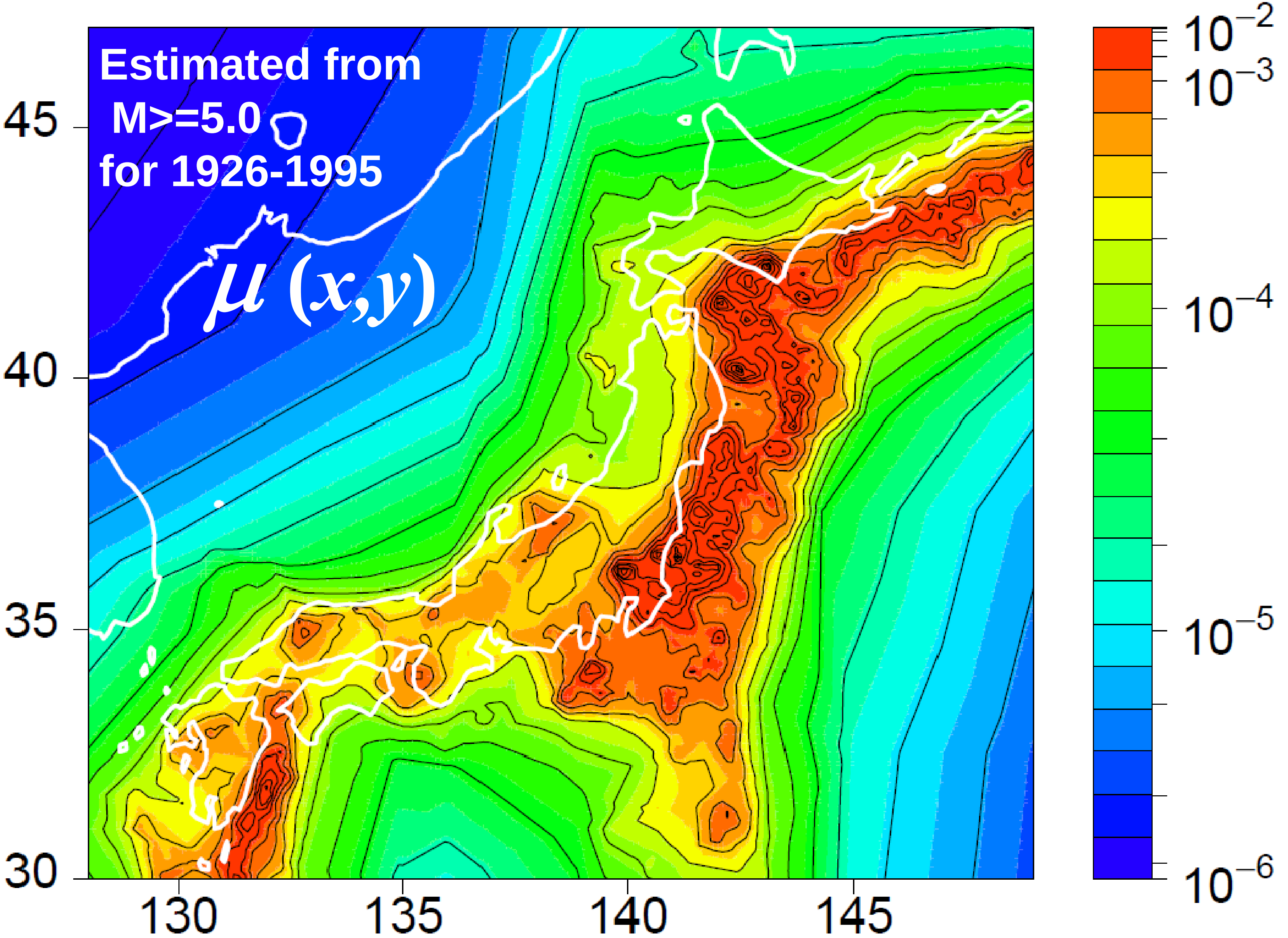
1926-1995 $M \geq 5.0$

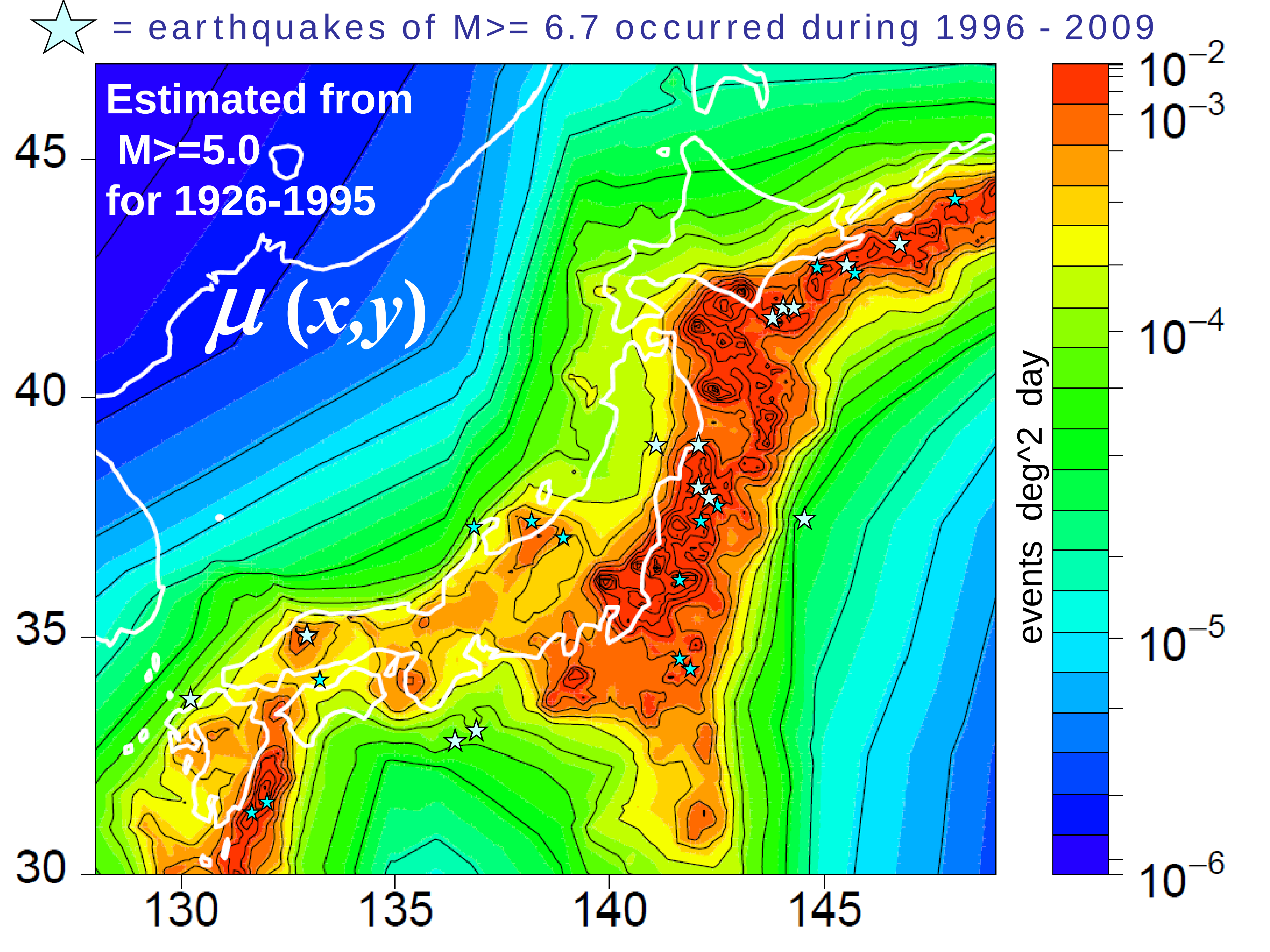
$\mu(x,y)$

$$\lambda(x,y) = \frac{1}{70}\int_{1926}^{1995} \lambda(t,x,y)\,dt$$



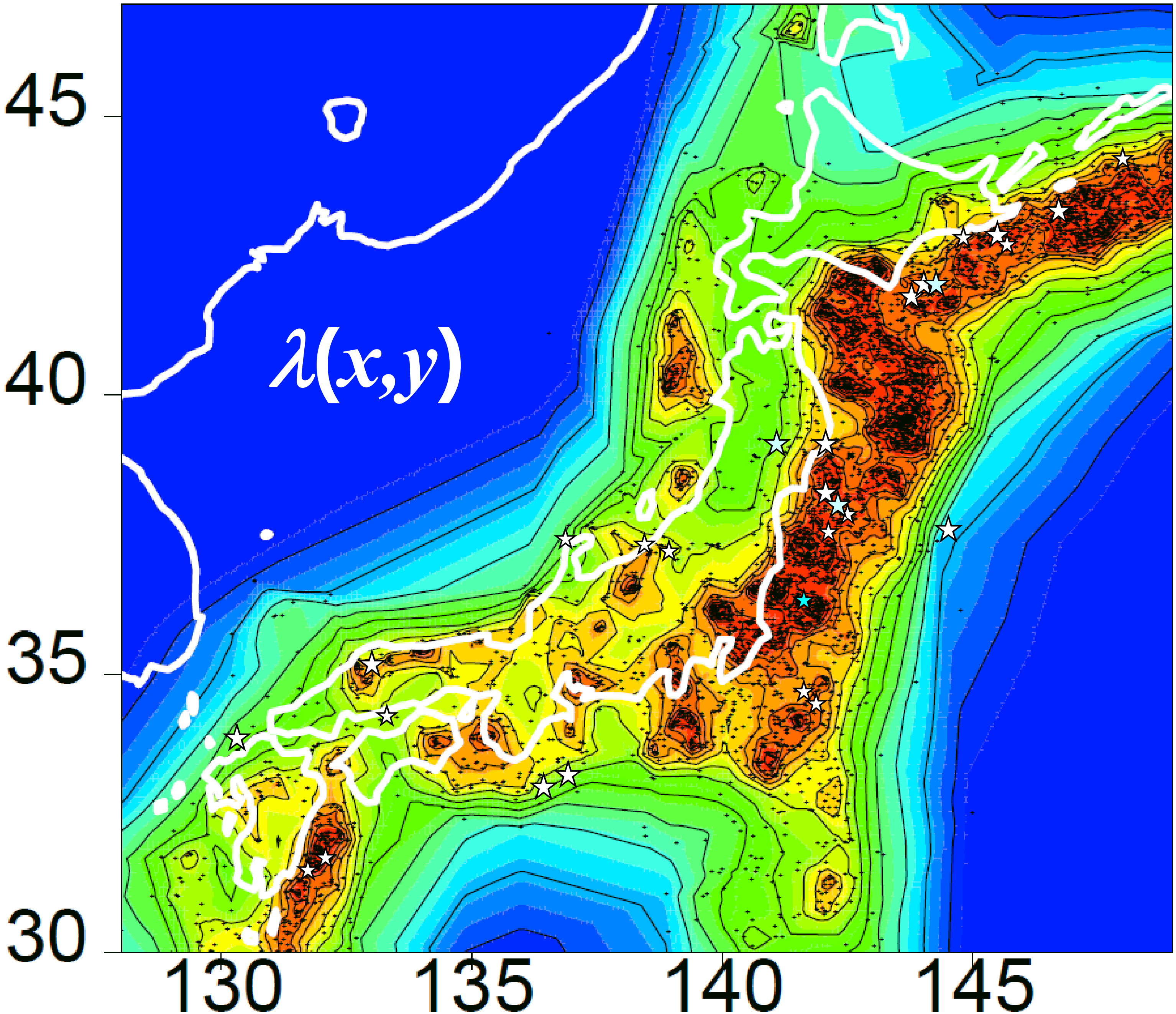



Estimated from
M>=5.0
for 1926-1995
$\mu(x,y)$
45
40
35
30
130
135
140
145
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}

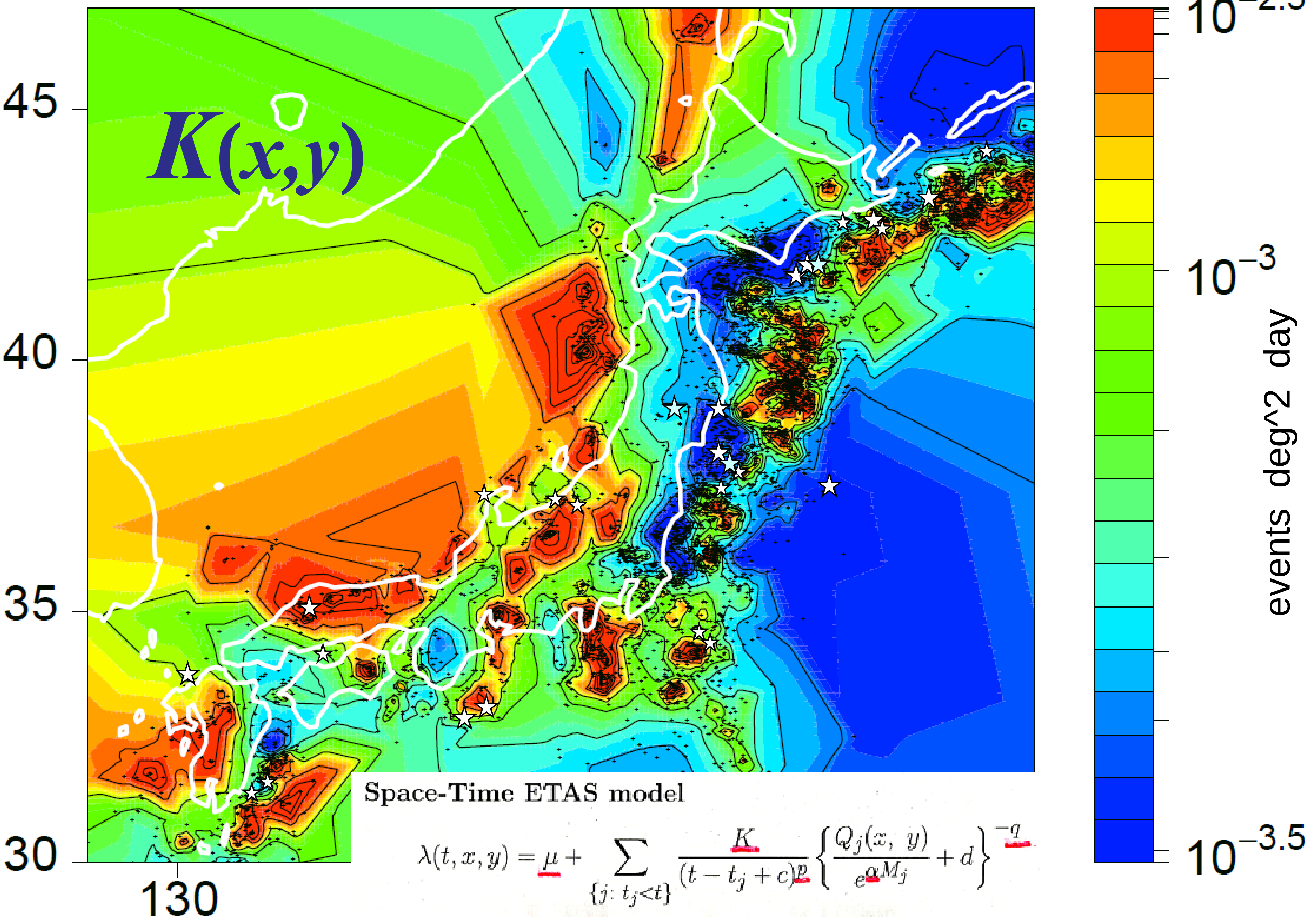


= earthquakes of M>= 6.7 occurred during 1996 - 2009
Estimated from
M>=5.0
for 1926-1995
μ (x,y)
45
40
35
30
130
135
140
145
10^-2
10^-3
10^-4
10^-5
10^-6
events deg^2 day

Earthquakes of M>= 6.7 occurred during 1996 - 2009



Earthquakes of M>= 6.7 occurred during 1996 - 2009



Space-Time ETAS model

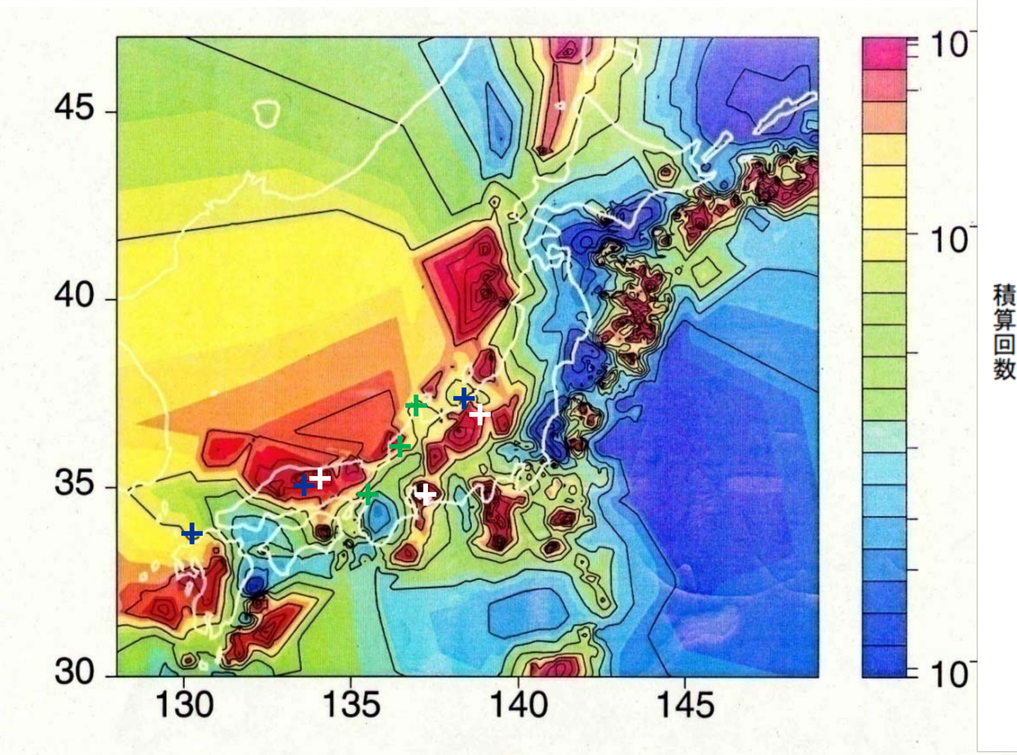
$$\lambda(t,x,y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t-t_j+c)^p} \left\{ \frac{Q_j(x,\ y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

Short term forecast of earthquakes, in and around Japan

$$\lambda(t,x,y)=\mu+\sum_{\{j:\,t_j<t\}}\frac{K}{(t-t_j+c)^p}\left\{\frac{Q_j(x,\,y)}{e^{\alpha M_j}}+d\right\}^{-q}$$

Mainshock M6.8~7.3

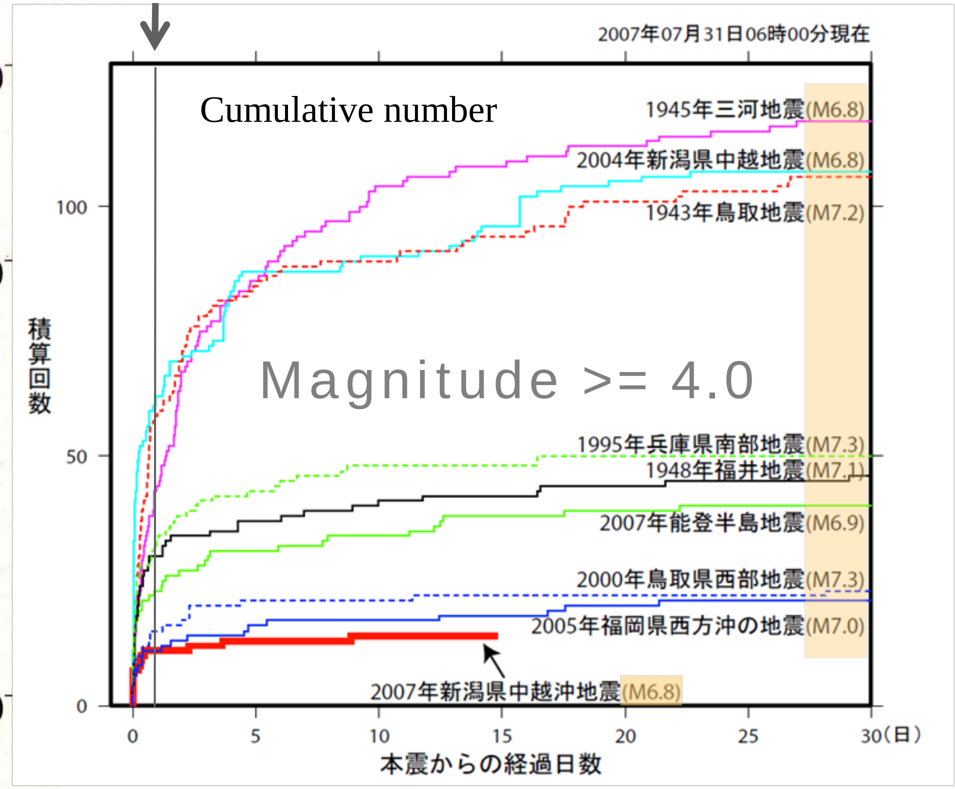
Due to the JMA report.



***K*-values**

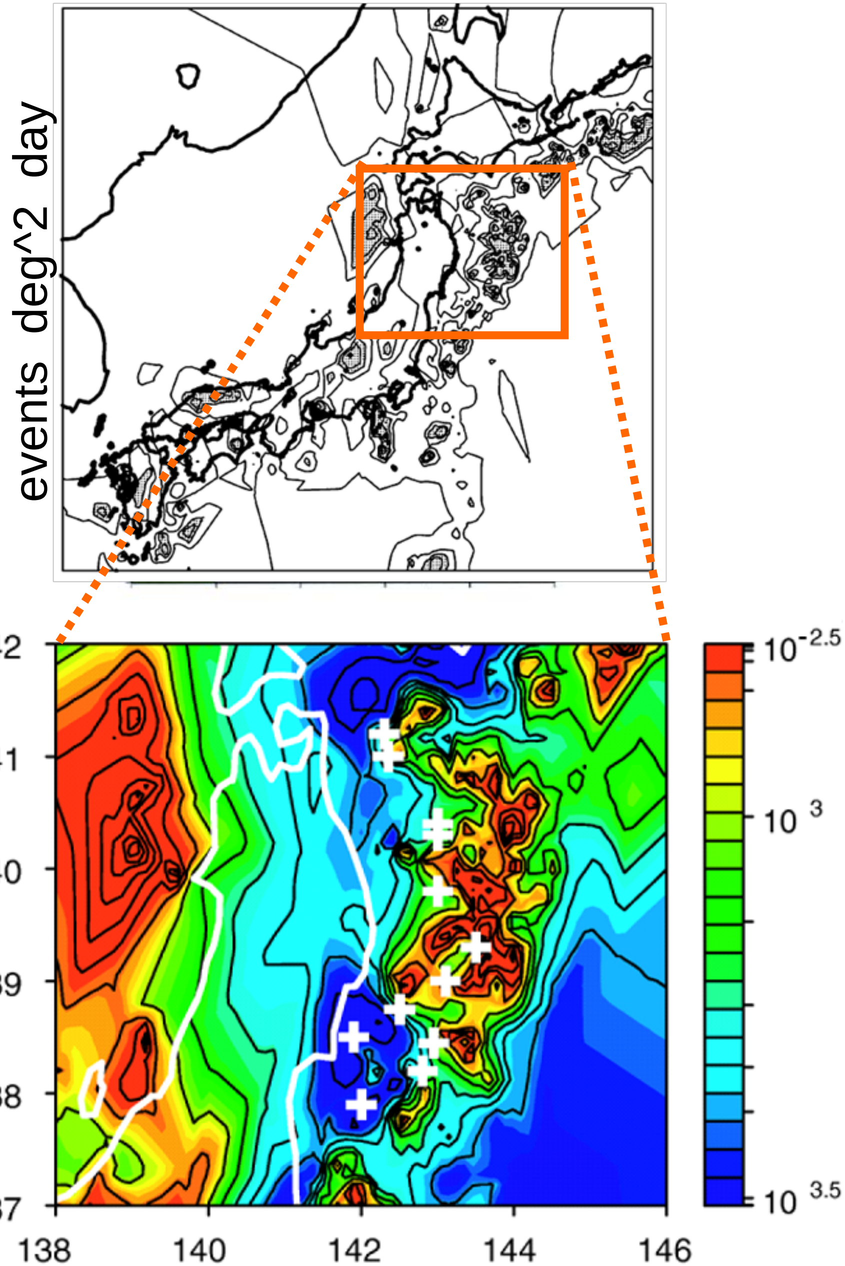
Estimated from M>=5.0 for 1926-1995

24 hours



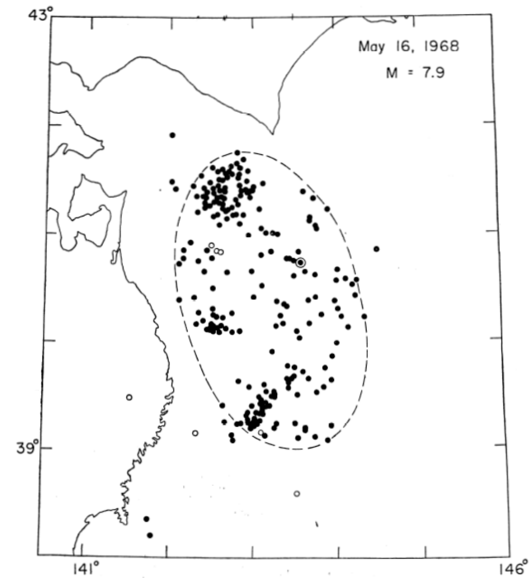
Elapsed time after main shock (days)

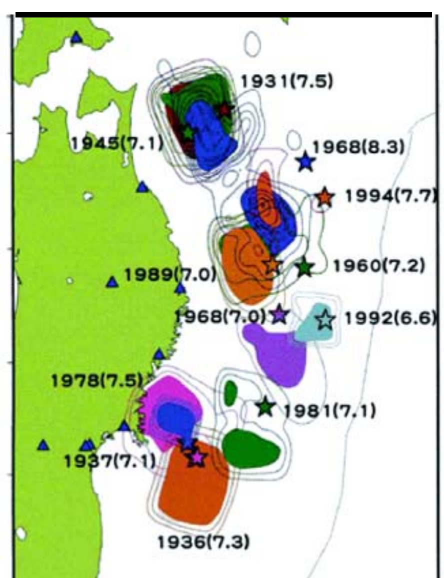
K-values

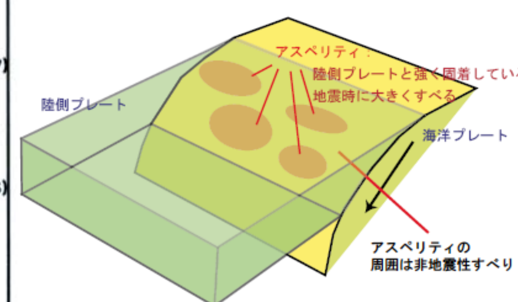


1968 M7.9 Tokachi aftershocks

Utsu(1969)

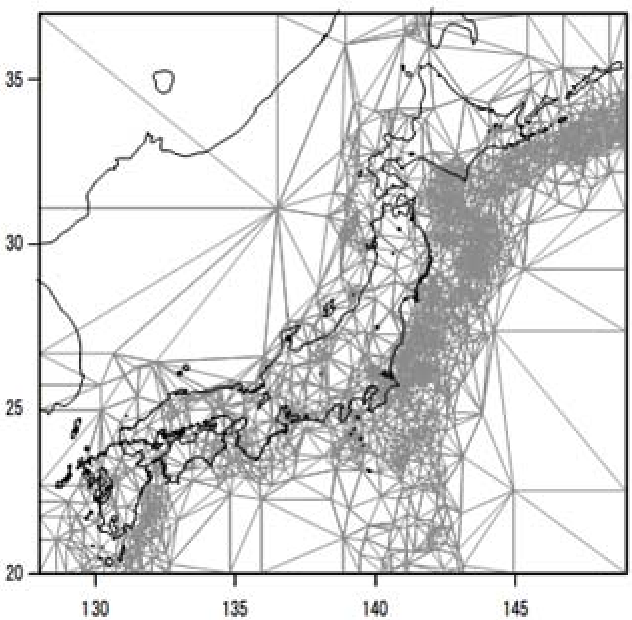






Asperities

Yamanaka & Kikuchi (2001)



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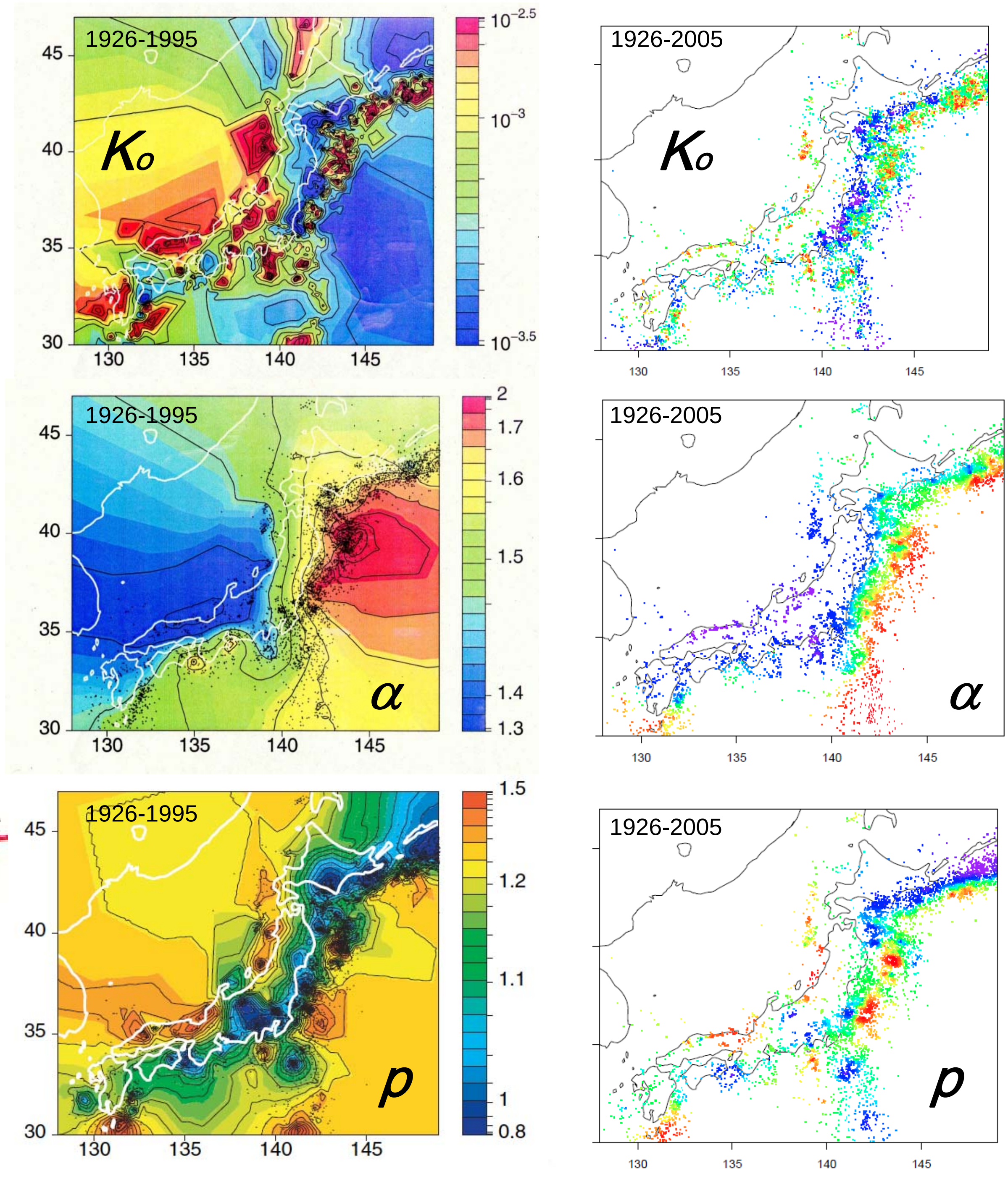
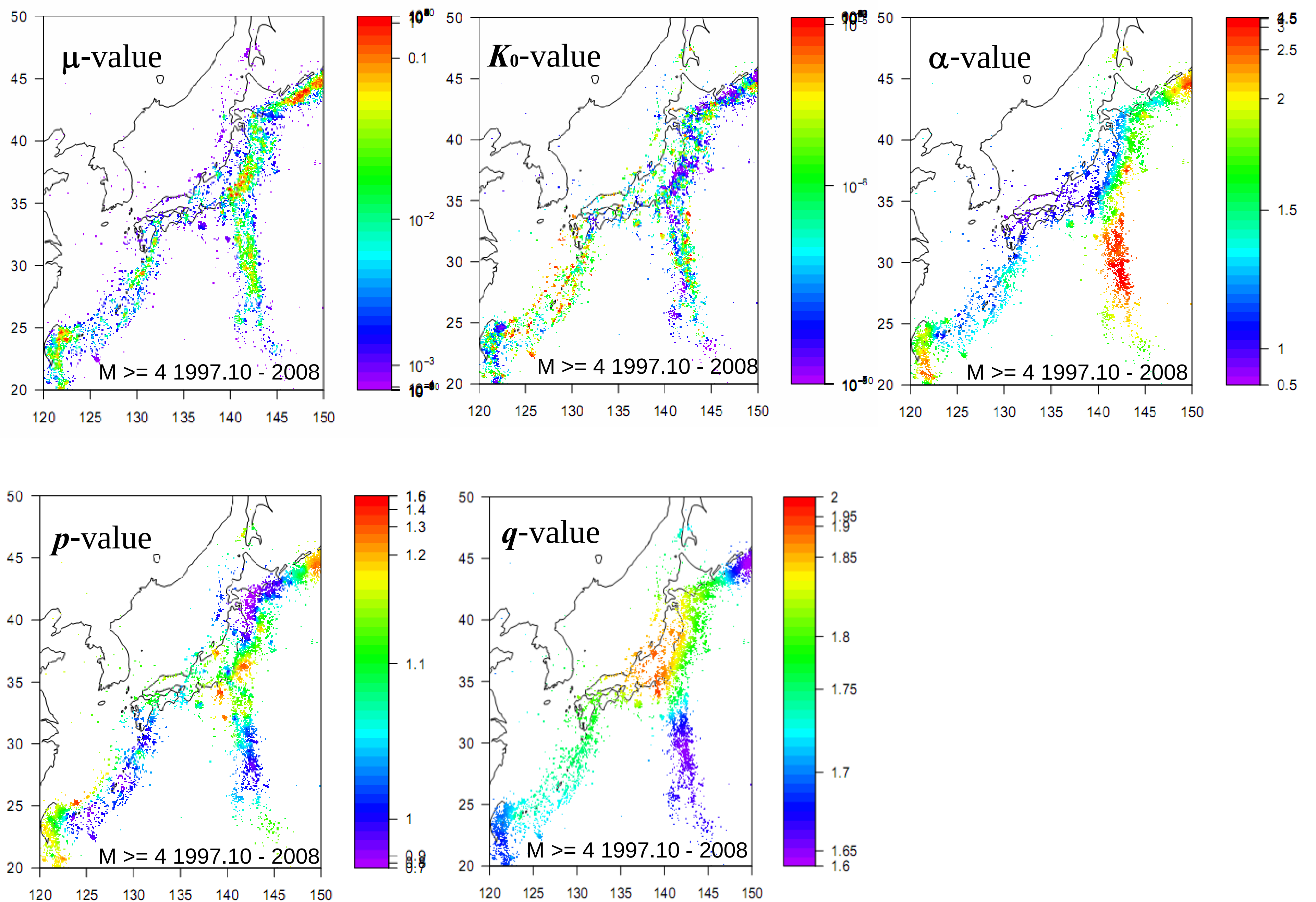
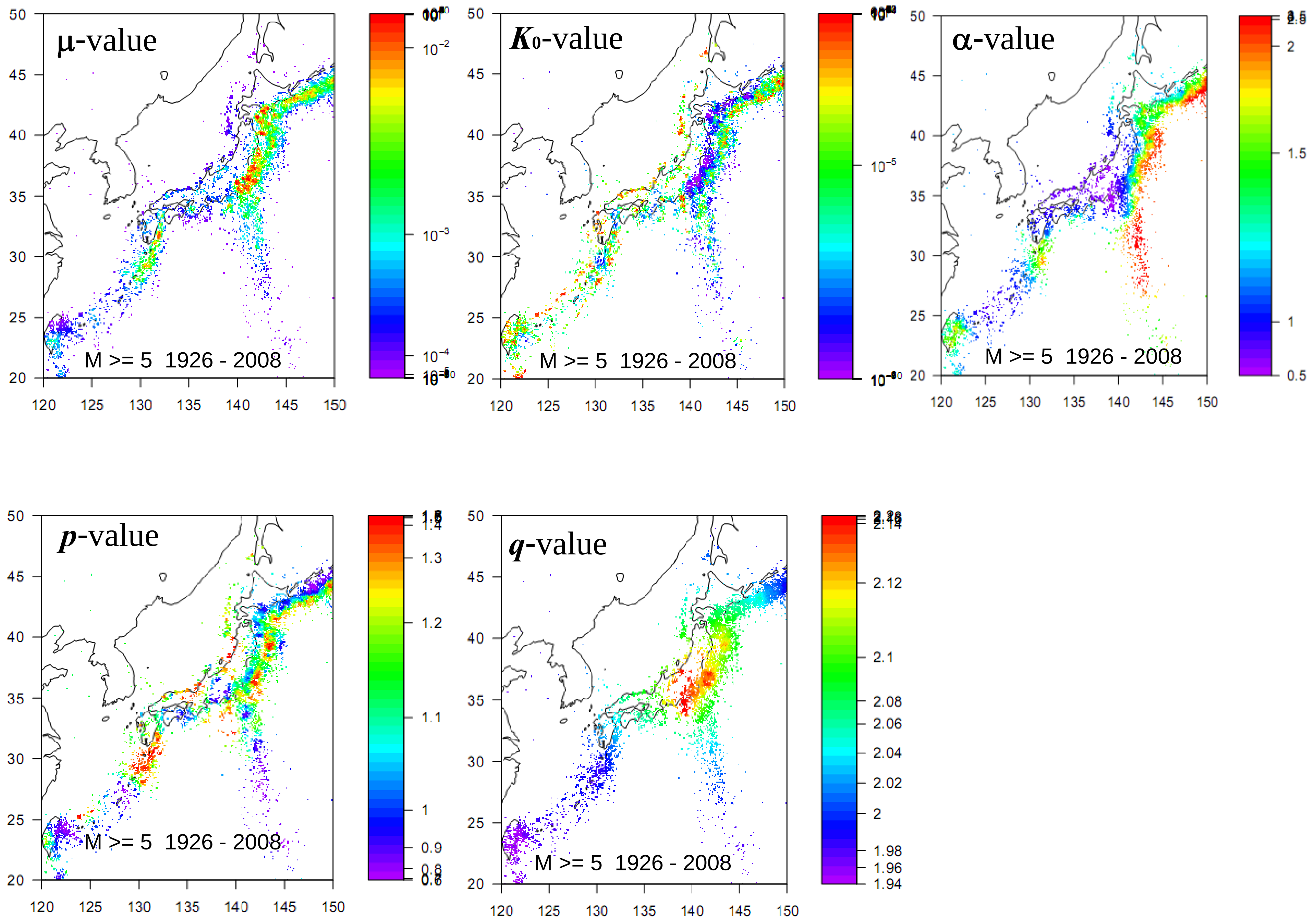
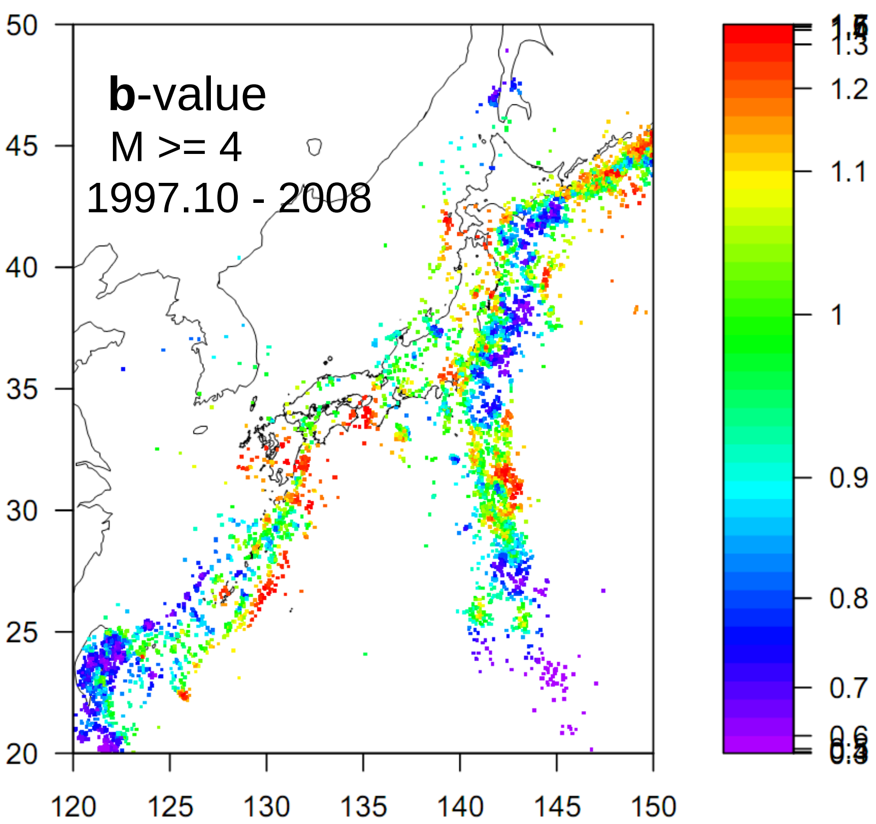


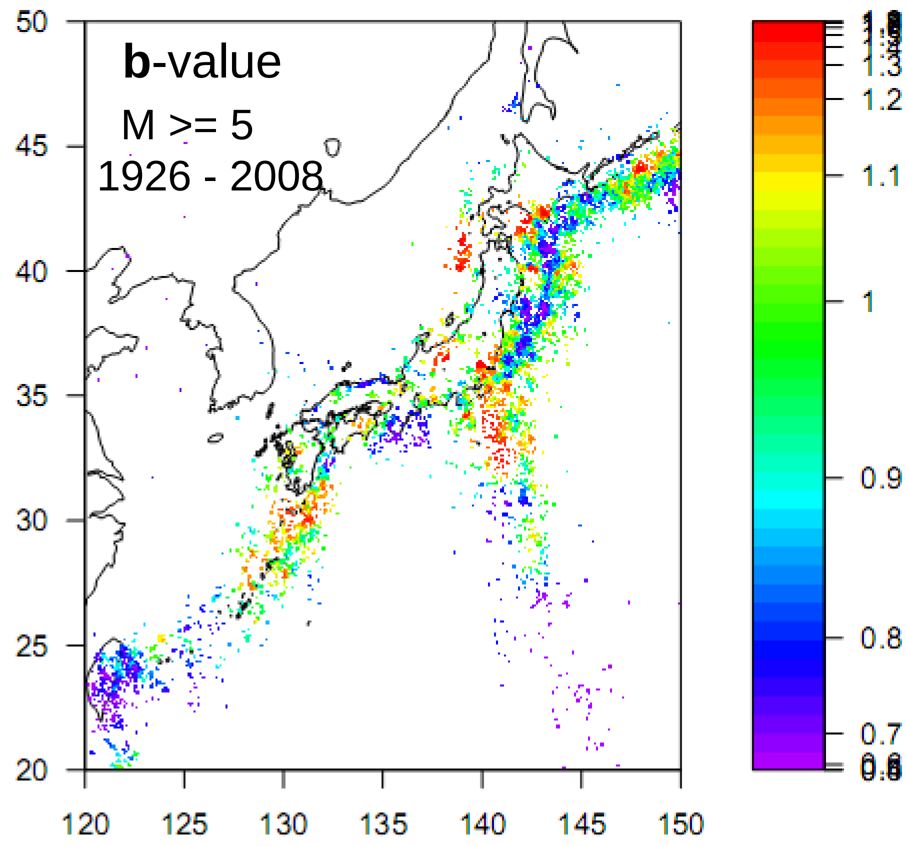


μ-value
M >= 4 1997.10 - 2008
0.1
10−2
10−3
K0-value
M >= 4 1997.10 - 2008
10−6
α-value
M >= 4 1997.10 - 2008
2.5
2
1.5
1
0.5
p-value
M >= 4 1997.10 - 2008
1.6
1.4
1.3
1.2
1.1
1
q-value
M >= 4 1997.10 - 2008
2
1.85
1.8
1.75
1.7
1.65
1.6


μ-value
M >= 5 1926 - 2008
10^{-2}
10^{-3}
10^{-4}
K_0-value
M >= 5 1926 - 2008
10^{-5}
α-value
M >= 5 1926 - 2008
2
1.5
1
0.5
p-value
M >= 5 1926 - 2008
1.4
1.3
1.2
1.1
1
0.9
q-value
M >= 5 1926 - 2008
2.14
2.12
2.1
2.08
2.06
2.04
2.02
2
1.98
1.96
1.94
50
45
40
35
30
25
20
120
125
130
135
140
145
150

Magnitude frequency





これで終わりです。お疲れさまでした。

Thank you very much for your attention